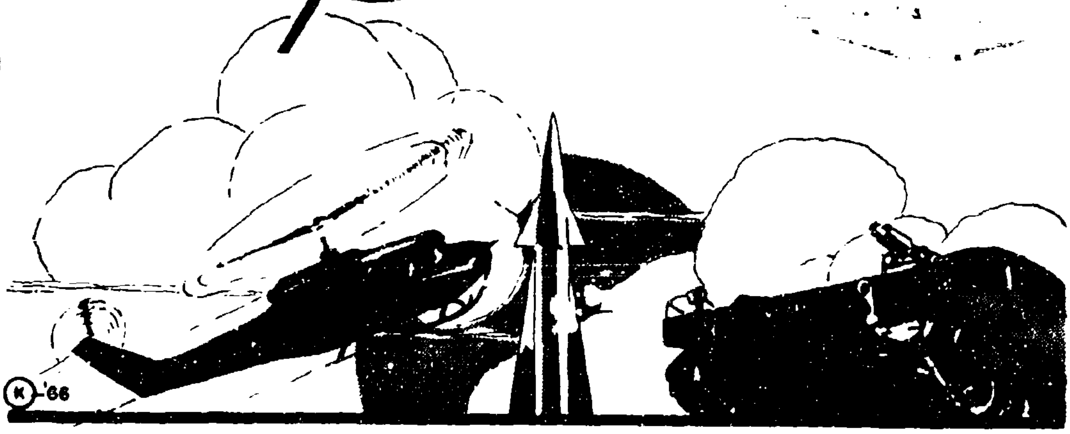
QUALITY ASSURANCE



RELIABILITY HANDBOOK

HEADQUARTERS, U. S. ARMY MATERIEL COMMAND

OCTOBER 1968

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UNITED STATES ARMY MATERIEL COMMAND
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QUALITY ASSURANCE

RELIABILITY HANDBOOK

This pamphlet is published for the information and guidance of all concerned.

(AMCQA-E)

FOR THE COMMANDER:

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DISTRIBUTION:
Special

FOREWORD

This handbook was prepared by the U. S. Army Management Engineering Training Agency under the technical direction of the Quality Assurance Directorate, Headquarters, AMC. It is intended to serve as a guide for project and commodity managers and professional personnel in the planning, direction, and monitoring of reliability programs. While not regulatory in nature, the material in the handbook is applicable to both in-house and contracted-for effort.

The format of the handbook is such that there are seven basic chapters with appendixes topically aligned to each. The material in the chapters is in narrative form and provides a simple, straightforward approach to the life cycle aspects of reliability without resorting to language of a mathematical or highly technical nature. Included in each chapter are topics which should be considered for that phase of the reliability program in the product life cycle. The discussion which follows each of these topics contains a brief explanation to provide guidance for the development, monitoring, or evaluation of reliability as it pertains to that element of total system performance.

The appendixes contain technical discussions and mathematical treatments of techniques as they apply to the narrative in the chapters. Examples, applications, and solutions are included. It is felt that this twofold approach to the subject lends itself to use by the manager and/or generalist, as well as the practitioner.

CONTENTS

CHAPTER 1

INTRODUCTION

Section I. RELIABILITY AS A PROGRAM ELEMENT

1-1. The importance of reliability to system effectiveness. a. Reliability is defined as the probability that an item[1] will perform its intended function for a specified interval under stated conditions. As it relates to Army systems, equipments, components, and parts, reliability is one of the important characteristics by which the usefulness of an item is judged.

b. Usefulness is meast . i in terms of an item's effectiveness for its intended role; therefore, reliability is one of the important parameters contributing to effectiveness. As roles and mission requirements become more sophisticated, items become more complex in the functional configuration necessary to satisfy increased performance requirements. As item complexity increases, reliability invariably becomes more problematical and elusive as a design parameter, and thus more difficult to assure as an operational characteristic under the projected conditions of use. These difficulties can never be completely eliminated, but may be reduced by means of the establishment and implementation of sound reliability program activities.

c. It is also now recognized that with the exercise of very deliberate and positive reliability engineering methods throughout the life cycle of the item--from the early planning sta ,es through design, development, production, and field use--the feasi..e reliability level can usually be attained. Like other system characteristics, reliability is a quantitative characteristic: predictable in design, measurable in test, controllable in production, and sustainable in the field. It follows that reliability may be achieved by introducing sound monitoring practices with corrective action criteria at key points throughout the life cycle.

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[1] In this document, the words item, equipment, and system are used interchangeably.

1-2. Purpose and scope of the handbook. a. This handbook provides procedures for the definition, pursuit, and acquisition of required reliability in Army systems, equipments, and components. The methods presented are generally applicable to all categories of items, including electronic, electromechanical, mechanical, hydraulic, and chemical. However, examples chosen to illustrate the application of specific procedures are drawn largely from experience with electronic and electromechanical systems because of the availability of documented experience with these systems.

b. The document is not intended to provide detailed instructions relative to any specific program or equipment, but is intended to fill three basic needs within the Army and its contractor facilities.

(1) Project management. General guidance for the implementation of selected reliability program functions at appropriate points in the item life cycle.

(2) Project engineering. Discussion of some procedures useful to the engineer in the actual performance of these reliability program functions.

(3) Design engineering. Identification of some important principles affecting reliability and some analytic techniques for predicting and measuring the reliability of a given design configuration.

1-3. Reliability as a growth process. a. As an item proceeds through the stages of the life cycle, reliability should be periodically predicted or estimated. These values, when plotted at selected points in the life cycle, result in a growth curve which reflects comparative reliability levels. This growth curve provides a source of information useful to decision makers relative to actions affecting reliability. Figure 1-1 indicates the relationship between certain key monitoring activities and a typical reliability growth curve. The slope of an actual reliability growth curve is dependent upon interactions among effectiveness characteristics. Consequently, a curve generated during a specific program may exhibit a pattern of growth different from that shown in figure 1-1.

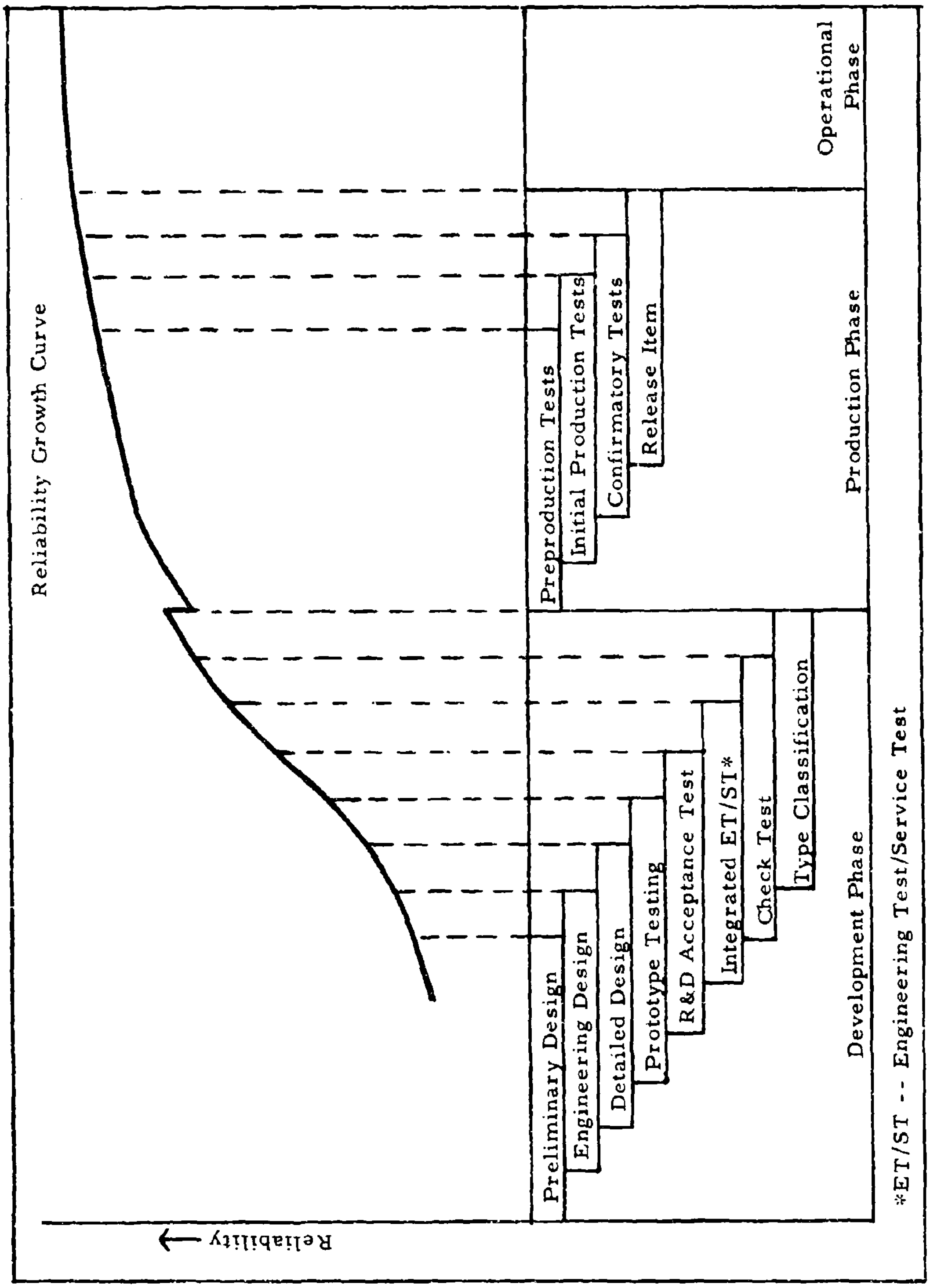


Figure 1-1
Reliability Growth Process

b. Desirable reliability growth results from planning, designing, testing, producing, and ultimately using the product according to a set of effectiveness-oriented procedures. Lack of reliability growth may result from overlooking or disregarding these same procedures at any single point in the growth process.

1-4. Organization and use of the handbook. Figure 1-2 identifies applicable chapters within the handbook corresponding to major reliability functions to be performed throughout the life cycle of a system. The figure may also serve as a basic checklist of things to be done in planning a new program. Not all of these functions are applicable for all materiel items, e.g., those items for which a Research and Technology Résumé (DD Form 1498) is used instead of a Technical Development Plan (TDP).[2]

RELIABILITY FUNCTIONS	CHAPTER/APPENDIX						
	1/A	2/B	3/C	4/D	5/E	6/F	7/G
Determination of feasibility				X			
Documentation of requirements		X	X	X			
Preparation of RFP		X	X	X			
Evaluation of proposal		X	X				
Prediction of reliability level				X			
Apportionment of reliability goals				X			
Formulation of design					X		
Conduct of design review					X		
Conduct of test and evaluation activities				X		X	X
Conduct of failure analysis							X
Utilization of a data feedback system							X
Conduct of appropriate training		X					
Planning a reliability program	X	X					
Monitoring a reliability program		X					
Managing a reliability program	X	X					

Figure 1-2
Reference Index for the Performance of
Some Specific Reliability Functions

[2]Where Technical Development Plan is used in this Pamphlet, System Development Plan (SDP) is also included.

Section II. RELIABILITY DOCUMENTS APPLICABLE TO THE MATERIEL LIFE CYCLE

1-5. The materiel life cycle. a. For purposes of discussion, the materiel life cycle is broken into the following six phases:

(1) Conceptual phase. The life cycle is initiated by a statement of general need for a particular capability. The general objective of this phase is to establish a feasible technical approach for satisfying the general requirements, to evaluate whether a specific approach is worth pursuing, or whether the military requirement should be satisfied in another manner. If the approach is found to be worth pursuing, the conceptual phase should:

(a) Provide explicit definition of effectiveness for the particular item under consideration; and

(b) Provide guidelines for item refinement in the definition phase.

(2) Definition phase. (a) During the definition phase, the detailed cost, schedule and technical design requirements of a program are defined and validated prior to development and production. Technological advances resulting from the conceptual phase are translated into design requirements to be met during development and production.

(b) The definition phase serves to refine the system definition to subsystem level based on the guidelines established during the conceptual phase. Thus, it enhances the probability of successful accomplishment of these requirements and allows development to proceed with minimum change. This phase provides the inputs to a request for proposal and the resulting contractor competition for development.

(3) Development phase. The development phase is the period during which design engineering and testing is performed to come up with an end item which satisfies the military requirement. The main product of the development phase is documentation of information for use in production of the end item for field use. Items produced during this phase generally serve to test the effectiveness of the research and the validity of the data. The design and configuration is determined during this phase, and the inherent reliability is established. Inherent reliability refers to the achievable reliability of the equipment under ideal environmental conditions.

(4) Production phase. The production phase utilizes the technical data package formulated during the development phase to produce, manufacture, and make engineering changes to the item under consideration. This phase includes production testing and arranging for facilities and logistic support.

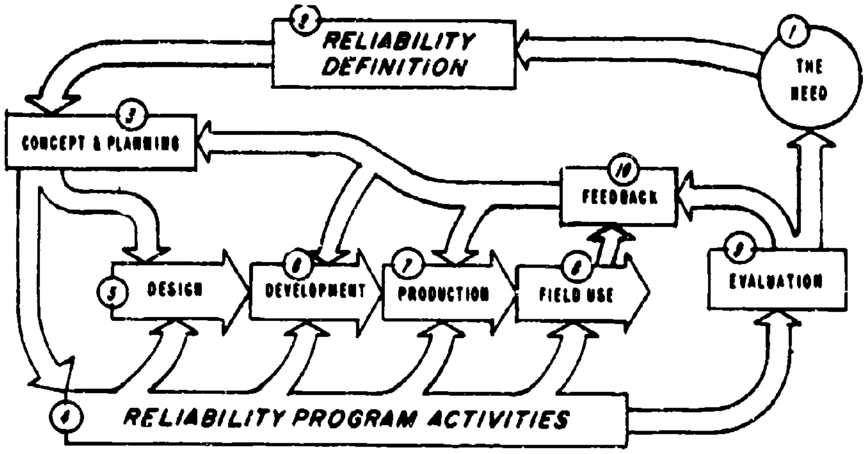
(5) Operational phase. This phase is characterized by rebuild, supply, training, maintenance, and materiel readiness operations while the system is being utilized by an operational unit. It is here that the results of all prior effort is put to the test in the field. However, this phase is not independent of preceding phases; e.g., inherent reliability established in design can be realized only if support activities are performed as specified. Feedback data from this phase can be utilized for improving reliability, either by engineering changes in the present system or in the development of new systems.

(6) Disposal phase. This phase is included in this document to complete the life cycle. It has to do with the removal of obsolete items from the inventory and consequently has little influence on reliability.

b. The major reliability system life cycle considerations are shown in figure 1-3.

c. A great many documents support the overall Army reliability program. These are intended to give assurance that each item ultimately satisfies the need initially anticipated. Figure 1-4 shows many of the documents related to the appropriate considerations in figure 1-3.

d. Some of these documents identify certain engineering or management procedures, test plans, and data requirements which are needed to fulfill contractual requirements. Similar requirements are implicitly defined in others. In general, they impose a responsibility upon the project office, contractor, or contracting agency to do certain things to assure ultimate realization of required reliability in the field. References which supplement the contents of the documents are identified in the documents themselves. Figure 1-5 is an abbreviated document directory. Opposite each document identification number are indicated those sections of this handbook that relate to these requirements.



(1) NEED for reliability must be anticipated.

(2) Qualitative Materiel Requirements (QMR) or Small Development Requirements (SDR) must reflect this need.

(3) Plans must be formulated to fulfill the reliability need, such as: (a) Reliability requirements defined and specified; (b) Reliability program plan formalized; (c) Requests for proposal (RFP) and contracts documented.

(4) Reliability program is implemented: Reliability is monitored continuously.

(5) Conceptual item is designed: Reliability is assessed in design review; design is revised to correct deficiencies; reliability is designed in by requirement.

(6) Prototype is developed according to the design: Reliability is evaluated by test; design is refined to correct deficiencies; reliability is validated by demonstration when practical.

(7) Item is produced: Parts, materials, and processes are controlled; equipment acceptability is determined by test.

(8) Item is deployed to the field: Operators and maintenance technicians are trained; operating and maintenance instructions are distributed; reliability is sustained by procedure.

(9) Item is evaluated to determine that the original need is met.

(10) Feedback loop completes the cycle: (a) to guide product improvements; (b) to guide future development planning; (c) to correct field deficiencies.

Figure 1-3
Reliability Considerations in the Materiel Life Cycle

CIRCLED NUMBERS CORRESPOND TO THOSE IN FIGURE 1-3

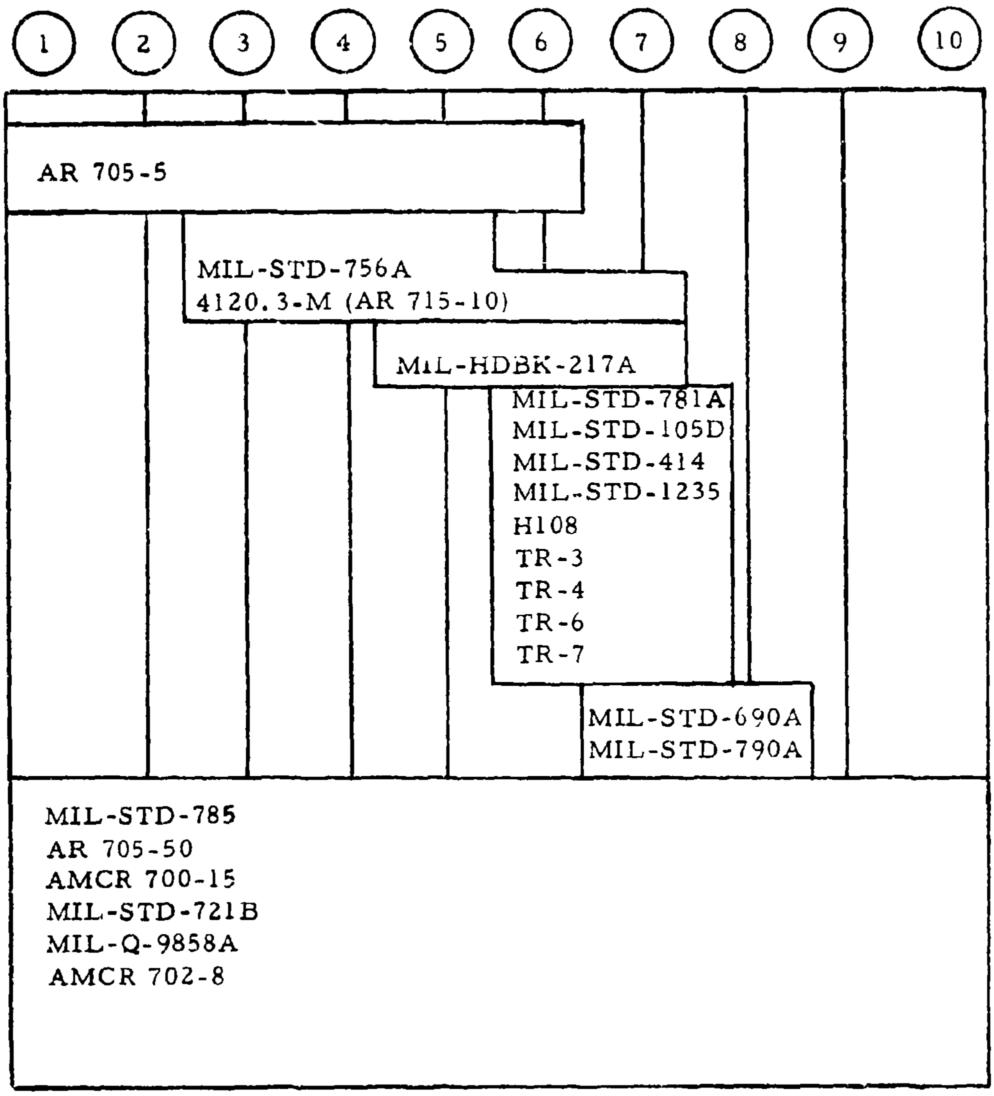


Figure 1-4
Documents Applicable to Materiel Life Cycle
Reliability Considerations

Note. See section V for document titles.

For Information Relative to

See

DOCUMENT	CHAPTER						
	1	2	3	4	5	6	7
MIL-Q-9858A		X					
MIL-STD-105D						X	
MIL-STD-414						X	
MIL-STD-690A						X	
MIL-STD-790A		X				X	
MIL-STD-721B	X						
MIL-STD-756A				X	X		
MIL-STD-781A						X	
MIL-STD-785	X	X	X	X	X	X	X
MIL-STD-1235						X	
AR 705-5			X	X			
AR 705-50	X	X	X	X			
AMCR 700-15	X	X	X	X	X	X	X
AMCR 702-8							X
4120.3-M (AR 715-10)			X				
MIL-HDBK-217A					X		X
H108						X	
TR-3						X	
TR-4						X	
TR-6						X	
TR-7						X	

Figure 1-5
Ready-Reference Index for Compliance with Specified Documents

Section III. RELATIONSHIP OF RELIABILITY TO SYSTEM EFFECTIVENESS

1-6. System effectiveness. a. The worth of a particular item is determined primarily by the effectiveness with which it does its job. Many characteristics, including reliability, contribute to system (item) effectiveness. For purposes of discussion, effectiveness-related characteristics may be grouped into three general categories:

(1) Those affecting response to a mission call.

(2) Those affecting endurance of item operation.

(3) Those comprising terminal results of the mission.

b. The contributions of these categories may be referred to as availability, dependability, and capability, respectively, (see figure 1-6). Then system effectiveness may be expressed as a function of availability, dependability, and capability.

(1) Availability is a measure of the degree to which an item is in the operable and committable state at the start of the mission when the mission is called for at an unknown (random) point in time.

(2) Dependability is a measure of the item operating condition at one or more points during the mission, including the effects of reliability, maintainability, and survivability, given the item condition(s) at the start of the mission. It may be stated as the probability that an item will enter or occupy one of its required operational modes during a specified mission and perform the functions associated with those operational modes.

(3) Capability is a measure of the ability of an item to achieve mission objectives, given the conditions during the mission.

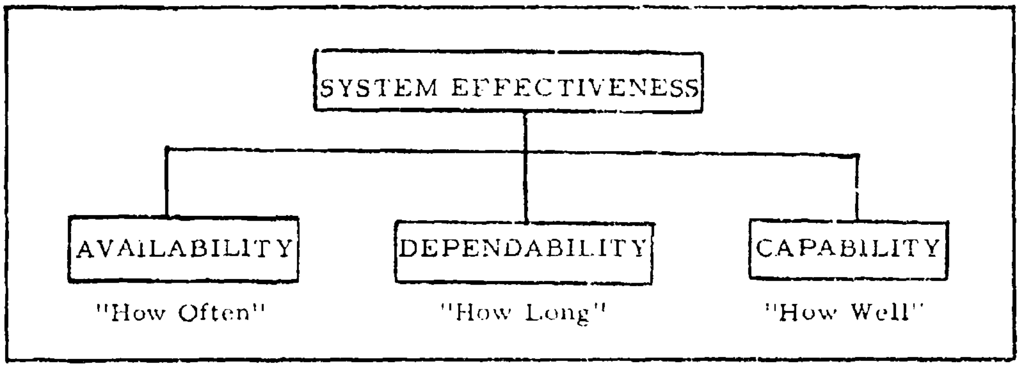


Figure 1-6
Definition of System Effectiveness[3]

c. Other factors, such as time, cost, and logistic supportability, enter into an evaluation of an item during system planning. Within the constraints imposed by such factors, effectiveness should be optimized by judicious balance among the characteristics comprising availability, dependability, and capability, taking care not to stress the importance of one at the exclusion of the others.

d. Reliability is an important part of the effectiveness model, especially in availability and dependability. With reference to availability, reliability pertains to the environment to which an item is subjected while awaiting initiation of its primary mission; e.g., storage, temporary use, war games, etc. As a contributor to dependability, reliability concepts pertain to the environment to which the item is subjected during its primary mission.

e. The descriptors, availability, dependability, and capability, have been chosen for discussion of effectiveness. Other system effectiveness approaches have been formulated using different descriptor categories.

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[3] The approach used here is that of WSEIAC Committee Reports.[4]
[4] See footnote 7, page 1-17.

Section IV. QUANTITATIVE DESCRIPTION OF A RELIABILITY LEVEL

1-7. Definitions of reliability. a. The reliability of an item is defined as the probability that the item will perform its intended function for a specified interval under stated conditions. When applied to a specific equipment or system, reliability is frequently defined as:

(1) The probability of satisfactory performance for specified time and use conditions; or

(2) The probability of a successful mission of specified duration under specified use conditions; or

(3) The probability of a successful event under specified conditions. This definition is particularly applicable to nontime dependent items.

b. Whenever the definition is worded to fit a particular item or device, it is always necessary to:

(1) Relate probability to a precise definition of success or satisfactory performance;

(2) Specify the time base or operating cycles over which such performance is to be sustained (except for nontime dependent items such as one shot devices); and

(3) Specify the environmental or use conditions which will prevail.

1-8. Reliability descriptors. A reliability level, and ultimately a reliability requirement, may be stated by using various descriptors. Any of the following may be used to specify a reliability requirement for a given mission time.

a. Both mission time and the reliability associated with that mission time; (i.e., the probability that the equipment will not fail during the required mission time). Such a requirement statement reflects the reliability definition.

b. Mean time between failures (MTBF).[5] This descriptor reflects a specific reliability level only if the relationship[6] between MTBF and reliability level is known. If different relationships apply to two items, it is highly likely that the same MTBF for both items will reflect different reliability levels. Thus, MTBF should be used with caution to express reliability requirements.

c. Failure rate. Failure rate may be used to express reliability requirements with the same type of precautions described for MTBF.

d. Probability of properly performing a specific function. This descriptor is useful for expressing reliability requirements for nontime dependent items.

Section V. DOCUMENTS APPLICABLE TO THE ARMY RELIABILITY PROGRAM

1-9. Synopses of reliability documents. A brief synopsis for each of the documents shown in figure 1-4 follows.

a. AR 705-50, Army Materiel Reliability and Maintainability. Sets forth concepts, objectives, responsibilities, and general policies for the Army reliability and maintainability program. This regulation identifies reliability and maintainability characteristics which must be specified for the design of materiel and must be considered and assessed throughout the life cycle.

b. MIL-STD-785, Requirements for Reliability Program (for Systems and Equipments). Provides general requirements for reliability programs, as well as guidelines for the preparation of reliability program plans. Particular attention is directed toward the topics of numerical reliability objectives and minimum acceptable requirements. Approval of or deviation from the proposed reliability plan, preplanned

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[5]For nonrepairable items, mean time to failure (MTF) may be used in lieu of MTBF. These terms are frequently used interchangeably.

[6]This relationship depends upon the probability distribution function of failure times. Some important probability distribution functions are summarized in appendix A.

program review check points, itemization of government-furnished or contractor-supplied equipment, which is to be integrated to provide a complete operational system, are also emphasized. In addition, human engineering design criteria reference documents, a list of items to be included in failure report form, milestones at which demonstration is to be performed, and the reliability test plan are included.

c. AMCR 700-15, Reliability Program for AMC Materiel. Establishes policies, procedures, and responsibilities concerning a reliability program for Army materiel. Included is a listing of essential factors to be considered in a reliability program, as well as essential phases during which reliability actions must be taken.

d. MIL-Q-9858A, Quality Program Requirements. Specifies requirements for an effective and economical quality program, planned and developed in consonance with the contractor's other administrative and technical programs. Design of the program shall be based upon consideration of the technical and manufacturing aspects of production and related engineering design and materials.

e. MIL-STD-105D, Sampling Procedures and Tables for Inspection by Attributes. Provides tabled acceptance sampling plans and general procedures for deciding whether a lot of components, subsystems, systems, etc., have an acceptable percentage defective when compared to specification limits or goals. Specification of a mission profile allows for usage for reliability acceptance plans.

f. MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective. Provides general procedures and sampling plans for determining acceptance of lots when quality is based on a characteristic which is measured on a continuous scale, and the measurements and the underlying distribution are normal. These plans may be applied to reliability tests if a mission time is specified.

g. MIL-STD-721B, Definition of Effectiveness Terms for Reliability, Maintainability, Human Factors, and Safety. Defines terms commonly used in reliability, maintainability, human factors, and safety.

h. MIL-STD-756A, Reliability Prediction. Establishes uniform procedures for predicting the quantitative reliability of aircraft, missiles, electronic equipment, and their subdivisions early in the development phases, to reveal design reliability weaknesses and to form a basis for apportionment of reliability requirements to the various subdivisions of the item. Graphically portrays the effects of system complexity on reliability to permit the early prediction of tolerance and interaction problems not accounted for in the simple multiplicative case, and provides appropriate factors by which to adjust MIL-HDBK-217A predictions for airborne and missile environments.

i. MIL-STD-781A, Reliability Tests, Exponential Distribution. Outlines a series of test levels and test plans for certain reliability acceptance tests and longevity tests. The test plans are based upon the exponential (or Poisson) distribution.

j. MIL-STD-690A, Failure Rate Sampling Plans and Procedures. Provides procedures for failure rate qualification sampling plans for establishing and maintaining failure rate levels at selected confidence levels and lot conformance inspection procedures associated with failure rate testing.

k. MIL-STD-790A, Reliability Assurance Program for Electronic Parts Specifications. Provides the controls and procedures a manufacturer must establish and continue to maintain in order to qualify parts to an established reliability level.

l. MIL-STD-1235, Sampling Procedures and Tables for Continuous Inspection by Attributes. Provides tabled acceptance sampling plans and general procedures for use where disposition of product is made on a unit-by-unit basis and production/rebuild is on a moving line.

m. AR 705-5, Army Research and Development. Specifies responsibilities and establishes policy and procedures for conducting research and development in the Department of Army. These procedures are classified into the three major categories of research, development, and special instructions pertaining to nuclear energy. Appendixes are included regarding the format for submitting QMR's and SDR's.

n. AMCR 702-8, Reliability Record and Status Report. Prescribes policies, procedures, and responsibilities for the preparation of quarterly reports on item reliability throughout the entire life cycle.

o. 4120.3-M (AR 715-10), Defense Standardization Manual. Establishes format and general instructions for the preparation of specifications, standards, handbooks, and maintenance manuals.

p. MIL-HDBK-217A, Reliability Stress and Failure Rate Data for Electronic Equipment. Provides the procedures and failure rate data for the prediction of part-dependent equipment reliability from stress analysis of the parts used in the design of the equipment. Must be used according to procedures outlined in MIL-STD-756A for estimates of MTBF and reliability at the system level, and to account for tolerance and interaction failures, and to adjust for the particular use environment.

q. H108, Sampling Procedures and Tables for Life and Reliability Testing (Based on Exponential Distribution). This document describes the general principles and outlines specific procedures and applications of life test sampling plans for determining conformance to established reliability requirements, assuming failure times to be exponentially distributed.

r. TR-3, Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Mean Life Criterion).[8] Provides procedures and tables of life test sampling plans for determining conformance to established reliability requirements (in terms of mean life) where the Weibull distribution describes failure times.

s. TR-4, Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Hazard Rate Criterion).[8] Provides procedures and tables of life test sampling plans for determining conformance to established reliability requirements (in terms of hazard rate) where the Weibull distribution describes failure times.

t. TR-6, Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Reliable Life Criterion).[8] Provides procedures and provides tables of life test sampling plans for determining conformance to established reliability requirements (in terms of reliable life) where the Weibull distribution describes failure times.

u. TR-7, Factors and Procedures for Applying MIL-STD-105D Sampling Plans to Life and Reliability Testing.[8] Provides a procedure and contains related tables of factors for adapting MIL-STD-105D sampling plans to reliability acceptance tests. The underlying distribution of failure times is assumed to be Weibull.

Section VI. SUMMARY

1-10. Elements of reliability achievement. a. The pursuit and acquisition of reliability objectives requires that management:

(1) Acknowledge and strive to attain established item effectiveness.

(2) Know and define the level of reliability desired.

(3) Recognize the disparity between the desired reliability level and that level which will probably be achieved unless proper controls are exercised to influence the reliability growth process.

(4) Understand the application of available approaches by which controlled reliability growth may be assured.

b. The remaining chapters of this document outline some of the planning considerations and describe some of the procedures that can be fruitful, both in the achievement of required reliability in specific programs and in the evaluation and monitoring of reliability on a program-wide basis throughout the system life cycle.

[7] Final Report of the Weapon System Effectiveness Industry Advisory Committee (WSEIAC). The documents listed below are available from the Defense Documentation Center, Cameron Station, Alexandria, Virginia 22314.
AFSC-TR-65-1, Requirements Methodology (AD-458453).
AFSC-TR-65-2, Prediction Measurement (3 volumes) (AD-458454, AD-458455, AD-458456).
AFSC-TR-65-3, Data Collection and Management Reports (AD-458585).
AFSC-TR-65-4, Cost Effectiveness and Optimization (3 volumes) (AD-458595, AD-462398, AD-458586).
AFSC-TR-65-5, Management Systems (2 volumes) (AD-461171, AD-461172).
AFSC-TR-65-6, Chairman's Final Report (AD-467816).

[8] See footnote 1, page F-57.

CHAPTER 2

RELIABILITY PROGRAM REQUIREMENTS, PLANNING, AND MANAGEMENT GUIDE

Section I. INTRODUCTION

2-1. General. a. Project and commodity managers are charged with the responsibility for delivering reliable systems to the field. This responsibility can be fulfilled only by giving due consideration to all characteristics, including reliability, in the early planning and feasibility study stages and continuing with a comprehensive program throughout the entire materiel life cycle. However, some programs do not provide adequate reliability control or monitoring prior to the operational phase. By then, it is usually too late to make modifications for improvement, since:

(1) The equipment is needed now for operational use (development time has been exhausted); and

(2) The money invested is too great to be written off because of poor reliability. Often it is considered more expedient to add funds in a desperate attempt to make product improvement.

b. This chapter sets forth reliability program activities deemed vital to development and production programs in general. Emphasis is placed upon reliability program planning, monitoring, and management review procedures. Appendix B contains a network diagram comprised of a suggested list of milestones for monitoring a reliability program. Among the primary purposes of a reliability program are:

(1) Focusing engineering and management attention on the reliability requirements;

(2) Insuring that reliability is treated as a design parameter of equal importance with other effectiveness parameters; and

(3) Alerting management, throughout the program, to reliability discrepancies which may require management decisions.

c. An adequate program must contribute to, and guide, an orderly and scientific approach to designing for reliability. It must help contractors and individuals overcome a lack of recognition that reliability must be a designed-for parameter with practical limitations. It must foster the realization that good conventional design may not result in the inherent reliability required to satisfy the Army.

d. A reliability program will not necessarily increase the effectiveness of an equipment, but an effectively monitored program will not permit an inadequate design to proceed into development, test, production, and field use without specific management approval. It is this effective monitoring that assists project and commodity managers to assess and pinpoint potential reliability problems in time to make adjustments.

e. The concept of a total reliability program, as generally endorsed by the DoD, has four major points:

(1) Quantitative requirements are stated in the contract or design specifications.

(2) A reliability program is established by the contractor.

(3) Reliability progress is monitored or audited by the responsible Army agency.

(4) Realistic requirements are stated in the Qualitative Materiel Requirements (QMR) and that they are included as one of the necessary requirements to be fulfilled for successful passing of acceptance tests. This applies to prototype or demonstration models prior to production approval and to production samples.

Section II. RECOMMENDED CONTRACTOR PROGRAM

2-2. <u>General</u>. Of specific interest is the reliability program required of the contractor. Those activities which experience has shown contribute to an orderly and scientific approach to designing for reliability are discussed below.

2-3. <u>Reliability organization</u>. The reliability function should be an integral part of the overall contractor organizational structure. Considerations for this function should include:

a. Proper placement within the overall organizational structure so as to have proper authority and effectivity.

b. Clear identification of the personnel responsible for managing the reliability program.

c. Clear definition of responsibilities and functions of those directly associated with reliability policies and implementation.

d. Integration of such functions as engineering, manufacturing, quality, and reliability.

2-4. <u>Reliability management, control, and monitoring activities</u>.

a. Management and control. The management of the reliability group should establish policies and maintain control of reliability functions. To assure these functions, the reliability program plan should include:

(1) Description of all tasks to be performed with a detailed list of specific tasks, including implementation and control procedures.

(2) Clearly defined authority and responsibility for carrying out each task.

(3) Schedule of activities indicating major milestones (network diagram) and estimates of manpower, equipment, facilities, time, and cost.

(4) A method for identification, effect analysis, and corrective actions for potential problems.

b. Contractor-established reliability monitoring activity. This activity provides analysis of reliability status relative to requirements, weaknesses, and follow-up on corrective action. Documentation of reliability assurance and monitoring procedures, such as checklists and instructional material normally used by the contractor, should be maintained so as to clearly delineate approach used and results obtained, and should be available for review by the procuring agency.

2-5. Program review. Contractor and procuring agency provisions for review of the reliability program status should include:

a. Establishment of major review points by procuring agency at time of program planning.

b. Criteria and information to be used for assessment of reliability progress.

c. Identification of the responsible group for carrying out the reviews.

2-6. Development testing. A main purpose of development testing is to determine how well design reliability requirements have been met and with what degree of confidence. Among the considerations necessary to accomplish this is a planned program, including:

a. Environmental tests based on extreme stress conditions.

b. Test-related procedures, including provisions for non-specified environmental criteria, nonavailable testing data, record keeping and a listing of items having critically limited useful life.

2-7. Integrating equipment. The reliability program plan should include provisions for use of equipment supplied by the government or other contractors. For such equipment, consideration must be given to:

a. Use of known or estimated reliability values.

b. Procedures for getting such data, if not available.

c. Procedures for handling of potential reliability problems introduced by such equipment.

2-8. Parts reliability improvement. The reliability plan should include procedures for identifying those parts, if any, needing improvement and for accomplishing the necessary improvement. Deficiencies in MIL-Specifications or inadequate parts resulting from such specifications should be reported.

2-9. Critical items. Procedures should be established for identifying and providing for critical items. Critical items are those:

a. The failure of which would prevent satisfactory operation of the system (of which it is a part) or create unwarranted safety hazards;

b. Which are of sufficient complexity to warrant special production techniques or controls;

c. Which require special treatment or handling during transport or storage;

d. Which impose a heavy maintenance and supply support burden; or

e. Which have a long production lead time.

2-10. Apportionment, prediction, and mathematical models. a. Methods should be established for developing mathematical models based on functional analysis for apportionment and prediction of reliability.

b. These models often provide the basis for periodic analyses of reliability achievement. These analyses should be scheduled to coincide with technical progress reporting requirements established by the contractor and should consider:

(1) Reliability estimates based on predictions and test data.

(2) The relationship between present reliability status and schedule progress.

(3) The changes in concepts and approaches that are necessary to accomplish the contract objective.

(4) The effects of changes made in design and manufacturing methods since the previous analysis.

(5) Criteria for success and failure, including partial successes (degraded operation) and alternative modes of operation.

(6) Production tolerances and techniques, including assembly test and inspection criteria and test equipment accuracies.

(7) Specific problem areas and recommended alternative approaches.

2-11. Contractor design reviews. Engineering design review and evaluation procedures should include reliability as a tangible operational characteristic of the equipment, assembly or part under review. Among reliability considerations during design reviews are:

a. Review of current reliability estimates and achievements for each mode of operation.

b. Review of potential design or production problem areas.

c. Analysis of mode(s) and effect(s) of failure.

d. Identification of the principal items inhibiting reliability achievement and proposed solutions.

e. The effects of engineering decisions and trade-offs upon reliability achievement.

f. Procedures to assure that appropriate personnel from the reliability organizations participate in the design reviews.

g. Documentations of design review results.

2-12. Subcontractor and supplier reliability programs. Provisions should be established to insure that subcontractors and supplier selection and performance are consistent with the reliability requirements of the contract. The prime contractor must extend the scope of his reliability program to the monitoring and control of subcontractors and suppliers. Considerations here are.

a. Incorporation of reliability requirements in subcontractor and supplier procurement documents.

b. Provision for assessment of reliability progress, including qualification and acceptance testing of incoming products.

c. Adequate liaison to insure compatibility among supplier products to be integrated into the end item.

d. Initial selection procedures for subcontractors and suppliers, which consider--in relation to the requirements--past performance, willingness to test and share test data, interest and response on feedback of deficiency information, test philosophy, and realism of cost and delivery schedules.

2-13 Reliability indoctrination and training. Provisions should be made to include reliability in the basic training and indoctrination of personnel with consideration given to:

a. Purpose, i.e., improvement of skills.

b. Skill level of personnel to be trained, e.g., manager, engineer, technician or worker.

c. Methods of instruction.

2-14. Statistical methods. Statistical analysis is a part of reliability assessment activities. The reliability plan should fully describe appropriate statistical techniques and where in the life cycle they are to be used.

2-15. Trade-off considerations. The prime purpose of any hardware development program is to get an effective item to the field. Fulfillment of this objective requires that the reliability plan provide for potential trade-offs between reliability and other disciplines, such as:

a. Maintainability.

b. Safety and human engineering.

c. Design configuration.

d. Production.

e. Cost and schedule.

2-16. Effects of storage, shelf life, packaging, transportation, handling, and maintenance. Provisions to prevent degrading reliability by improper storage, packaging, shipping, handling, and maintenance of parts, units, subsystems, and systems should be established. The plan should include procedures for:

a. Periodic inspection and tests to determine effects of storage, shelf life, packaging, transportation, handling, and maintenance on the reliability of the product.

b. Identification of major or critical characteristics of items which deteriorate with age, environmental conditions, etc.

c. Maintenance or restoration of equipment.

2-17. Manufacturing controls and monitoring. Manufacturing controls and monitoring are required to assure that the reliability achieved in design is sustained during production. Detailed consideration should be given to:

a. Integration of reliability requirements into production process and production control specifications.

b. Production environments induced by handling, transporting, storage, processing, and human factors.

c. Quality standards from incoming piece-part inspections.

d. Calibration and tolerance controls for production, instrumentation, and toolin

e. Integration of reliability requirements and acceptance tests into procurement activities.

f. Identification and correction of production control discrepancies.

g. Production change orders for compliance with reliability requirements.

h. Life tests of production samples to verify quality standards.

2-18. Failure reporting, analysis, and corrective action. A formalized system for the reporting, analysis, correction, and data feedback for all failures should be a part of the contractor reliability program. A mechanism for failure data feedback to engineering, management, and production activities in accordance with contractual requirements is an integral part of such a program. Complete reporting provides data on such things as accumulated operating time, on-off cycling, adjustments, replacements, and repairs related to each system, subsystem, component, and critical part. The analysis of all failure reports by an analysis team formally designated by management determines the basic or underlying causes of failures in parts, assemblies, and end items. These results provide for assignment of corrective action and follow-up responsibilities.

2-19. Reliability demonstration. a. A plan should be included for demonstrating achieved reliability at specified milestones. A demonstration plan normally includes:

(1) Number of test articles.

(2) Accept/reject criteria (or other quantitative decision criteria).

(3) Confidence levels.

(4) Subsystem vs. system level testing.

(5) Plans for handling of invalid data.

(6) Duration of test.

(7) Condition of test.

b. Provisions for periodic and final reports of demonstration results as specified by the procuring agency are a necessary part of such a plan.

c. Reliability demonstration tests are, in general, statistically designed experiments with consideration given to confidence levels and experimental error. Unless proof of adequacy can be substantiated by other available data acceptable to the procuring activity, all items of equipment of higher order designations should be tested in order to verify that reliability is achievable with the proposed design. If it is not, problem areas which prevent its attainment should be isolated and defined. The test program should include tests of questionable areas where reliability experience is not available, particularly new or unique concepts, materials, and environments.

d. The extent of the test program is determined by weighing the cost of testing against the degree of assurance required that the product will have a given level of reliability.

e. In addition to those tests performed specifically for reliability demonstration, all formally planned and documented tests which are performed throughout the contract period should be evaluated from a reliability viewpoint to maximize the data return per test dollar. Data which are obtained should facilitate prediction of reliability on the basis of individual and accumulated test results and the determination of performance variabilities and instabilities that are induced by time and stress.

Section III. PROGRAM MANAGEMENT AND MONITORING

2-20. Program implementation. Effective implementation requires that both the procuring agency and the contractor fulfill obligations and responsibilities in a cooperative framework toward the common objective of reliable equipment in the field. The following steps are presented as a guide in this implementation.

a. Step 1: Specify reliability requirements. The procuring agency should state the reliability requirements in design specifications or procurement documents (including Requests for Proposals). Format and details for including the requirements as part of the specification are provided in Defense Standardization Manual 4120.3-M (AR 715-10) and appendix C of this document.

b. Step 2: Establish schedules. The procuring agency should establish schedules for reliability reporting and monitoring, to include:

(1) Reliability report(s). Delivery dates for such reports may be specified on either a calendar or a program-phase basis.

(2) Test plans. The detailed test plan should be submitted well in advance of test initiation in order to allow sufficient time for Army review and approval.

(3) Progress evaluation schedule. Progress evaluations for effective monitoring are scheduled to correspond with major milestones rather than at fixed time intervals.

c. Step 3: Prepare Request for Proposal (RFP). The procuring agency should include desired proposal coverage of reliability in the Request for Proposal. A clause similar to the following, inserted in the RFP, aids in obtaining desired reliability: Proposals responsive to this RFP shall, in addition to the requirements listed in MIL-STD-785, contain the following:

(1) A narrative of the contractor's interpretation of the requirements to demonstrate that the requirements are understood.

(2) Proposed technical and management approach toward achievement within the stated or implied limitations (if the bidder deems the requirement unrealistic, that which he considers realistic and achievable should be stated).

(3) Supporting evidence for the above, including reliability predictions of the proposed concept and approach; source and applicability of data; experience of bidder with similar programs; specific ways and means of attainment; assumptions and noncontrollable dependencies upon which the approach is based.

(4) Description of the proposed reliability program, including specific technical activities; responsibilities and authorities within the proposed organizational structure (including list of key personnel, together with background and experience); proposed schedule of reliability activities; recommended monitoring points and major milestones (including cost milestones); and proposed reliability development test program."

d. Step 4: Prepare proposal. The prospective contractor should prepare a proposal in response to the RFP. Specifically, the proposing contractor should:

(1) Analyze the reliability requirements and make a preliminary prediction to determine feasibility for a given time and cost.

(2) Establish and cost the reliability activities and integrate them into the total program.

(3) Schedule in-house reliability activities and monitoring which become part of the master schedule.

(4) Plan development reliability tests. The contractor should evaluate the design approach and planned developments to determine which assemblies and components will require test demonstration.

(5) Prepare his total reliability plan.

e. Step 5: Evaluate proposals. (1) The procuring agency should evaluate proposals for their response to the specific task requirement in source selection evaluation procedures.

(2) The proposal review should give particular attention to specific proposed reliability activities rather than stress the contractor's organizational structure.

(3) Figures 2-1.a, 2-1.b and 2-1.c provide guidance for evaluating proposals with respect to reliability.

f. Step 6: Review contractual documents. The procuring agency should review contractual documents prior to contract negotiation. Changes in the reliability requirements, program, or acceptance tests that are recommended in the proposal submitted by the successful bidder must be reflected in the design specifications, references, or contractual documents. When the recommendations are not accepted, the prospective contractor should be notified early in the negotiation period in order that his cost and time estimates may be adjusted prior to final negotiation.

g. Step 7: Implement reliability program in development contract. Both contractor and procuring agency should implement and monitor the reliability program during design and development. The contractor is committed to perform in accordance with the specifications in the contractual documents. The milestones of appendix B provide a guide for monitoring a reliability program.

h. Step 8: Implement reliability program in production. Implementation and monitoring of the reliability program during production is a key step. A suggested list of review points is provided by the milestones in appendix B. Reliability records should include:

(1) Design changes in order to insure that each production engineering and design change is given the same reliability considerations and approvals as the original design.

(2) Procurement of parts and assemblies in accordance with appropriate reliability requirements.

(3) Evidence that each step in the production process has been evaluated for its possible detrimental effect upon reliability.

(4) Effectiveness of production inspections and collection, analysis, and feedback of test data in maintaining design quality.

(5) Summaries of qualification, environmental, and other test data.

(6) Compliance with the production acceptance tests requirements.

REQUIREMENTS ANALYSIS

Is the reliability requirement treated as a design parameter?

Has the requirement been analyzed in relation to the proposed design approach?

Is there a specific statement that the requirement is, or is not, feasible within the time and costs quoted? If not feasible, is an alternative recommended?

Is there evidence in the proposal that the reliability requirement influenced the cost and time estimates?

Are initial predictions and apportionments included in sufficient detail (data sources, complexity, block diagram, etc.) to permit Army evaluation of its realism?

Are potential problem areas and unknown areas discussed? Or, if none are anticipated, is this so stated?

If the requirement is beyond that which presently can be achieved through conventional design, does the proposal describe how and where improvements will be accomplished?

Is consideration given to conducting trade-offs between reliability and other technical parameters?

RELIABILITY PROGRAM AND MONITORING

Does the proposed program satisfy the requirements of the RFP?

If the contractor has indicated that certain of the reliability activities requested are not acceptable to him, has he suggested satisfactory alternatives?

Figure 2-1.a
Proposal Evaluation Guide

RELIABILITY PROGRAM AND MONITORING
(continued)

Is the program specifically oriented to the anticipated needs of the proposed equipment? Is it in sufficient detail?

Are program activities defined in terms of functions and accomplishments relating to the proposed equipment?

Does the proposal include planned assignment of responsibilities for reliability program accomplishments?

Is it clear by what means the program may influence development of the proposed equipment?

Have internal "independent" reliability assessments been scheduled?

Does the proposal provide justification (data derived from testing or other experience) for the exclusion of specified items from demonstration testing?

Is the proposed documentation of activities, events, and analyses designed for ease in monitoring, ease of data retrieval, and use on future programs?

Are planned activities and events scheduled and documented?

Does the proposal include a controlled corrective action program for reliability data?

Figure 2-1.b
Proposal Evaluation Guide

BACKGROUND ORGANIZATION AND EXPERIENCE

Does the bidder have an established program whereby past experience is made available to engineers and designers?

Does the bidder have a designated group (or individual) to whom designers can turn for technical reliability assistance?

Does the assignment of responsibilities include reliability activities?

Do (or will) company standards manuals or other documents set forth standard reliability operating procedures?

Does the bidder provide for appropriate reliability training for management, engineering, and technical personnel?

Does the bidder implement and conduct planned research programs in support of line activities, seeking new materiels, new techniques, or improved analytical methods?

ACCEPTANCE TESTING

Has the bidder agreed to perform acceptance tests and included the costs and time within his proposal?

If acceptance test plans were not included in the request for proposal, has the bidder recommended any?

Does the proposal contain a positive statement concerning the bidder's liability in the event of rejection by the acceptance tests?

Figure 2-1.c
Proposal Evaluation Guide

Section IV. RELIABILITY TRAINING

2-21. General. a. The concept of reliability in system development is not new. Only a few of the fundamental principles need be understood by project management and engineering in order to put quantitative measurements on this system parameter. It is true that the complexities of redundancy, statistical test design, sampling, and many other aspects of reliability assessment are difficult concepts, and an effective training program must include consideration of all levels of personnel involved with the reliability program. The technical content of a training course must be tailored to the personnel to be trained; e.g., a survey course for management and detailed technique courses for engineers and technical personnel.

b. The training problem is to prepare and present highly practical courses in the fundamentals of reliability, tailored to fit the needs of individual groups within the Army. Thus, the course must be dynamic in its flexibility and adaptability. It must be well documented with examples and "tools of the trade."

c. Training courses available at DoD schools and private schools and conferences sponsored by various technical societies provide valuable means of meeting training needs at minimum cost.

2-22. <u>Guidelines</u>. Ideal training activities include classroom instruction, supplemented by on-the-job application of the subject materials. The following questions are helpful in planning or selecting training courses. Do they:

a. Reflect the needs of attendees in terms of the scope of the course to be presented?

b. Include separate training programs and materials to specifically meet the needs of management and technical personnel?

c. Include management practices and engineering methods utilized throughout the entire life cycle?

2-23. Course content. The following suggested course outline can be adapted to specific needs drawing on appropriate sections of this document.

a. What should be known about basic concepts of reliability as a measurable product characteristic? How, for example, do you:

(1) Define characteristics for specific equipments?

(2) Graphically and mathematically visualize these characteristics?

(3) Express reliability in terms of confidence statements?

b. What should be known about specifications pertaining to reliability? How do you:

(1) Determine reliability requirements for parts, equipments, and systems?

(2) Specify the requirements?

(3) Specify tests for compliance with given confidence levels?

c. What should be known about reliability as an engineering function? How do you:

(1) Predict reliability feasibility of new design concepts?

(2) Predict reliability achievement during the development phase?

(3) Evaluate the described reliability problem areas for correction in early design?

d. What should be known about reliability assurance? How do you:

(1) Control reliability?

(2) Demonstrate reliability achievement?

e. How do you review and develop specific equipment and system program plans and specifications? Include:

(1) Program requirements.

(2) Quality assurance provisions for reliability.

f. How do you review development status of specific systems? Include:

(1) Reliability apportionment.

(2) Problem areas.

g. What should be known about contractor reliability programs? How do you:

(1) Evaluate a program?

(2) Specify program requirements?

(3) Monitor contractor programs for compliance?

h. What should be known about reliability monitoring and failure diagnosis:

(1) In design, development, production, and field use?

(2) To assure earliest practicable correction?

i. What specific steps can you take to assure higher reliability in systems? These include review of:

(1) Requirements analysis and specifications.

(2) Demonstration and acceptance.

(3) Procurement documentation.

(4) Monitoring and follow-up (including feedback).

CHAPTER 3

TECHNICAL REQUIREMENTS ANALYSIS AND DOCUMENTATION OF RELIABILITY REQUIREMENTS

Section I. INTRODUCTION

3-1. General. a. Early system development plans are not complete if they do not quantitatively define the required characteristics of the product or system proposed for development. While in the past the characteristics of a new equipment or system have been adequate to guide development effort toward full realization of performance requirements, they often have not been sufficiently descriptive of the reliability characteristics required for system success under field use conditions. These important success characteristics must be planned for and designed into the system. They cannot be added as an afterthought. This chapter outlines procedures for the definition and documentation of reliability requirements in essential planning documents, specifications and contractual task statements.

b. The problem is one of first stating system requirements for reliability in the Qualitative Materiel Requirements (QMR). These constitute the basis for the preparation of the Technical Development Plan (TDP) to accomplish CDC objectives. Required is the definition and documentation of requirements in the TDP and the definition baseline in order to give the system concept a clean entry into its development cycle. This is intended to insure that an operationally suitable system evolves as a result of good planning followed by effective pursuit of planned objectives.

Section II. CONTENTS OF QMR'S and SDR'S

3-2. General. a. Among the most important phases of the system life cycle are the concept and definition phases, where system requirements are analyzed and translated into well-defined technical objectives and detailed plans are laid to assure successful achievement of these objectives.

b. In general, there are three closely related analyses required in order to generate the essential descriptive information needed for preparation of technical development plans, design specifications, request for proposals, and contractual task statements. These are:

(1) Analysis and definition of the operational requirements -- performance, reliability, and maintainability -- necessary for the desired level of system effectiveness.

(2) Prediction of the feasibility of achieving these requirements by conventional design in order to assess the practical difficulty of the development job.

(3) An equitable method of initial apportionment (allocation) of requirements and supporting R&D effort among subsystems.

c. The last two of these analyses are discussed in chapter 4. The first is discussed in this section. It pertains to the formulation of a QMR/SDR based upon national defense objectives, intelligence estimates, and concept or feasibility studies which determine the requirements for a new capability and the need for a new item. The QMR/SDR expresses Department of Army requirements for new equipment or for major innovations or improvements related to research and development as developed from new concepts.

d. The QMR is a Department of Army approved statement of a military need for a new item, system or assemblage, the development of which is believed feasible, and is directed toward attainment of new or substantially improved materiel. It is stated at the earliest time after the need is recognized and feasibility of development has been determined.

e. The SDR is used to state a DA need for development of equipment of proven feasibility which can be developed with less effort. Because of low cost and simplicity of development, such equipment does not warrant the establishment of a QMR.

f. The QMR/SDR goes through four stages before final approval is given. These are:

(1) Initial draft proposed QMR/SDR .

(2) Draft proposed QMR/SDR.

(3) Proposed QMR/SDR .

(4) Department of Army approved QMR/SDR .

3-3. Reliability information in QMR's and SDR's. a. Reliability requirements should be stated in terms appropriate to the item considering its intended purpose, its complexity, and the quantity expected to be produced. In addition, these requirements must be clear, quantitative, and capable of being measured, tested for, or otherwise verified. QMR's/SDR's must include detailed essential reliability requirements. Statistical confidence levels and risks associated with demonstrating achievement of these requirements are to be stated in documents describing test requirements, but not in the QMR or SDR. Specifically, the information to be included in requirements is as follows:

(1) Reliability. The overall reliability requirement must be quantitatively expressed as a probability of success for one (or more) specified operational and environmental cycle(s) or functional sequence(s). Reliability may be apportioned for major phases of the mission. When an operational profile is not well defined (e.g., continuous operation), the closely related attribute, MTBF, may be specified instead of probability of success. Normally, one or the other attribute, but not both, is specified. Reliability requirements should be stated for two or more operational profiles, if appropriate.

(2) Reliability after storage. This must be specified so as to indicate the amount of deterioration which can be tolerated during storage. Length of storage, storage environment, and surveillance constraints should be identified for planning purposes.

b. Of the above requirements, only those that are appropriate for the item or equipment in question should be used. A more detailed discussion with examples of how the above requirements are to be stated in QMR's/SDR's is given in appendix C.

c. The discussions in this section represent an approach to determination of feasible requirements of the proposed system. Quantification of the above elements provides input for the development of realistic and meaningful contractual documents and specifications.

Section III. DOCUMENTATION OF RELIABILITY REQUIREMENTS IN TECHNICAL DEVELOPMENT PLANS (TDP's)

3-4. Role of the TDP and the research and technology resume in system development. The technical development plan (TDP) is expected to outline plans for development and provide guidance, goals, and specific direction necessary to assure that effectiveness will be achieved. The inclusion of statements delineating performance, reliability, and maintainability in TDP's is aimed at this goal. The TDP is applicable to those major development projects and tasks selected by the Chief of Research and Development and announced by separate correspondence. In order that the Army Research, Development, Test, and Evaluation (RDTE) Program be extended to all major equipment, the Research and Technology Résumé (DD Form 1498), applies to projects not covered by TDP's.

3-5. TDP format. In order to highlight the existence and adequacy of a reliability program, a separate section is included in the TDP for this program. An outline of the TDP format is shown in figure 3-1.

Cover Sheet
Introductory Sheet
Table of Contents
Section I - Narrative Summary
  Statement of Requirements
  Scope and Objectives
  Development Plan
Section II - Detailed Development Plan
Section III - Reliability and Maintainability Program
Section IV - Detailed Development Funding Plan
Distribution
Responsible Project Officer

Figure 3-1
TDP Contents

3-6. Documentation of reliability in TDP's. a. Each TDP must include information on the reliability program for that project and on the interface of these characteristics with other characteristics, as follows:

(1) Information listed in QMR's/SDR's.

(2) The plan for achieving reliability goals.

(3) The plan for conducting the reliability program.

(4) Reliability inputs to the costs and scheduling portions of the development plan.

(5) The plan for life cycle assessment of reliability characteristics.

(6) The plan for development of compatibility with multipurpose maintenance equipment and of system peculiar maintenance equipment.

b. Information contained in development plans is expected to be more detailed than that normally found in objectives/requirements documents (e.g., QMR); therefore, direct extracts from QMR's are not adequate. In those instances where the development plan is preliminary, listing of significant elements without detail will suffice; e.g., the fact that apportionment and prediction will be part of the program. Subsequent revisions must become increasingly more explicit and detailed, and must include updated reliability status information for comparison with requirements.

Section IV. DOCUMENTATION OF RELIABILITY REQUIREMENTS IN PROCUREMENT DOCUMENTS AND SPECIFICATIONS

3-7. General. a. The specification is, "...a document intended primarily for use in procurement, which clearly and accurately describes the essential and technical requirements for items, materials, or services including the procedures by which it will be determined that the requirements have been met." [Defense Standardization Manual 4120.3-M (AR 715-10)]

b. Reliability specification requirements consist of three distinct but related areas of coverage:

(1) Detailed quantitative requirements.

(2) General program requirements.

(3) Quality assurance provisions (Test and Evaluation Requirements).

c. These three areas may be included in the overall design specification for a product (Method A) or covered under a separate reliability specification (Method B).

(1) Method A. Integrated specifications: Reliability as a design parameter is logically specified in section III of the design specification (both detailed and general coverage) and the quality assurance provisions integrated into the overall provisions of section IV.

(2) Method B. Separate specifications: This alternative is recommended only when clarity and simplicity can be greatly enhanced. A reliability specification must follow approved specification format, consisting of the following:

(a) Scope.

(b) Applicable documents.

(c) Requirements.

(d) Quality assurance provisions (Test and Evaluation Requirements).

(e) Preparation for delivery.

(f) Notes.

3-8. Types of documents and specifications required. In order to maintain control throughout the materiel life cycle, it is necessary to have a given plan which requires documentation of item requirements. Throughout AMC, control is accomplished by means of the concept known as configuration management in conjunction with project management or commodity management (see AMCR 11-26).

3-9. Essential reliability features of specifications. a. The content of military specifications is prescribed in Defense Standardization Manual 4120.3-M. Important features of the specifications are the numerical requirements for equipment characteristics and the compliance requirements. These are given, respectively, in the sections labeled Requirements and Quality Assurance Provisions (Test and Evaluation Requirements).

b. Basically, the section of the specification outlining requirements for system and/or development descriptions contains performance and design requirements. Requirements for the test and evaluation methods to be used to check on conformance with these requirements are stated separately.

(1) The introductory paragraph consists of descriptive and introductory material, while quantitative requirements are stated and explained in detail as separate parts of the section. The paragraph specifying reliability requirements must be in agreement with those stated in the QMR/SDR and TDP and must be in quantitative terms. In order to assure that these reliability requirements are properly specified, system operational requirements, use conditions, the time measure or mission profile, reliability design objectives, quantitative reliability requirements, and reliability program requirements should be considered as sources of information for preparing the specifications.

(a) System operational requirements. Reliability is a system characteristic in the same sense that speed, range, and maneuverability are system characteristics. To have full understanding of the reliability requirement, operational requirements expressed in QMR's and TDP's must be described as well. The description provides a dividing line between what constitutes satisfactory and unsatisfactory equipment. To clearly make this distinction, it is necessary to include both design objectives and minimum acceptable values as a lower tolerance limit on the performance parameter.

Example: A radar design specification may desire the system to detect 1 sq. meter targets at 300,000 meters. The quantitative requirement might be stated as follows: The design objective shall be to detect 1 sq. meter targets at 300,000 meters. The system shall be considered unacceptable if 1 sq. meter targets are not detected at 225,000 meters.

(b) Use conditions. The conditions under which the item must perform should be stated in standard terminology. Use conditions refer to those conditions under which specified reliability is to be obtained, including temperature, humidity, shock, vibration, pressure, penetration/abrasion, ambient light, mounting position, weather (wind, rain, snow), operator skills and other conditions covered in AR 705-15, Operation of Equipment Under Extreme Conditions of Environment. In order to prevent undue equipment costs, stated use conditions should not be overly stringent, nor should unnecessary conditions be specified for equipment which will be used under controlled or limited climatic conditions. Use conditions are stated in both narrative and specific formats, with mission profiles included where environmental changes are expected through the operating period.

Example: Narrative. The XXX Tractor must be capable of operating as specified in climatic and weather conditions ranging from temperate to arctic and must be resistant to fungus, humidity, water, condensation, and icing.

Example: Specific. The XXX Tractor must operate as specified under any or all of the following environmental conditions: temperature, -65° F. to 160° F.; humidity, up to 100%; and water depth, traverse up to 3 feet.

Example: Mission Profile. The ABC system shall meet its performance requirements when subjected to a mission temperature profile similar to that shown in figure 3-2.

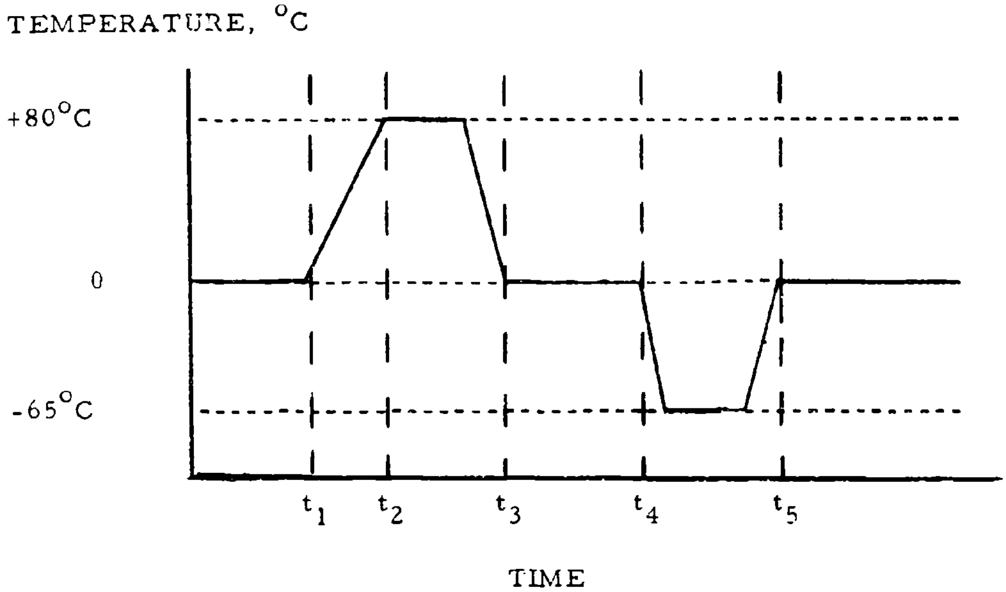


Figure 3-2
Mission Temperature Profile

(c) Time measure. System usage, from a time standpoint, plays a large part in determining the form of the reliability expression. Figure 3-3 is a representation of a typical operational sequence. In those cases where a system is not designed for continuous operation, total anticipated time profile or time sequences of operation should be defined either in terms of duty cycles or profile charts.

Example: The mission reliability for the "x" missile fire control system shall be at least 0.9 for a 6-hour mission having the typical operational sequence illustrated in figure 3-3.

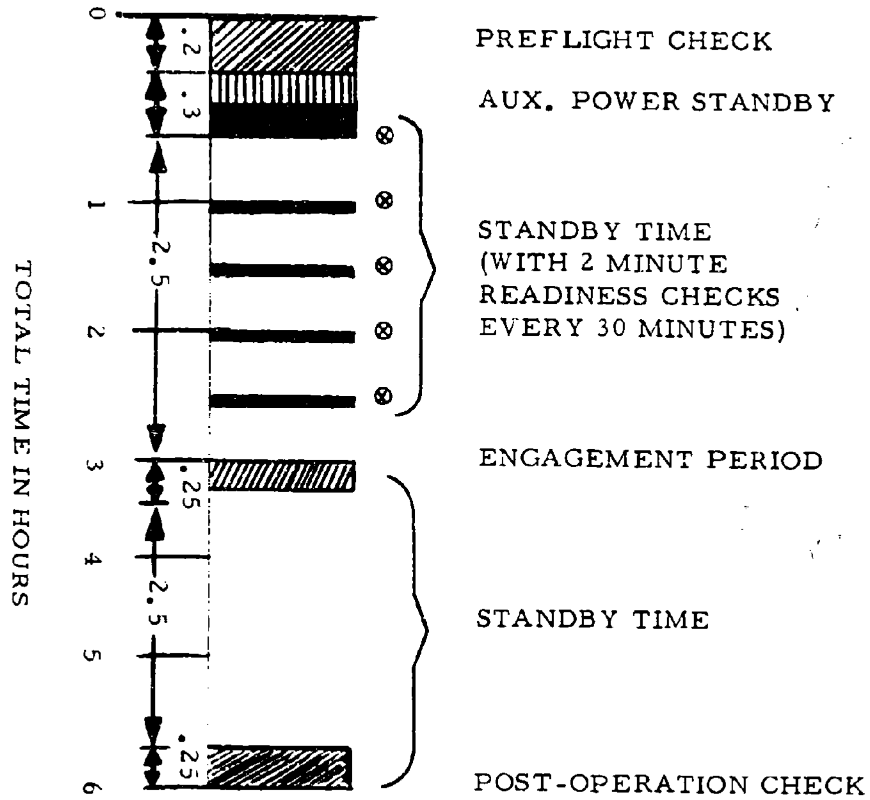


Figure 3-3
Typical Operational Sequence for
Missile Fire Control System

(d) Reliability design objectives and requirements. The specific functions for which reliability improvement is sought should be clearly spelled out. It is desirable that both the specific functions to be improved and the nature and extent of the improvement be described in enough detail that prospective designers have the advantage of earlier feasibility analysis.

Example: An improvement in the firing reliability of the XXX Howitzer is sought as a design objective. Specifically, it shall be the objective to reduce stoppages resulting from faulty extraction of cartridge cases from 1 per 10,000 to 1 per 50,000 rounds.

(e) Quantitative reliability requirements. 1. The specific values of reliability on which the success of the conceptual system is based should be quantitatively defined at one or more points to establish the desired reliability characteristics. Four common ways of defining reliability requirements are: mean time between failures (MTBF); probability of survival for a specified mission time; failure rate over a specified period of time; and probability of success, independent of time. Further discussion of these and other methods are included in chapter 1 and appendix A.

Example: A complex radar has both search and track functions. It is also possible to operate the search function in both a low and a high power mode. The reliability requirements for this system may be expressed as: "The reliability requirements for this system shall be at least: Case I, high power search, 28 hours MTBF; Case II, low power search, 40 hours MTBF; and Case III, track, 0.98 probability of satisfactory performance for 1/2 hour." The definition of satisfactory performance must include limits for each case. This can be conveniently tabulated for inclusion in the specification. A sample of the satisfactory performance table for the radar is shown as figure 3-4.

System Characteristic	Units	Performance Limits		
		Case 1	Case 2	Case 3
Range	Meters	300,000	120,000	120,000
Resolution - Range	Meters	± 50	± 50	± 10
- Velocity	m/Sec.	±100	±100	± 25

Figure 3-4
Satisfactory Performance Limits

2. The specified reliability requirement should also be defined in terms of nominal or minimum values. This can be done by either identifying a NOMINAL value with which the user would be satisfied, along with a minimum that must be exceeded, or simply a MINIMUM value below which the user would find the system unacceptable.

(f) Reliability program requirements. 1. The characteristics of the proposed program should be described in such a way that the fulfilling of these requirements will provide for controls and decision points necessary to assure the development of an item which will meet desired reliability requirements. For discussion of reliability programs, see chapter 2.

2. In the requiring of a reliability program, the following points should be kept in mind:

a Do not expect a reliability program to provide unlimited reliability. On the contrary, expect the program to provide realistic appraisals of progress, status, and potential of the overall program.

b. Avoid specifying, as part of the reliability program, organizational or internal (contractor) responsibilities which would limit or constrain the contractor's individual approach.

c. Reliability analyses or assessments are primarily design guides and monitoring techniques and should not be used as acceptance criteria in lieu of acceptance testing.

(2) Test and evaluation requirements. The reliability requirement is of little value if a method for measurement is not included in the specification. Conformance to the requirement is demonstrated by tests such as research and development acceptance tests, engineer design tests, engineering service tests, and environmental tests. The requirements for conducting the tests for each item performance and design characteristic must be included in section 4 of the specification test and evaluation requirements. It should be remembered that test data and test results may provide multipurpose information. Therefore, formal tests and analysis oriented

primarily to demonstrate reliability should be limited to those tests which provide reliability information not otherwise available.

c. Production description requirements and quality assurance provisions are summarized below.

(1) Requirements. The requirements section of the production description provides the same information for production contracts as the requirements section of a development document provides for development contracts. This section uses the drawings and specifications for the item to be produced as well as description of the processes needed for production. The relationship of reliability to the production description is primarily one of insuring that the level of reliability designed into the item is maintained during production and can be realized only if a successful transition from design to production of hardware is achieved.

(2) Quality assurance provisions (test and evaluation requirements. The specification must, in addition, set forth methods by which product acceptability can be determined. This involves types of tests to be conducted, inspection provisions and test methods and procedures. Quality assurance provisions should contain descriptions of preproduction, initial production, confirmatory acceptance and product improvement tests. These test provisions provide for lot formation, classification of characteristics, and acceptable quality levels as well as number of failures per sample, treatment of failures, preparation of specimens, apparatus and/or reagents, and decision making criteria. Further discussion of these provisions is included in chapter 6 and appendix F.

CHAPTER 4

RELIABILITY MODELING, PREDICTION, AND APPORTIONMENT TECHNIQUES

Section I. INTRODUCTION

4-1. General. a. Certain reliability analyses involve relating system reliability to subsystem or component reliability. This chapter contains a general method for constructing models relevant to such an analysis. Such models are useful for predicting system reliability from subsystem reliability data and for apportioning system reliability requirements among the subsystems.

b. In addition to the discussion of models, this chapter discusses the last two of the three analysis techniques mentioned in Chapter 3, Section II, which are used to generate the essential descriptive information needed for the preparation of TDP's, design specifications, requests for proposals, and contractual tasks statements. These techniques are prediction and apportionment.

Section II. RELIABILITY MODELS

4-2. General. a. The reliability model relates equipment or system reliability to subsystem and/or component reliability. These models are used for reliability prediction and apportionment. The particular form taken by the model is dependent upon the functional configuration of the system considered and thereby depicts the effect of failure on the system.

b. The types of models are as numerous as there are types of systems. However, all systems can be reduced to combinations and/or modifications of basic configurations. These configurations and combinations thereof, which are discussed in detail in Appendix D, are:

(1) Series configurations;

(2) Parallel (redundant) configurations;

(3) Mixed (series and parallel) configurations;

(4) Partially redundant configurations; and

(5) Standby redundancy configurations.

4-3. Procedural steps. The basic procedural steps for constructing a reliability model may be stated as follows:

a. Step 1. Completely define the components and subsystems and their relationship to system success.

b. Step 2. Construct a block diagram which indicates the function of each component or subsystem, including redundancy considerations. The block diagram is constructed not as a physical appearance of the system, but to indicate the function of each subsystem relative to system function. In general, a reliability block diagram represents a systematic arrangement of functions that must be performed and, when appropriate, the sequence in which they must be performed for system success. For example, the diagram contained in figure 4-1

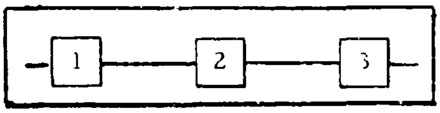


Figure 4-1

Series Block Diagram

indicates that all subsystems (1, 2 and 3) must function properly if the system is to be successful; and the diagram contained in figure 4-2

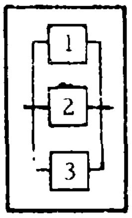


Figure 4-2

Parallel Block Diagram

indicates that the system will be a success if at least one of the subsystems (1, 2 or 3) function properly.

c. Step 3. From the block diagram, develop the mathematical model as shown in appendix D.

Section III. RELIABILITY PREDICTION

4-4. General. a. Reliability prediction is the process whereby a numerical value is assigned indicating the ability of a design or proposed design to perform in accordance with specified reliability requirements.

b. The primary objective of reliability prediction is to provide guidance relative to expected inherent reliability of a given design. Information obtained from prediction techniques is most useful early in the life cycle; specifically, during the conceptual, definition, and development phases. Basically, the purpose of reliability prediction includes feasibility evaluation, comparison of alternative configurations, identification of potential problems, logistics support planning, determination of data deficiencies, tradeoff decisions, apportionment of requirements, etc. Some important uses of reliability prediction include:

(1) Establishment of firm reliability requirements in QMR's and SDR's, TDP's, preliminary design specifications and requests for proposals, as well as determination of the feasibility of a proposed reliability requirement.

(2) Comparison of the established reliability requirement with state of the art feasibility for guidance in budget and schedule decisions.

(3) Providing a basis for uniform proposal preparation and evaluation and ultimate contractor selection.

(4) Evaluation of reliability predictions submitted in technical proposals and reports in precontract transactions.

(5) Identification and ranking of potential problem areas and the suggestion of possible solutions.

(6) Apportionment of reliability requirements among the subsystems.

(7) Evaluation of the choice of proposed parts, materials, components and processes.

(8) Conditional evaluation of the design for prototype fabrication during the development phase.

(9) Provides a basis for trade-off analysis.

4-5. Feasibility prediction procedure. The feasibility prediction procedure is useful during the conceptual phase from initial design concept to its preliminary paper design. Details of the product are, at this time, usually restricted to those which may be derived from existing products having similar functional and operational requirements. The procedure may be defined by the following steps:

a. Step 1. Define the product.

(1) Determine its purpose, intended function, or mission.

(2) Determine, in terms of performance requirements, the conditions which constitute product failure.

(3) Determine functional and physical boundaries of the system for which the prediction will be made.

b. Step 2. Establish the reliability model.

(1) Construct a reliability block diagram to the lowest identifiable function, showing the relationships necessary for successful system operation. Clearly indicate alternate modes of operation.

(2) Establish a mathematical model relating system reliability to the functional blocks in the block diagram.

c. Step 3. (1) Establish the functional complexity.

(a) Estimate the complexity of each block in terms of the number and use of functional parts.

(b) Determine failure rates associated with these parts.

(c) Combine part failure rates to establish a predicted failure rate for the block.

(2) Many electronic part failure rates may be found in MIL-HDBK-217A. For electronic parts not covered in this handbook, as well as nonelectronic parts, use available existing data and identify the source or estimate the failure rate from experience with existing equipment of similar design and function.

d. Step 4. Compute predicted system reliability by utilizing the mathematical model and the functional block predictions.

4-6. Design prediction procedure. This procedure is useful during the design phase which may continue through construction of prototype, preproduction, and production models. During this time, detailed schematics, breadboards, mockups, firm part selections and description, and complete functional block diagrams are developed. In addition, information pertaining to environmental, storage, final packaging, and handling conditions become available. Thus, prediction at this time is more dependable than that of the feasibility study. Predictions during this portion of the product cycle shall be made at intervals specified by the procuring agency for information regarding reliability growth. The steps for the procedure are:

a. Step 1. Define the system in the same manner as indicated by the feasibility prediction procedure.

b. Step 2. Establish the reliability model in the same manner as indicated by the feasibility prediction procedure.

c. Step 3. Determine the part population for each functional block. In addition to compiling a list of individual parts, include detailed information on factors that are pertinent to reliability.

d. Step 4. Determine appropriate stress factors for each part in its particular application.

e. Step 5. Assign applicable failure rate to each part.

f. Step 6. Compute reliability for each functional block utilizing the mathematical model and the predicted reliability of the parts.

g. Step 7. Compute predicted reliability for the system using the mathematical model and the predicted block reliability.

4-7. Specific techniques of reliability prediction. There are several sources of guidance for prediction procedures. Three techniques are discussed herein.

a. AGREE technique for design phase prediction. This technique recommends a procedure for design stage prediction of reliability of new equipment (electronic). The reliability function for this procedure uses the exponential distribution. The technique is summarized by the following steps:

(1) Step 1. Define the equipment explicitly and uniquely in terms of its functions, boundary points, operating conditions and performance characteristics.

(2) Step 2. Specify the components within the system. Components must be uniquely identifiable without duplication and must be selected in such a way as to take into account any redundancy and independence of operation.

(3) Step 3. Select the parts which have a dominant effect on system reliability, either because of their large number or because of their large failure rate, etc.

(4) Step 4. Determine a failure rate for each part or class of parts used in each component of the system. If parts are grouped and not analyzed singly, then classification of parts could be made in terms of homogeneity of failure rate, such as: tubes with high temperature of operation; tubes with low temperature; tubes that can deteriorate to the life test end point; tubes that can deteriorate well below life test end point; condensers with high voltage applied; resistors with high power rating; etc. From data obtained from MIL-HDBK-217A, or other available sources, the failure rate as related to the various stresses applied to the parts will be estimated. In the case of new parts or applications, it may be necessary to obtain new data through special investigations.

(5) Step 5. Determine a failure rate for each component within the equipment. Add the failure rates for all parts in each component of the equipment as determined in Step 4 to obtain the figures for component failure rates.

(6) Step 6. Determine the failure rate for each component.

(7) Step 7. Determine a failure rate for the equipment. Add the failure rates for all independent components within the equipment to obtain the figure for the equipment failure rate.

(8) Step 8. Determine the predicted reliability function for the equipment. The reliability for the equipment is based on the exponential failure distribution and is dependent upon failure rate and mission time.

(9) Step 9. Determine the predicted mean time between equipment failures (MTBF). The predicted mean number of hours between malfunctions is the reciprocal of failure rate.

b. ARINC technique of predesign reliability prediction. A predesign reliability prediction for ground electronic systems has been developed by ARINC Research Corporation. It was intended to provide prediction of reliability during the early planning stages and, as a consequence, is based on general information which can only be estimated.

The procedure provides a method for estimating confidence intervals associated with the predictions.

c. NAVSHIPS offers four methods for obtaining reliability predictions for electronic items. Each method pertains to a different category configuration.

(1) The first method deals with system reliability prediction from "typical" equipment failure rates. It pertains to systems which will be comprised of subsystems similar, in parts count, to equipment which has been used in the past.

(2) The second method applies to nontypical equipment in terms of the number of parts employed. It utilizes a factor to be multiplied by the number of parts to obtain a prediction of failure rate.

(3) The third method utilizes parts failure rate by part category in prediction of reliability. Employment of this method requires a count of the various type parts included in the design.

(4) The fourth method is the most detailed of the group and deals with reliability prediction of equipment or circuits from parts rates with severity function. It requires not only a parts count but the design must be carefully analyzed to determine the severity of stress to which each part will be subjected.

Section IV. RELIABILITY APPORTIONMENT

4-8. General. a. Reliability apportionment (allocation) represents the assignment of reliability goals or requirements to subsystems in such a manner that the system reliability goals or requirements will be satisfied. Whereas prediction utilized the reliability model to obtain system reliability from subsystem reliability values, apportionment makes use of the same models by proceeding from system reliability goals to compatible subsystem goals.

b. Apportioned reliability requirements will prove useful for directing reliability effort along profitable channels and keeping the development effort compatible. A few important uses of reliability apportionment are listed.

(1) During the conceptual phase, apportionment of proposed reliability requirements will provide an aid in determination of feasibility.

(2) When various subsystems are being developed by different contractors, apportionment will provide compatible contractual reliability requirements.

(3) Apportionment will provide the prime contractor, as well as government monitors, with a means of evaluating subcontractor reliability achievements.

(4) Apportioned reliability requirements may be used as developmental goals for parts and subsystems. Consequently, reliability growth progress can be monitored for subsystems with the result that problem areas may be discovered and such problems alleviated by reallocation of resources and efforts or initiating appropriate reliability trade-offs.

4-9. <u>Considerations for reliability apportionment</u>. The ideal apportionment would be the allocation of requirements resulting in the most economical use of resources, including time and cost. Apportionment of reliability is a trade-off between the reliabilities of units to achieve a specified system reliability. By imposing high requirements on those units in which high reliability is easier to attain, and lower requirements on those in which high reliability is more difficult and more costly, the overall cost of system development may be reduced. A few important factors for consideration follow.

a. The complexity of the system will have an effect on the achievable reliability. The more complex the system, the greater the number of subassemblies and modules, the more difficult and costly it is to achieve a high reliability. Imposing an unrealistically high reliability on the more complex systems increases the cost disproportionately when compared with the effect of increasing the reliability requirement for simpler systems.

b. The amount of development and research required to produce the system will greatly influence the time and cost of development. Imposition of a high reliability requirement on a system under development will increase the development time, number of tests required to obtain the reliability, and the cost.

c. The intended operational environment will have an effect on the achievable reliability. A system to be used in a rugged environment will tend to cost more to develop to an equal reliability than a similar one to be used under less severe conditions.

d. The length of time the equipment is required to perform will influence the achievable reliability. It will require more development effort and cost to produce a system capable of operating for a long period of time without failure than to develop one for a shorter period of use.

e. The need for high reliability in a component is based on the importance of its operation. A component whose failure would not jeopardize the accomplishment of the mission need not be highly reliable. To the extent that failures can be tolerated, lower reliability requirements should be imposed.

4-10. Specific techniques of reliability apportionment. Several techniques of reliability apportionment have been discussed in the literature. Those to be presented herein are the equal apportionment method, the AGREE method, a method by ARINC Research Corporation, and two methods for minimizing total effort expended. More detail is found in appendix D.

a. Equal apportionment technique. In the absence of definitive information on the system, other than the fact that n subsystems are to be used in series, equal apportionment to each subsystem would seem reasonable. In this case, the n^{th} root of the system reliability requirement would be apportioned to each of the n subsystems.

b. AGREE apportionment technique. A method of apportionment for electronics systems is outlined in the AGREE. This technique

takes into consideration both the complexity and importance of each subsystem. It assumes a series of k subsystems, each with exponential failure distributions. The apportioned reliability goal is expressed in terms of MTBF.

c. ARINC apportionment technique. This method assumes series subsystems with constant failure rates such that any subsystem failure causes system failure and that subsystem mission time is equal to system mission time. This apportionment technique requires expression of reliability requirements in terms of failure rate.

d. Minimization of effort algorithm. This algorithm considers minimization of total effort expended to meet system reliability requirements. It assumes a system comprised of n subsystems in series. Certain assumptions are made concerning the effort function. It assumes that the reliability of each subsystem is measured at the present stage of development or is estimated, and apportions reliability such that greater reliability improvement is demanded of the lower reliability subsystems.

e. Dynamic programming approach. If all subsystems are not equally difficult to develop, dynamic programming provides an approach to reliability apportionment with minimum effort expenditure when the subsystems are subject to different but identifiable effort functions.

CHAPTER 5

RELIABILITY DESIGN AND REVIEW

Section I. INTRODUCTION

5-1. **General.** During development, a design is formulated to meet quantitative requirements previously defined. The results of these activities provide inputs for all future actions. The importance of designing in the required degree of reliability cannot be overemphasized; for once the design is approved, inherent reliability is fixed. Less than perfect compliance with required actions from this point may result in an achieved reliability level less than the fixed inherent level. This concept is illustrated in figure 5-1.

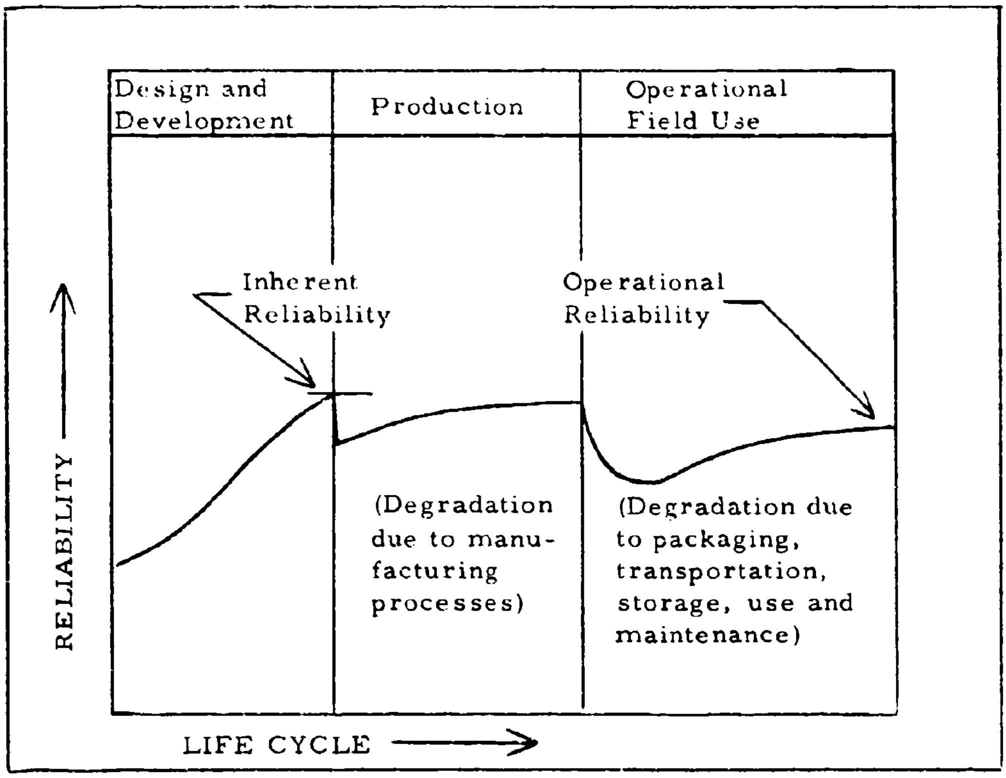


Figure 5-1
Reliability Growth During System Life

Section II. SOME BASIC PRINCIPLES OF RELIABILITY DESIGN

5-2. General. This chapter deals with identification of some basic principles of reliability design of which the designer should be aware and with the concept of design reviews. Each is discussed briefly in terms of its role in the design of reliable equipment.

5-3. Simplicity. Simplification of item configuration can contribute to reliability improvement mainly by reducing the number of possible failure modes. A common approach toward design simplification, especially structural, is that of component integration, i.e., the use of a single part to perform multiple functions.

5-4. Use of proven components, preferred circuits, and preferred design concepts. a. If reliability is to be designed into an item, the reliability of the individual components must be predicted or estimated. When working within time and cost constraints, it is wise to use proven components where possible, thus minimizing analysis and testing.

b. Information is available concerning reliability of certain component configurations and circuits. There are electronic design handbooks available, for example, illustrating standard circuitry which should be used in preference to unique designs. Just as with electronic designs, proven mechanical and fluid system design concepts can be categorized and proven configurations given first preference.

c. Existing standards must be constantly reviewed and updated. The establishment of new hardware standards must be preceded by thorough reliability verification. Some areas in standardization which are important to reliability design are standard values, parts, components, systems and subsystems. Another area of importance is that of analysis techniques. Accepted methods, such as reliability prediction, can be established for use by design and reliability engineers. These analysis methods are useful provided their limitations are recognized.

5-5. Stress/strength design. a. The classical approach to design is to give every part enough strength to handle the worst stress it will encounter. Several references, such as MIL-HDBK-5 are available providing data on the strength of materials, and some of these provide

limited data on strength degradation with time, resulting from fatigue. However, when designing for a specified reliability, the traditional and common use of safety factors often is inadequate. Effective design procedures should provide for evaluating alternative configurations with respect to reliability. Since failure is not always related to time, the designer needs techniques for comparing stress vs. strength. These include:

(1) Derating. Use parts rated higher than expected stress.

(2) Reliability margin. Measure the separation between stress and strength.

(3) Stress/strength testing. Determine the stress and strength distributions.

b. The concept of stress-strength in design recognizes the reality that loads or stresses and strengths of particular items subjected to these stresses cannot be identified as a specific value but have ranges of values with a probability of occurrence associated with each value in the range.

c. The reliability of an item may be defined as the probability that the strength of that item will never be less than the stress to which it will be subjected (see appendix A for analytic methods of determining reliability using stress and strength distributions). There are four basic procedures the designer may use to increase reliability.

(1) Increase average strength. This approach is tolerable if size and weight increases can be accepted or if a stronger material is available.

(2) Decrease average stress. Occasionally the average stress on a component can be reduced without greatly affecting its capability.

(3) Decrease stress variation. The variation in stress is usually hard to control. However, the stress distribution can be effectively truncated by putting limitations on use conditions.

(4) Decrease strength variation. The inherent part to part variation in strength can be reduced by improving the basic process, holding tighter control over the process, or by utilizing tests to eliminate the less desirable parts.

5-6. Redundancy. Redundancy provides more than one way to accomplish a function. In some design situations, reliability improvement may be achieved by introducing redundant subsystems or components. For details of various evaluation models, see appendix D.

5-7. Local environment control. Often it becomes apparent during design that a severe local environment would prevent achievement of the required component reliability. The design engineer is faced with the choice of improving the component to withstand the environment or, if possible, changing the environment to satisfy the component. Such local environment control may add weight, space and cost; so tradeoffs must be evaluated on the basis of overall effectiveness. Sometimes overlooked is the harmful effect of transportation and installation as opposed to operation. Improved packaging and special handling instructions may be necessary to preserve reliability. Some typical environmental problems are:

a. Shock and vibration. There are two approaches that may be taken when shock or vibration are present. Either isolate the equipment or build it to withstand the shock or vibration. The problem with isolation is that effective, simultaneous control of both shock and vibration is difficult. When only one or the other is present, special mountings are often used.

b. Heat. In virtually any kind of system, heat buildup increases the possibility of failure. Commonly used methods of heat transfer include free convection, forced air cooling, liquid cooling, conduction, radiation and vaporization cooling.

c. Corrosion. The following design considerations are used to provide protection against corrosion:

(1) Use corrosion resistant materials.

(2) Use plating and protective finishes.

(3) Avoid dissimilar metal contact.

(4) Control the environment (prevent water entrapment, remove atmospheric moisture, etc.)

5-8. Identification and elimination of critical failure modes. a. Failure mode and effect analysis (FMEA) is an effective technique for revealing design deficiencies and potential hazards. The FMEA team, to be most effective relative to reliability, should include a reliability and a design engineer.

b. FMEA is nothing more than a thorough analysis of the questions: "How could ... fail?" and "What would happen if ... fails?" FMEA is more than a check on a design. The design concept can be developed using this technique as follows:

(1) Start with a functional block diagram to determine importance of each function.

(2) Based on this importance, certain configurations may be developed.

(3) Analysis of the resulting design for compatibility between failure effects and predicted failure probability.

5-9. Self-healing. A design approach which has possibilities for future development is the use of self-healing devices. Perhaps the simplest example of a self-healing device is [illegible] tire with a layer of sealing compound which will seal any small punctures. A similar technique is used in some aircraft fuel cells. In this case, a puncture exposes a layer of uncured rubber which swells to seal the leak. Automatic sensing and switching devices represent a form of self-healing.

5-10. Detection of impending failure. Achieved reliability in the field may be facilitated by the introduction of methods and/or devices for detecting impending failures. Such devices may be used for:

a. Screening of parts and components.

b. Periodic maintenance schedules.

c. Monitoring of operating equipment.

5-11. Preventive maintenance. a. For continuously operating repairable items, preventive maintenance procedures may greatly enhance reliability when recommended practices are followed. Because such practices may be difficult to comply with, the designer may add to achieved reliability by avoiding the need for preventive maintenance to the highest degree possible.

b. When the need for preventive maintenance cannot be avoided, the design should provide for the longest possible period between such maintenance, and above all must be consistent with the overall maintenance policy, the availability of skills, and accessibility.

c. Finally, the technical manuals must emphatically call out the schedule and importance of such maintenance to reliability; and it may be desirable to place prominent labels containing maintenance instructions directly on the equipment.

5-12. Tolerance evaluation. A design is not considered complete until it has been determined that the different types of tolerances cannot combine in such a way as to interfere with the intended function. In a complex item, it is necessary to consider the expected range of manufacturing process variance, operational environment, and all stresses, as well as the effect of time. Tolerances resulting from environment (temperature, etc.) and time must be added to manufacturing tolerances in order to determine the real operational distribution. Some methods of tolerance evaluation consist of:

a. Worst case tolerance analysis. Determining whether the equipment can perform properly with all parts simultaneously at their tolerance limits and in such a direction as to produce the greatest deviation of nominal performance.

b. Statistical tolerance analysis. A statistical procedure determining the manner in which individual parts tolerances affect the overall tolerance. This avoids the inherent pessimism of the worst case approach.

c. Marginal checking. A quantitative method for stating what sensitivity a given circuit has to variations in its components.

5-13. Prediction and apportionment. a. The role of reliability prediction during design is that of providing an evaluation of a proposed design and to provide a comparison of alternative designs. Similarly, apportionment provides an approach for identifying reliability goals for subsystems in such a combination so as to afford design feasibility. Generally, these procedures may be useful in the following applications:

(1) As a planning tool for the initial establishment of reliability requirements.

(2) As a design tool to guide the designer in the choice of parts to meet the specified reliability requirement.

(3) As a design review tool by management for the evaluation of design adequacy to meet the reliability requirement and to point up potential reliability problem areas for design correction.

(4) As a monitoring tool for the assessment of development program progress toward established goals to predict and circumvent oncoming problems before the hardware stage.

b. Design reliability assessments can be divided into two phases:

(1) The conceptual or design proposal phase. A prediction is based on the design concept as reflected in development specifications and early design documentation.

(2) The design or development phase. Predictions are based on the actual design.

c. For prediction and apportionment methodology, see chapter 4.

5-14. Human engineering. Mistakes by people often result in failure of an item to perform its function. Therefore, human activities and limitations may be very important to item reliability. The reliability of people can be influenced by the design engineer by considering the factors which directly refer to human aspects, such as:

a. Human factors.

b. Man-machine interface.

c. Evaluation of man in the system.

d. Human reliability.

5-15. Mean life ratio. Frequently a design engineer wishes to evaluate the life of a new product or process in relation to the old. In making a decision, he should perform a statistical evaluation on the variation between the two sample results; or, in other words, he would state that the new product or process is better only when he could make the statement with a statistically high confidence of being correct. A common method for determining the confidence is the mean life ratio approach.

Section III. RELIABILITY REVIEWS

5-16. General. a. A reliability review is defined as planned monitoring of a product to assure that it will meet the expressed and implied performance requirements of the equipment during operational use. Such a review provides periodic appraisal of the design effort to determine the progress being made in achieving the design objectives and systematically brings to bear specialized talent on specific problem areas. In this manner, an overall evaluation is made to identify specific reliability problems that may be encountered later in the development and production cycles.

b. Realization of the full worth of reliability design reviews requires that system program personnel actively participate in the design review process on all development programs which result in items entering the military inventory.

5-17. Basic review philosophy. Reviews may be profitably applied at any point during design activities ranging from concept to production. Design chang s during the early design reviews generally require very little engineering effort since they usually involve only paper changes of a part, dimension, or value; although redesign of components might at times be mandatory. Design changes occurring

during subsequent design reviews involving changes to drawings, modifications, replacement of existing hardware, replacement of field personnel, etc., will be considerably more costly. The periodic review of reliability at key points in the development program facilitates detection and correction of actual or potential problems prior to the final design.

5-18. Required review points. A review schedule should include the time-phased events representing the appropriate milestones at which formal reviews are made at major decision points. The number of critical decision points will vary according to the type of development program underway. The reliability management milestone guide in appendix B covers the basic reliability review points in the materiel life cycle. As reliability reviews are normally repetitious, it is recommended the review points be expanded and reoriented to conform to each unique program.

CHAPTER 6

DEMONSTRATION AND TESTING

Section I. INTRODUCTION

6-1. General. a. The design and proper use of adequate test and demonstration methods and procedures is of prime importance if the user is to be assured a reliable equipment for field use. This chapter is devoted to a general description of such methods and procedures which are applicable to a wide variety of equipments and components.

b. The purpose of reliability demonstrations and tests is to determine current item reliability level. The demonstration of high reliability with a high level of confidence usually requires testing of a greater number of items than are available, especially when concerned with complex or expensive items. Thus, it is important to design tests in such a manner that maximum information can be obtained from a minimum amount of testing. The efficient use of statistical techniques is often essential.

c. A major problem in the formulation of adequate tests is that of simulating a realistic use environment. During its lifetime, an equipment may be subjected to many environmental factors or stresses such as temperature, vibration, moisture, acceleration, rough handling, etc., and these stresses may be encountered singly, simultaneously, or sequentially.

d. The ideal test program should provide continuity of reliability assessment activities from prediction through end item demonstration and testing activities.

Section II. RELIABILITY DEMONSTRATION AND TEST PROCEDURES

6-2. General. a. Reliability demonstration and test procedures are applied in order to gain information concerning failures and their frequency of occurrence.

b. Demonstration of reliability may be accomplished by the testing of sample items and may be defined as the process of placing an item or product under a specified set of conditions and observing the results. Such tests may be applied to systems, subsystems, or components.

c. Developmental tests are quite useful for estimating the values of certain reliability parameters, or deciding whether the reliability parameters have reached an acceptable level at the particular stage of development. Statistical estimation techniques and statistical tests of hypotheses, respectively, are utilized for these purposes.

d. Acceptance tests are used for deciding whether the reliability of an item is at an acceptable level. This is merely an adaptation of the above-mentioned tests of hypotheses. Statistical analysis will provide measures of the risk involved in these inferences.

e. Life tests may assume any of several different test disciplines. The test may be terminated after a preassigned time has elapsed, or it may be terminated when a preassigned number of failures have occurred. For either termination criteria, the test may be conducted either by replacing or by not replacing items as they fail. A major difference between the test disciplines is that time terminated and nonreplacement tests usually are simpler to conduct than are failure terminated and replacement tests.

f. Several types of analysis may be applicable to a particular testing situation. The determination of the appropriate type must be taken into account during pretest planning. Among the common techniques to be discussed include: parameter estimation, testing of hypotheses, acceptance testing, regression analysis, accelerated life testing, and stress-strength testing.

6-3. Parameter estimation. a. For each population of components, subsystems, or systems, there exists one or more numerics which describe the entire population. These numerics are called population parameters. If the important reliability parameters of a population were known, reliability testing would not be required. However, in practice, they are not known, and we must resort to submitting sample items to tests in order to estimate these parameters.

b. A population may be a group of existing items, or it may pertain to items potentially generated by a process. The difference is reflected by the interpretation of the estimated parameters. In the first case, the estimated parameter describes existing items or equipments; and in the second case, the estimated parameter describes the capability of the process for generating reliable items.

c. An estimate of a reliability parameter may take the form of either a point or an interval to which a degree of confidence may be attached. A point estimate is merely a single number which is defined as the estimated value of the parameter of interest, e.g., the population MTBF may be estimated as 100 hours. The confidence interval consists of the statement that the parameter falls between two numerical values with the associated degree of confidence. The confidence interval is, in effect, a measure of precision.

d. The estimation of reliability parameters is facilitated if the underlying distribution of failure times is known. Analysis techniques which do not depend on a known failure time distribution are known as nonparametric techniques. In general, nonparametric reliability analysis has the advantage of providing information without prior knowledge of the failure distribution, but it also has the disadvantage of less precise information than that obtained when the underlying distribution is known.

e. Knowledge of the failure distribution usually depends upon historical information about similar items or upon a relatively large sample by which the hypothesized distribution can be tested. Since most development tests are based upon a small sample of prototype models, and in many cases no historical information exists for similar items, nonparametric analysis may be necessary.

f. Appendix F contains several analysis procedures pertaining to the estimation of reliability parameters. These apply to:

(1) Unknown distribution of failure times.

(2) Determination of the underlying distribution of failure times.

(3) Normal failure times.

(4) Exponential failure times.

(5) Weibull failure times.

6-4. Tests of hypotheses. The preceding comments on test analysis were concerned with estimating the value of certain parameters. It is sometimes more meaningful to decide whether or not the parameters are at acceptable levels. Such decisions may be accomplished by hypothesizing a value for the parameter of interest and using the test results to decide whether the hypothesis should be accepted or rejected. These procedures are referred to as tests of hypotheses and are discussed in detail in appendix F.

6-5. Acceptance testing. The test of hypothesis, when used as a basis for accepting material, is sometimes referred to as an acceptance test. There are a number of government documents which contain reliability test plan tables. These, too, are discussed in appendix F.

6-6. Regression analysis. Regression analysis sometimes may be used to determine a reliability parameter for various stress levels or design characteristics, such as determination of the mean time to failure at different levels of stress, e.g., determination of the rocket bursting pressure for different wall thicknesses. The determinations are useful for evaluating equipment design, identifying trouble areas, and potential corrective activities, etc. Regression analysis techniques are sometimes used to generate this type information from test results. For these methods, see appendix F.

6-7. Accelerated life testing. a. Life tests, conducted at or near normal operating stresses, have proven useful for evaluating an equipment with regard to reliability and for providing data to be used in reliability improvement activities. The extreme test duration time poses a serious problem when conventional life testing procedures are used to demonstrate very high reliability. This life test duration time can sometimes be shortened, however, by utilizing the functional relationship between life characteristics and variable stress conditions. The technique of inducing failures by subjecting test items to excessive stresses is known as accelerated life testing.

b. The primary purpose of accelerated life testing is to reduce the time required to obtain failure data. This data, however, is not representative of reality and must be transformed to failure data pertaining to normal stress conditions by means of a functional relationship between stress level and failure occurrence.

c. Accelerated life testing can be successful only if the functional relationship is available from existing sources or if it can be determined experimentally, and if additional modes of failure are not introduced.

d. MIL-HDBK-217A, Reliability Stress and Failure Rate Data for Electronic Equipment, provides data concerning stress levels versus failure rate for certain electrical components. The data in this handbook were intended for reliability prediction purposes, but the included adjustment factors would allow failure rate transformation from one stress level to another.

6-8. Stress-strength testing. a. Stress-strength testing techniques may be used for evaluating reliability in instances when time or duration of mission does not contribute significuantly to failure, e.g., mechanical devices and one-shot devices. Analysis techniques for evaluating reliability for stress and strength are discussed in appendix A.

b. Stress testing generally involves simulated usage of the item of interest to determine the stress distribution. The stresses incurred are determined by using such things as strain gages, plastic models, polarized light, etc. The results are as dependable as the accuracy of simulation of manufacturing variations, operational environment, external stresses, time effects, and other important variables.

c. Strength testing usually involves some variation of testing under increased stresses until failure occurs. The strength distribution can be determined by a number of tests to failure caused by continually increasing the stress load. For such tests, it is extremely important that failure be precisely defined before testing.

d. The method of increasing stress of an item until it fails is applicable only if there is no degradation of strength due to the preceding stress level. In this case, one-shot testing may be performed where an item will be subjected to a given stress level; but if it does not fail, it will not again be tested at an increased stress level. By properly selecting the test stress levels and recording whether the item failed, it is possible to determine the stress distribution.

6-9. Reliability testing and the total test program. In general, tests should not be conducted solely for determination of reliability characteristics, but should include consideration of other technical characteristics. Thus, reliability personnel should be acquainted with the overall Army test program. To fulfill the AMC testing concepts, an item of Army materiel must be tested at appropriate points throughout its life cycle. The reliability considerations associated with life cycle testing are shown in appendix B.

Section III. TEST DESIGN

6-10. General. This section furnishes guidance in the application of the Army testing concepts to test planning and design. In addition, a test matrix is discussed as an approach for development of an effective test program.

6-11. Test procedures. Test conditions and methods of data analysis are preplanned on the basis of engineering requirements, test methodology, and statistical considerations. The following cycle must be completed if effective and unbiased test results are to be achieved:

a. Define the problem.

b. State test objective.

c. Establish test requirements.

d. Design test.

e. Implement test.

f. Analyze results.

6-12. Importance of technical characteristics. a. The approved statement of the military need for a new end item or system is contained in the QMR/SDR. A statement of essential characteristics may be derived from this document to provide the basic elements of a design matrix. Such a matrix serves as guidance for development of the item or system in response to a stated military need.

b. In order to develop an efficient test design, it is essential that key performance parameters be identified to assure that the test program is comprehensive and complete. Some important determinations relevant to an efficient test design follow.

(1) Definition of the overall mission of the system undergoing test and evaluation.

(2) Breakout of the overall mission into major system characteristics.

(3) Further breakout of each major system characteristic into a related set of subsystem characteristics.

(4) Identification and definition of required subsystem characteristics necessary for each element of the system requiring evaluation.

(5) Determination of critical high risk characteristics which are essential to successful system performance.

c. These determinations provide an orderly breakout of performance characteristics such that the test results may be evaluated against some given standards or performance criteria. The performance criteria must have been based upon an associated rationale traceable to intended tactical performance.

d. End item performance criteria are incorporated into the overall test plan through application of appropriate prior data, use of mathematical modeling and simulation techniques, use of statistical techniques, and engineering analysis.

6-13. Design of test programs. a. The following discussions apply to test programs planned during any portion of the materiel life cycle. It should be kept in mind that program emphasis changes as the item moves from conception to obsolescence.

b. The relationship between stated performance characteristics, performance criteria, criteria rationale and sample requirements are outlined in figure 6-1. This figure represents the requirement for a generalized test matrix upon which a comprehensive set of development and test objectives can be based.

c. Figure 6-2 provides an example of a partial engineering and service test matrix. The matrix for other tests may be developed in a similar manner.

d. A test matrix provides a ready outline for the development of a comprehensive set of test objectives. Detailed test objectives provide the basis for a test plan. Each objective should discuss the primary purpose of the test, the relationship of the specific test to the purpose of the overall test program, and the test standards which require satisfaction. In all cases, the performance criteria associated with each performance characteristic should be included. This information, coupled with the test method to be employed in the execution of the test and the data to be obtained from the test, constitutes the major portion of any test plan.

e. Verification of technical performance with a reasonably high level of confidence requires a well-designed test program. In conjunction with engineering analysis and test methodology determination, modeling and statistical analysis techniques are useful for development of a test program.

(1) Modeling. The application of modeling is a valuable engineering tool which provides a means of analyzing dependent system characteristics to identify maximum stress conditions. Modeling techniques provide preliminary performance estimates which can be subsequently verified through test; thus reducing the empirical element in test planning. Some reliability modeling approaches are treated in appendix D.

(2) Statistical techniques. Some techniques for analysis of test results are shown in appendix F.

Performance Characteristics	Performance Criteria (Standard of Performance)	Standard of Performance Rationale	Minimum Sample Requirements	Sample Rationale
This column will provide an outline of identifiable minimum and/or nominal level of technical performance characteristics required by the user. They will normally be extracted or derived from statements appearing in the QMR/SDR.	The performance standard is a direct statement of user criteria (e.g., the item must be air transportable for use in Phase I airborne operations), or a technically derived standard based upon user requirements (e.g., engagement of the postulated threat requires a tracking rate of a certain value).	This column is a statement of reason(s) justifying the cited performance standards. For example studies are usually performed to determine anticipated levels.	The number of test items necessary to perform each test will be defined. It is recognized that one group of test items may support several tests, however, the allocation and use of test items will be identified.	The rationale behind selection of test items will be specified.

Figure 6-1

General Matrix Requirements

Performance Characteristics	Performance Criteria (Standard of Performance)	Standard of Performance Rationale	Minimum Sample Requirements	Sample Rationale
Weight 3500# max required	Air transportability requirement for Phase I airborne operations	Helicopter design limitations 3500# @5000' altitude	1	Measurement will be provided for **Reliability Evaluation**
Tracking Rate 5^{o}/Sec required	System must track targets at ___ range and ___ velocity	Characteristic of Postulated threat.	1	Characteristic verified by design analysis. Operational check obtained during reliability evaluation
Safety Hazard from Electromagnetic radiation will be minimal	Power level at 600' range from antenna shall not exceed ___.	Concept of Tactical Performance. Surgeon General's Report on EMR hazards	0	Development data subjected to analysis. Performance verified
Reliability mean time between failures (MTBF) shall be no less than 800 hours (nominal), 400 hours (minimum)	Requirement is consistent with performance of similar items scheduled for fielding in this time frame	The system must be capable of operating under field conditions without extensive logistic support for 24 hrs	6	These 6 test items are intended to verify the stated reliability required with 90% confidence. This sample size assumes that preliminary reliability data for critical components are provided
Operation required at +165°F.	System must operate in desert and tropic areas	Required by AR 705-12	1	Stability of components verified during development testing. Measurement will be made on samples provided for Reliability Evaluation

Figure 6-2
Typical Partial Engineering and Service Test Matrix

CHAPTER 7

RELIABILITY EVALUATION, FAILURE ANALYSIS AND CORRECTION -- THE FEEDBACK LOOP

Section I. INTRODUCTION

7-1. General. a. Reliability improvement may be characterized by five policy type objectives in order to concentrate the project resources effectively. These are represented by a reduction of: safety hazards; catastrophic equipment hazards; failure rate of highly replicated system components; relatively high subsystem failure rates; and failures which produce very high support costs. Implementation of these objectives is brought about through increased technical understanding and improvement of the design. Data analysis is used by the engineer as a tool to identify those areas where greater technical understanding must be developed. The nature and true value of the reliability improvement program thus lies in the conscientiousness and rigor with which reliability personnel investigate problems or weak areas and follow up with corrective action. The determination of which problems to pursue, to what lengths and by what means, should be based upon thorough understanding of the system, Army policies, contractual limitations, and experience with previous problems. In areas not covered by established Army policy, procedures or experience, it is necessary to pursue whatever areas seem to promise the most benefit to overall item reliability.

b. Field operation, in addition to development testing, can be viewed as an extremely important -- albeit costly -- source of reliability data. A failure reporting program should never be implemented before making a careful analysis of what data is to be measured, how the data is to be analyzed and interpreted, and what can be done to correct the system faults as a result of such interpretation. The purpose of this chapter is to provide guidance for planning such a testing and feedback information program.

c. In planning for data collection, it is just as important to collect data on successful or satisfactory operations as it is to collect data on failures. The use of statistical analysis techniques should be explored

since programs may sink from their own weight where requirements for data collection are excessive. In addition to collecting data, care must be taken to assure that the lessons learned from experience are recorded and that failure modes are identified. Data collection should include plans for incorporating proper statistical procedures for evaluating the data. Decisions resulting in corrective action should be made with careful regard for the system mission requirements and the effective use of available resources.

Section II. OBJECTIVES OF A POSITIVE MATERIAL FAILURE ANALYSIS AND CONTROL SYSTEM

7-2. Objectives. In order to develop a positive material failure analysis and control system (FACS), the following objectives should be adhered to.

a. Provide only the pertinent facts needed to evaluate the criticality of a failure by:

(1) Collecting both success and failure data.

(2) Using accepted statistical analysis techniques to provide a confidence level and assure the precision of the data.

(3) Taking care to gather data pertaining to all failure modes.

b. Timely distribution of failure data and information to all organizational elements needing such data through the use of a simple and quick response data collection and reporting system.

c. Provide for the cause and effect of failures to be established and evaluated by the proper organizational element in a methodical manner which uses appropriate statistical techniques and quantitative application of engineering principles.

d. Assure that cost-effective and timely corrective action is taken by:

(1) Preplanned and scheduled steps for handling an identified reliability problem.

(2) Requiring that changes be made with due regard for the stated mission-responsive requirements of the system.

(3) Requiring that cost-effectiveness principles and the official guidelines on this subject be adhered to.

e. Closing the loop on each action using the methods of data collection and evaluation provided by the system to verify and evaluate the effectiveness of the action.

Section III. METHODS OF DATA COLLECTION AND DOCUMENTATION

7-3. General. a. Data collection involves some method of placing data on individual events into source documents or records. Documentation for reporting purposes involves assembling data on individual events into composite reports which present the information in a meaningful and usable form. Masses of data improperly collected and assembled will not provide needed information. Thus, the requirements for what data is to be collected, how it is to be collected, and how it is to be reported are fundamental and tasks which must be approached with great care in planning. If this is not done, a tremendous amount of effort and resources may be expended on an effort which has relatively little value.

b. Basically, data requirements consist of two factors: data elements and data reports. The data elements form the basis for devising individual event source document forms. Choice of data elements must be based on the requirements of the reliability reports program. Data reports reduce the many individual data source documents to manageable and meaningful form which communicates pertinent information to decision makers. The requirements for these reports must be based on a detailed, planned concept of how reliability analysis and evaluations are to be performed. From such a plan, the minimum information needed in each report and the report format can be constructed.

c. After establishing the requirements for the data elements and data reports, the method of implementation is selected. Implementation may consist of using an existing Army data collection and reporting system if it meets the requirements, or the procuring agency may elect to develop its own data collection and reporting system if resources are available.

7-4. Reliability data sources. The design engineer is dependent upon data feedback of part performance and failure data from a wide range of applications and use environments if he is to optimize design reliability. Some specific sources of such data follow:

a. MIL-HDBK-217A, Reliability Stress Analysis for Electronic Equipment. This handbook provides a source of parts failure rate data for standard electronic and electromechanical parts. Catastrophic part failure rates observed over wide ranges of electrical and thermal stresses have been analyzed and presented in a form which permits determination of the most likely failure rate for a given set of stresses.

b. The Army Equipment Record System (TAERS). The TAERS system is designed to provide field commanders, commodity command managers, project managers, and top-level headquarters with problem-solving data for improved materiel readiness. It is an official Army method for reporting information necessary for control of operation and maintenance support of Army equipment.

c. Tri-Service and NASA Failure Rate Data (FARADA) Program. The purpose of the FARADA Program is to provide part/component failure rate and failure mode data to reliability engineers and design engineers engaged in the design, development and production of hardware for the entire spectrum of military and space applications. The information presented in the FARADA Handbooks has been obtained from operational experience on military and space equipments from many tri-service and NASA contractors and government agencies. As a result of applying engineering and statistical techniques to failure rate data, the program provides design and reliability engineers with ready access to analyzed, summarized, and

descriptive statistics of failure rates at the component/part levcl. If properly applied, the information will provide a means of numerically assessing the probability of survival (reliability) of an item prior to or simultaneously with the construction of hardware. As experience in the use of this method is gained, refinements can be made, and improved design should result. In detail, the program covers:

(1) Stress analysis: to assist designers in performing quantitative reliability stress analyses by providing operational stress data on parts/components.

(2) Environmental factors: to provide data on various operating modes influencing failure rates and highlighting the critical functional environmental stresses of each mode.

(3) Application factors: to provide data which modify the basic failure rate in order to allow for different applications of the parts/components.

(4) Performance degradation: to provide data on stability or degradation of parts/components under a specific set of application conditions.

The FARADA Program is directed by the Navy and is administered and implemented by the U. S. Naval Fleet Missile Systems Analysis and Evaluation Group (FMSAEG) at Corona, California.

d. Inter-Service Data Exchange Program. (1) IDEP is a tri-service program for the exchange of part test reports to assist system designers in the selection and application of reliable part types. The test data exchanged includes, but is not limited to, that obtained from: qualification or certification tests; production acceptance tests; diagnostic or design and development tests; general or comparative evaluation tests; reliability, exaggerated stress, and life tests; controlled data collection and sampling programs.

(2) The IDEP exchange program does not summarize or edit test reports; instead, the three distribution centers (one for each service) act as clearing houses. Contractor test reports are forwarded to their appropriate service distribution center where they are reproduced and forwarded to other participants in the program.

(3) The Army IDEP contact is IDEP Office, Redstone Scientific Center, U. S. Army Missile Command, Redstone Arsenal, Alabama.

7-5. Reports. a. Reports are the products resulting from the data elements. They summarize for management the status on various system parameters such as: reliability, availability, maintainability, parts usage rates, capability, system effectiveness, etc.

b. Within the scope of official guidelines, the procuring agency has freedom of action for developing its own methods to meet the stated requirements, as well as the restraints imposed on it by resources, etc.

c. For the reliability portion of the program, reliability data files should be established for Army materiel and that the following technical type data should be recorded where appropriate:

(1) Critical design or manufacturing features.

(2) Applicable specifications or standards.

(3) Modes of failure.

(4) Causes of failure.

(5) Stresses at failure.

(6) Methods of detection or test.

(7) Type of failure distributions.

(8) Recommended necessary preventive or corrective action.

(9) Estimate of reliability for various applications.

(10) Prime manufacturer and alternate sources.

d. The above technical type data is to be collected during the development and testing phases. Such items provide the quantitative and engineering data upon which a decision for actions may be based.

e. In addition to technical type data, tactical and operational data accumulation is required and should be organized so as to provide background information for combat development purposes. Specific items of data must be patterned according to the nature of the item. Where appropriate, the data should include information concerning:

(1) Mission reliability with respect to the overall mission assigned to the field unit.

(2) Reliability data for tactically or operationally significant phases of the overall mission.

(3) Data for environmental and operational conditions varying from the normal.

7-6. Selection of data elements for data collection forms. a. In selecting or developing data collection forms for use in a reliability program, the following data elements are suggested:

(1) Using unit.

(2) Equipment identification (aircraft tail number, gun tube number, etc.).

(3) Data of failure.

(4) Identification of failure (part number, subsystem, etc.).

(5) Result of failure (red-X, mission abort, launch hold, item not available, etc.).

(6) Total system time (flight hours, equipment hours, miles, rounds fired, etc.).

(7) Number of previous failures of this type on the equipment.

(8) Time to each previous failure of this type.

(9) Characteristic of the failure (cable jammed, receiver intermittent, heavy vibration, out of specification limits, etc.).

(10) Environment in which the failure occurred (use conditions).

b. It should be kept in mind that the above list of elements is by no means complete. However, these are basic to reliability analysis and status reporting. In order for a reliability program to effectively utilize data source documents, a system must be established to handle the paper flow and reduce it to a compact and comprehensible form. The basic requirement is to determine the minimum needed data elements and then synthesize these into a composite form from which the required reliability analysis and reports may be generated. Obviously, an alternative to using established forms and systems is to develop a data collection system tailored strictly to the project. The latter method is probably the most efficient, relative to a specific project.

Section IV. FEEDBACK CYCLE

7-7. General. a. A basic failure analysis and corrective action feedback loop should determine: what failed; how it failed; why it failed; and when it failed.

b. Failure data provides information to determine the first two factors. The third, essential to corrective action, usually requires information which can be obtained only by laboratory study and/or engineering analysis of the problem areas uncovered by failure analysis.

c. A well planned failure reporting program provides important inputs for reliability improvement. Such a closed loop feedback cycle is illustrated in figure 7-1. Data collection is only one task of several

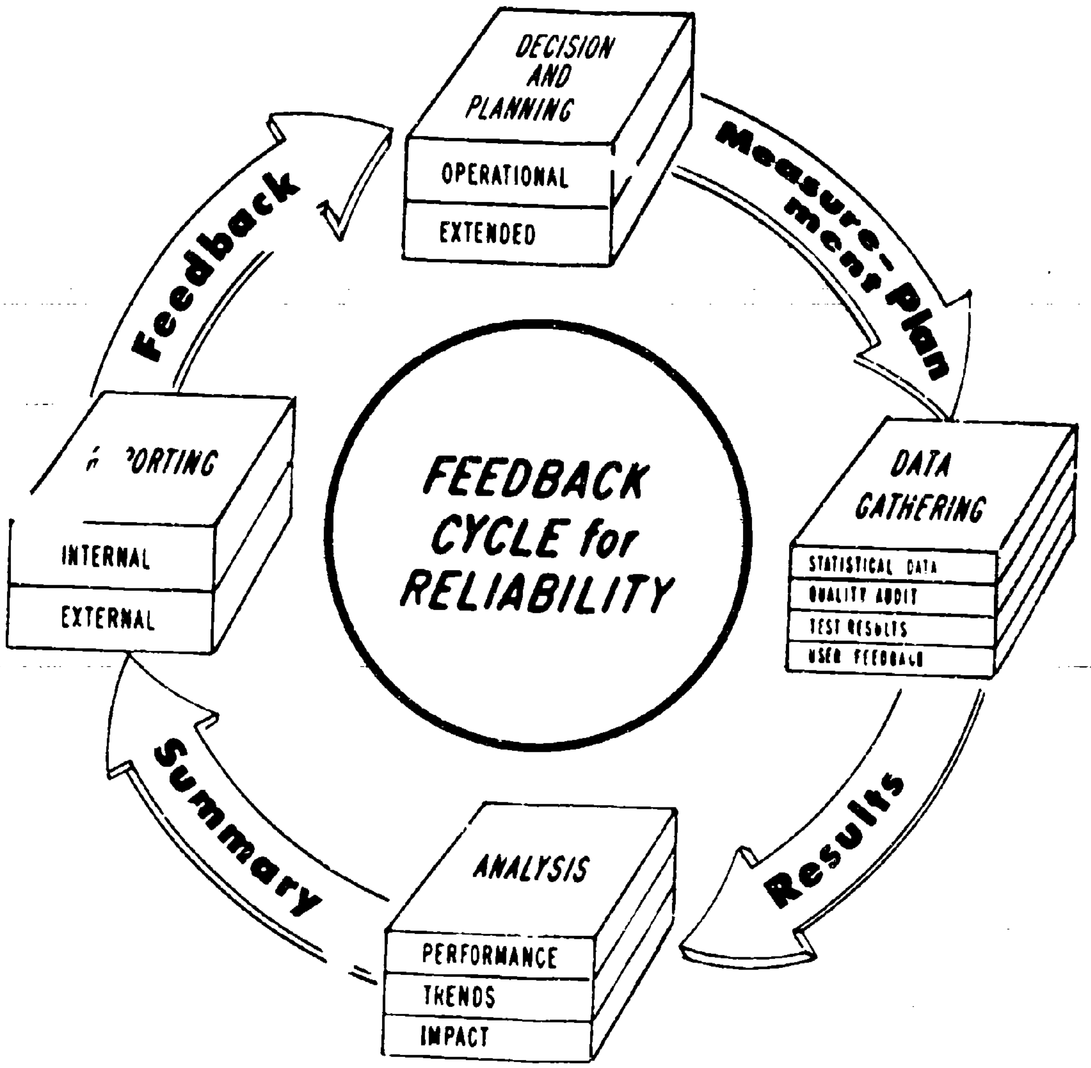


Figure 7-1

Feedback Cycle for Reliability

in a well conceived reliability program plan. Contained herein are some basic requirements of a data collection system, data sources, and data uses. This discussion is concerned with three major phases of an overall program; namely, the design and development phase, the manufacturing or production phase, and the operational or field evaluation phase.

7-8. Design and development phase requirements. a. It is during this phase that system inherent reliability is planned and established. Various test programs are conducted at this time. Examples of these tests are those conducted at the part level, breadboard and prototype assembly and subassembly levels, and many times, at the prototype system level.

b. Some very meaningful reliability data results from initial tests performed in the engineering laboratory under either room ambient or controlled environmental conditions. The collection system should provide for the routine collection of these data, either by completion of failure report forms by test personnel, or by lifting the desired data from the test logs by the reliability personnel, or a combination of both. It is very important that due consideration be given to the total planned test program -- not only those tests that are to be performed during the design and development phase, but for all phases of the overall program as sources for reliability data. It is at the beginning of a proposed program that the reliability engineers should plan and coordinate with other activities for their total data needs and the manner in which these data will be time phased as inputs for use during the performance of the other reliability tasks.

7-9. Manufacturing or production phase requirements. a. As sources of data, the procuring agency reliability personnel should look to the areas and agencies responsible for the preservation of reliability; namely, manufacturing (production), handling, storage, maintenance, and test.

b. Thus, data gathered can be separated into broad categories as quality data and reliability data. Quality data includes records of

inspection and testing; e.g., go no-go tests, measurements of variables such as resistance and capacitance to determine conformance to established technical requirements contained in specifications, drawings, and purchase orders. Reliability data on equipment is developed during preproduction stages in order to detect equipment weaknesses before release to production and to obtain a quantitative estimate of equipment reliability. Reliability data on parts and/or components is developed during the production stages to assure that the equipment inherent reliability is not unduly degraded by manufacturing processes. When the data indicates excessive failure rates, corrective action should follow.

7-10. Operational or field evaluation phase requirements. a. Obtaining timely, accurate and complete reliability data from the field is probably the most difficult. People who are more concerned with getting the equipment to function (their prime mission) may be lax in reporting data (failure and success). An initially well conceived data collection plan which is properly coordinated should reduce data collection to a routine activity.

b. Data sources include operational logs, contractor's report forms, and reports associated with the Army equipment failure reporting system.

c. The types and evaluation of field failure and repair data are much the same as those for other phases of the equipment life cycle. However, greater emphasis is given to operational malpractices and incompatibility between inplant performance specifications and operational specifications. During the operational phase of a given program, the reliability engineer should be exerting a great deal of effort to uncover the causes for equipment and system unreliability by searching out both quantitative and qualitative information pertaining to failures.

Section V. STEPS FOR UTILIZATION OF FAILURE DATA

7-11. Procedural steps. Of the many questions which may be asked of a failure reporting system, and among the most useful when answered, is:

What, within an equipment, contributes most to its unreliability? The following represents a step by step approach for analyzing present failure reports, whether originating in the field, at a test facility, or in a contractor's plant, designed to answer this question.

a. Step 1, Organize the data. Reliability data should be evaluated at planned periods throughout the program and should be tailored to each specific phase. A reliability data center is useful in the realistic assessment of current reliability levels. It is a tool which enables failure data to be used in indicating where design improvement and support is needed and is a necessary storehouse of vital information to be used in engineering, manufacturing, quality, and service activities. Here, the data is generally arranged first by identifiable subassemblies within the subject equipment; then by subsystem or part reference designation. (This step is easily accomplished by machine sorting of data when information is transcribed on punched cards or tape.)

b. Step 2, Frequency analysis. The data center can be interrogated to provide failure data; failure times; accumulated operating time on the system or equipment; and total accumulated number of failures for a selected subsystem, assembly, or component. Continuous updating of reports will provide continuous management visibility of the reliability program. Information may be arranged as to frequency of failure occurrence vs. subsystem for the purpose of identifying those subsystems causing the most trouble. This procedure can then be repeated through descending levels to identify most troublesome assemblies, components, or parts.

c. Step 3, Selection of vital failures. Failure types considered vital should be sought and can be recognized as those failure modes, failure parts, and problem areas to which reliability improvement effort can be profitably applied. Specific components and subsystems which fit into this category are:

(1) Those whose failure markedly affects the safety of the system both in terms of human safety and equipment destruction.

(2) Those which appear in large numbers in the system.

(3) Those whose failure results in high support costs.

(4) Those whose reliability level is relatively low with respect to the rest of the system.

d. Step 4, Problem evaluation. Identification of vital failures within troublesome subsystems, assemblies, etc., is not enough to satisfy the requirement for a sound reliability intelligence system. Corrective action can only be accomplished if the cause can be determined. This is an engineering job consisting of such things as quality audits, laboratory tests, engineering evaluation, etc. It is through these methods, coupled with the routine observation, interview, and data evaluation, that failure causes can be isolated and necessary corrective action initiated.

e. Step 5, Determine corrective action. Corrective action must be carried out with the objective of providing a design change or modification which mitigates the causes of failure. In generating a technique for handling corrective action, prime consideration should be given to a system which would prevent discrepancies from escaping detection, tap the many available sources of data, and be rapid and comprehensive in its closure action. Figure 7-2 is a schematic of a typical corrective action system. All changes, including corrective actions, shall undergo engineering-reliability analysis. This approach is applicable to all phases of the life cycle.

f. Step 6, Implement. Implementation of corrective action involves the developing of a new design or modification of an existing design within a system. Once the problem area has been recognized and defined, the cause identified, and corrective action initiated, there must be a means for implementing this change in the program. This implementation can be accomplished in many ways, such as: procedures, engineering change proposals (ECP), or modification work orders (MWO). A point to remember is that implementation must take place in a timely manner in order for it to be effective. This action complete the cycle for a closed loop feedback system.

g. <u>Step 7, Follow-up</u>. The approach illustrated in the preceding steps will be useful and effective only if changes conceived, tested, and introduced into existing systems or used to develop new systems are evaluated and monitored to assure compliance with the intent. Follow-up should also provide checks to determine whether the problem has been eliminated, and review to see whether new problems have been introduced as a result of the corrective action.

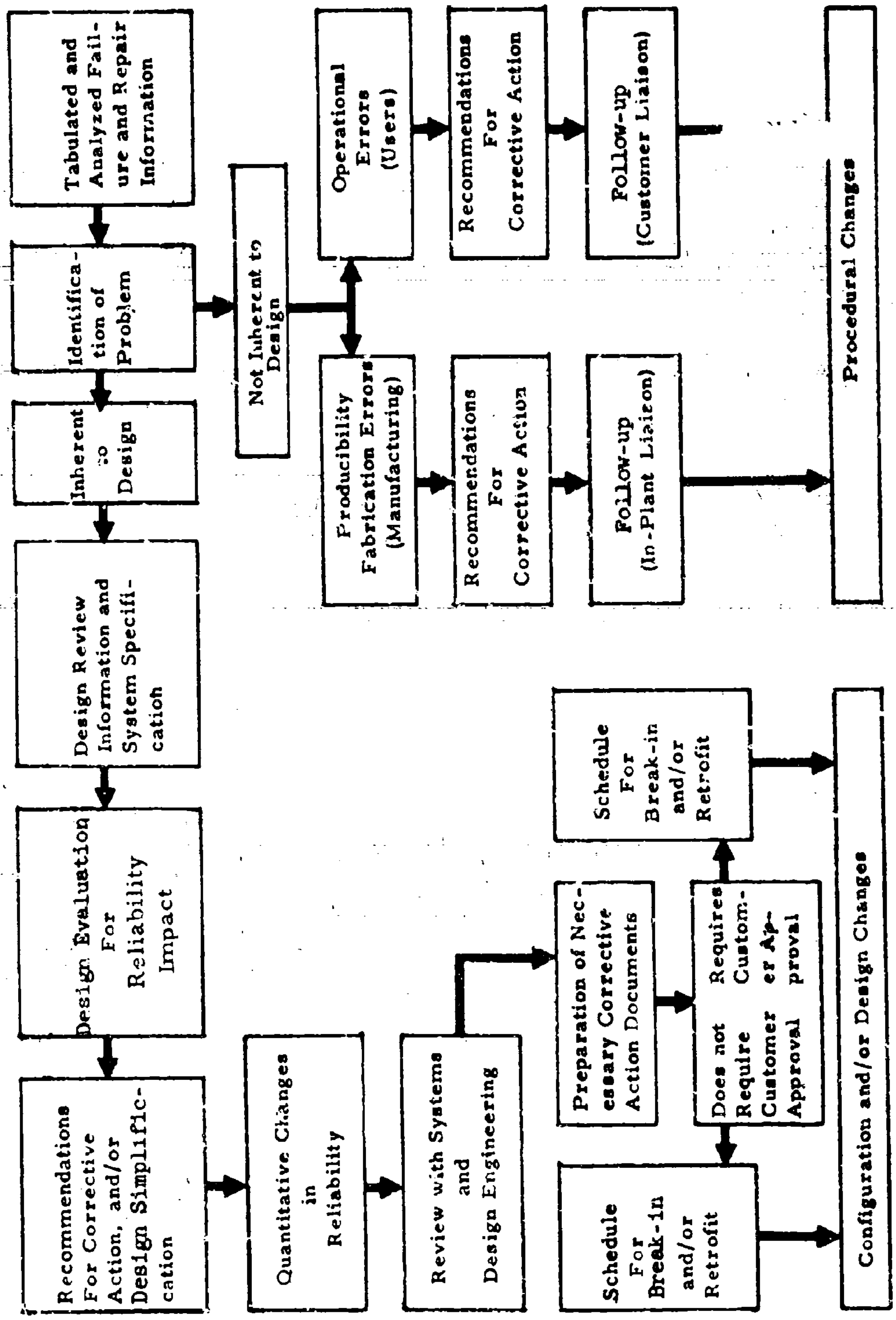


Figure 7-2
Typical Corrective Action Flow Process

APPENDIX A

CONCEPTS OF RELIABILITY QUANTIFICATION

Section I. INTRODUCTION

A-1. General. Quantitative expression and measurement of reliability requires an understanding of the concepts of probability and statistics. Probability serves as a measurement scale by which reliability is expressed and, as such, is a measure of the likelihood or chance that an item will survive a required mission time for a specified intended function and use environment. This appendix reviews a few basic rules, symbols, and concepts necessary for quantification of reliability. The review does not constitute an exhaustive coverage of the necessary materials. Further coverage may be found in various textbooks, professional journals, etc.

Section II. PROBABILITY

A-2. Definition. a. In general, the probability that an event A will happen is the portion of time the event will occur over a large number of trials. When only a single trial is to be encountered, the probability that event A will happen is merely the relative chance of its happening.

b. The statement which follows provides a more formal definition of probability. Given an experiment, if an event may happen in "a" ways and fail to happen in "b" ways, and all of these ways are mutually exclusive and equally likely to occur, the probability of the event happening is

$$\frac{a}{a+b}$$

i.e., the ratio of the number of favorable ways to total number of ways the event can happen. Symbolically, the probability that the event A will happen is expressed: P(A).

c. The numerical expression of probability operates along a dimensionless, continuous scale extending from 0 to 1. If P(A) = 0,

the event A will not happen. If $P(A) = 1$, the event A will always happen. If $P(A) = .5$, the event A would be expected to occur in 50% of a large number of trials. In general, high frequency events will be assigned a probability value near one, and low frequency events will be assigned a probability value near zero.

d. To illustrate the definition of probability, consider an experiment consisting of a single toss of an "honest" die. Find the probability that the upturned face will show an odd number.

$$P(\text{odd number}) = \frac{\text{Total number possible outcomes resulting in odd number}}{\text{Total number possible outcomes}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

This means that in a large number of tosses, about half of the tosses would result in an odd number. The interpretation for a single toss is that there is a 50-50 chance that the outcome would be odd.

A-3. Concept of a set. a. General. A brief investigation of set notation and operation will facilitate the discussion of probability. A set is defined as a collection of objects having certain specified properties. Each object belonging to the set is called an element. The set that contains the totality of all elements that may appear in our investigation is called a space. A space has neither dimension nor volume, but is comprised of a complete set of elements.

b. Definitions of specific sets.

(1) Infinite set. Set having an infinite number of elements.

(2) Finite set. Set having a finite number of elements.

(3) Empty set. Set having no elements.

(4) Subset. Set consisting of several elements of another set. A subset is, of course, a set and the operations on sets will be applicable to subsets.

c. Examples of sets and subsets. The set of all prime numbers is (1, 2, 3, 5, 7, 11, ···). This is an infinite set. The set of all planets is (Earth, Mars, Jupiter, Venus, Pluto, Saturn, Mercury, Neptune, Uranus). This is a finite set with nine elements. The set of all prime numbers less than 11 is (1, 2, 3, 5, 7). This is a finite set and a subset of all prime numbers. The set of all integral quotients greater than one obtained from dividing the prime numbers by 3 will contain no elements, i.e., is the empty set. The set of all elements on the real line between 0 and 1 is an infinite set. This infinite set is called a non-countable infinite set. The infinite set typified by the prime numbers is called a countable infinite set.

d. Operations on sets.

(1) Let a space S be given and consider various sets in S. Let A and B be the subsets of S. This may be written

$$A \subset S, \quad B \subset S$$

which is read: the set A is contained in the set S; the set B is contained in the set S.

(2) The set $A \cup B$ called the union of A and B is the set of all elements belonging to at least one of the sets A and B. To help in visualizing these operations, Venn diagrams will be used for illustration. The rectangle represents the space S, and the circles represent the sets A and B. In figure A-1, the shaded area represents the union of A and B.

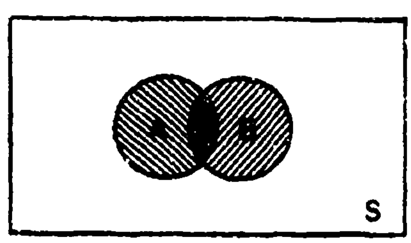


Figure A-1
Union of A and B

(3) The set $A \cap B$ or AB,[1] called the intersection of A and B, is the set of all elements belonging to both the sets A and B. In figure A-2, the shaded area represents the intersection of A and B.

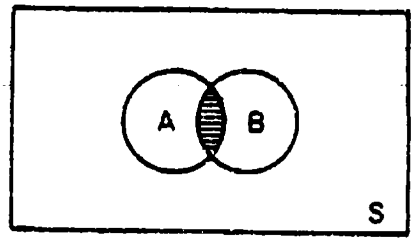


Figure A-2
Intersection of A and B

(4) The difference of A and B, designated by A is the set of all elements that belong to A but not to B. In figure A-3 the shaded area represents the set A-B.

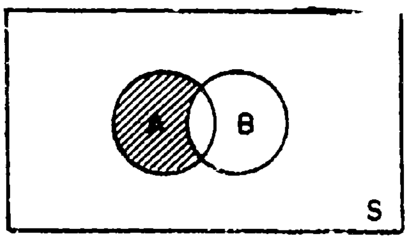


Figure A-3
Difference of A and B

[1] $A \cap B$ and AB are used interchangeably in this document.

(5) The set $\overline{A}$, called the complement of A, is the set of all elements in S that are not contained in A. The complement of A is represented in figure A-4 by the shaded area.

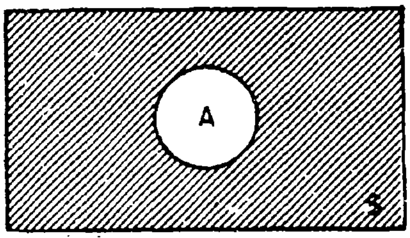


Figure A-4
Complement of A

(6) The following example illustrates the union, intersection, difference, and complement of sets. Let

$S = (2, 3, 4, 8, 9, 11, 17, 20)$

$A = (2, 3, 4, 8)$ $\quad B = (3, 8, 9, 11)$

$A \cup B = (2, 3, 4, 8, 9, 11)$

$A \cap B = (3, 8)$

$A - B = (2, 4)$ $\quad B - A = (9, 11)$

$\overline{A} = (9, 11, 17, 20)$

$\overline{B} = (2, 4, 17, 20)$

A-4. Probability function. a. In connection with a random phenomenon or a real or conceptual experiment, there will be certain possible outcomes. If the experiment is repeated under identical (or more practically, under nearly identical) conditions and the outcomes are recorded, intuition

tells us that relative frequency of the possible outcomes will tend to a fixed value after a large number of repetitions. These considerations lead us to assign a number (weight or measure) to the random outcomes and to talk of the probability of an outcome.

b. Define a sample space S as the set of all possible outcomes of an experiment or random phenomenon, and the probability of an outcome as a rule that assigns a real number to each element of the sample space. A sample space, together with the assignment of probability numbers is called a probability space. An event is defined as a subset of a sample space, i.e., a definite collection of sample points. The event A is said to have occurred on a trial of the experiment if the experiment results in an outcome that is one of the sample points of A. There clearly are many possible events associated with an experiment (i.e., sample space). The aggregate of all subsets of S, plus the unions, differences, intersections and complements of these subsets, are the events associated with an experiment.

c. We define then the probability function as a rule or function that assigns a real number to each element of a set of objects, (i.e., the outcomes of an experiment). The probability of an event A, called P(A), is defined to be the sum of the numbers (or weights) assigned to each of the sample points contained in A. Some basic properties of the probability function follow.

(1) $0 \leq P(A) \leq 1$

(2) $P(A) = 1$ if $A = S$. In other words, if A is the set of all possible outcomes, then the occurrence of one of the elements of A is certain.

(3) $P(A) = 0$ if A is the empty or null set, denoted by ϕ. This implies that the set A contains none of the possible outcomes of the experiment; hence, the occurrence of an element of A is impossible.

(4) $P(\bar{A}) = 1 - P(A)$. This is known as the complementation principle.

(5) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ for every pair of events A, B.

d. In many probability situations when the outcomes of a random phenomenon are finite in number and the outcomes are equally likely, we assign equal probabilities to the possible outcomes. For example, in the experiment of tossing a coin, S = (H, T), the possible outcomes are heads and tails and each is equally likely and has probability 1/2. In general, if

$$S = A_1 \cup A_2 \cup A_3 \cup \cdots \cup A_r \quad \text{where } A_i \cap A_j = \phi$$

and the probability

$$P(A_1) = P(A_2) = \cdots = P(A_r), \text{ then } P(A_i) = 1/r.$$

For any event $E = A_1 \cup A_2 \cup \cdots \cup A_k$ where $k \leq r$ and $A_i \cap A_j = \phi$,

the probability of E is

$$P(E) = P(A_1) + P(A_2) + \cdots + P(A_k)$$

$$= \frac{1}{r} + \frac{1}{r} + \cdots + \frac{1}{r} = \frac{k}{r}$$

Sometimes probability is defined using this concept where k is the number of equally likely ways favorable to the event E and r is the total number of possible outcomes of the experiment or random phenomenon. The probability of the event E is defined as the ratio k/r.

e. To exemplify the assignment of probabilities, consider an experiment consisting of tossing a coin twice. The sample space S is defined as S = (HT, HH, TH, TT), consisting of four outcomes where HT denotes heads on the first toss and tails on the second, and so forth. Since each is equally likely, we might assign the numbers 1/4, 1/4, 1/4, 1/4 to each of these sample points.

(1) Let E = event of a head on the first toss

$$P(E) = P(HT) + P(HH)$$

$$= 1/4 + 1/4 = 2/4$$

(2) Let E_1 = event head on first toss

E_2 = event tails on second toss

E_1 = (HH, HT) $\quad\quad$ $P(E_1) = 2/4$

E_2 = (HT, TT) $\quad\quad$ $P(E_2) = 2/4$

$E_1 E_2$ = (HT) $\quad\quad$ $P(E_1 E_2) = 1/4$

To determine probability of $E_1 \cup E_2$, we sum the probabilities of each of the sample points favorable to event $E_1 \cup E_2$.

$E_1 \cup E_2$ = (HH, HT, TT) and $P(E_1 \cup E_2) = 1/4 + 1/4 + 1/4 = 3/4$;

or, using the relationship,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 E_2) = 2/4 + 2/4 - 1/4 = 3/4.$$

A-5. Independent and dependent events. a. Conditional Probability. Given two events, A and B, the conditional probability of the event B, given the event A, denoted by P(B/A), means the probability that B will occur knowing that the event A has already occurred (or will occur). This probability is defined as

$$P(B/A) = \frac{P(AB)}{P(A)}$$

This definition has intuitive appeal as may be seen from the Venn diagram in figure A-5.

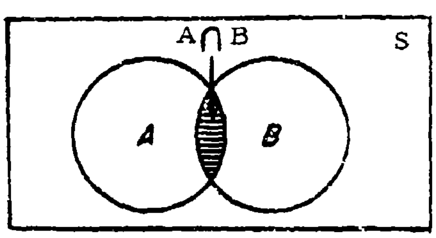


Figure A-5
Illustration of Conditional Probability Relationship

(1) Knowing that the event A has occurred, our attention is turned to the set A. The elements in A favorable to the event B is the set AB, represented by the shaded area. The set A may now be considered in a sense our sample space, and the set AB, the set in A favorable to the event B. Therefore, the probability of the event B/A is given by the ratio of the two probabilities, P(AB) and P(A).

(2) Note that this form also defines the joint probability of the event AB.

$$P(AB) = P(A)P(B/A) = P(B)P(A/B)$$

(3) For example, consider the problem of drawing without replacement samples of size 2 from an urn containing 3 white and 2 red balls. Let A be the event the first ball drawn is white and B the event the second ball drawn is white. Determine P(B/A) the probability the second ball drawn is white given the first ball drawn is white. By definition,

$$P(B/A) = \frac{P(AB)}{P(A)} .$$

Three outcomes are favorable to the event AB, namely (w_1, w_2), (w_2, w_3), (w_1, w_3), where w_i is white ball i. Since there are $\binom{5}{2} = \frac{5!}{2!3!} = 10$ possible outcomes, we assign the value 1/10 to each outcome and, hence P(AB) = 3/10. Relating the event $A = (w_1, w_2, w_3)$ to the 5 possible outcomes of the first draw gives P(A) = 3/5. Thus, P(B/A) = (3/10)/(3/5)=1/2.

b. Independence. Let A and B be events defined on the same probability space. The events A and B are defined to be independent if

$$P(AB) = P(A)P(B)$$

Events that do not satisfy the above relationship are said to be dependent. The concepts of independence and conditional probability may be defined for n events

$$A_1, A_2, \cdots, A_n .$$

The conditional probability of A_n given that the events $A_1, A_2, \cdots, A_{n-1}$ have occurred is given by

$$P(A_n/A_1, A_2, \cdots, A_{n-1}) = \frac{P(A_1, A_2, A_3, \cdots, A_n)}{P(A_1, A_2, \cdots, A_{n-1})}$$

and the n events are mutually independent if

$$P(A_iA_j) = P(A_i)P(A_j)$$

$$P(A_i A_j A_k) = P(A_i)P(A_j)P(A_k)$$

$$P(A_1A_2 \cdots A_n) = P(A_1)P(A_2) \cdots P(A_n)$$

for all combinations $1 \leq i \leq j \leq k \cdots \leq n$.

A-6. Basic rules of probability. Certain basic rules of probability will be useful for reliability analysis activities. Some of these follow.

a. Multiplication. (1) Consider two events A and B with respective probabilities of occurrence of P(A) and P(B). Then the probability of occurrence of both A and B is

$$P(A \cap B) = P(AB) = P(A)P(B/A)$$

$$= P(B)P(A/B)$$

(a) To illustrate the above relationship, consider drawing two cards (without replacement) from a well-shuffled, 52-card deck. What is the probability that both cards will be aces?

(b) Let A be the event of an ace on the first draw and B the event of an ace on the second draw. Then

$$P(A) = \frac{4}{52} \text{ and } P(B/A) = \frac{3}{51}$$

$$P(AB) = P(A)P(B/A) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$$

(2) If the events A and B are independent, the above relationship reduces to

$$P(AB) = P(A \cap B) = P(A)P(B)$$

because

$$P(B/A) = P(B) \text{ and } P(A/B) = P(A).$$

Then A and B are defined as statistically independent events.

(a) To illustrate this special case, consider an electronic assembly consisting of two independent subsystems: A and B, functionally connected in series. Both subsystems must function properly in order for the system to function properly. Suppose that the probability of A working properly = 0.9 and that the probability of B working properly = 0.8. Compute the probability that the system will function properly.

$$P(A) = 0.9 \quad \text{and} \quad P(B) = 0.8$$

(b) The event that the system will work properly is the intersection of A and B.

$$P(AB) = P(A)P(B) = (0.9)(0.8) = 0.72$$

b. Addition. (1) The probability that at least one of two events, A and B, occurs, (i.e., either A or B or both) is

$$P(A \cup B) = P(A) + P(B) - P(AB)$$

where $A \cup B$ is defined as A union B.

(a) For example, a certain operation can be performed by either one of two systems, A and B. Assume that the systems A and B operate completely independently and that the probability of A functioning properly is 0.8 and that the probability of B functioning properly is 0.7. Compute the probability that the operation is performed successfully by at least one of the two systems, A, B. Then P(A) = 0.8 and P(B) = 0.7.

(b) The event that the system will work properly is the union of A and B and

$$P(A \cup B) = P(A) = P(B) - P(AB) = 0.8 + 0.7 - (0.8)(0.7) = 0.94.$$

(2) If A and B are mutually exclusive, i.e., $P(AB) = 0$, the above relationship becomes

$$P(A \cup B) = P(A) + P(B)$$

(3) To illustrate, consider drawing one card from a well-shuffled deck. Find the probability of that card being either a club or a diamond. Let A = the event of a club and B = the event of a diamond.

(b) Then $P(A) = \frac{13}{52}$, $P(B) = \frac{13}{52}$ and $P(AB) = 0$;

i.e., A and B are mutually exclusive.

$$P(A \cup B) = P(A) + P(B) = \frac{13}{52} + \frac{13}{52} = \frac{26}{52}$$

c. Complementation. (1) The last of the probability relationships to be discussed at this time is that of complementation. If P(A) is the probability that the event A will occur, then $P(\overline{A})$ is the probability that the event A will not occur, and

$$P(A) + P(\overline{A}) = 1$$

(2) To illustrate, consider the toss of a single die. Let A be the event of a six appearing and $\overline{A}$ the event of a six not appearing. Then

$$P(A) = \frac{1}{6} \text{ and } P(\overline{A}) = \frac{5}{6} \text{ and } P(A) + P(\overline{A}) = \frac{1}{6} + \frac{5}{6} = 1 .$$

d. Summary of probability rules.

(1) <u>Multiplication of probabilities when events are not independent; conditional probabilities.</u> If E and F are not independent,

(i.e., occurrence of event E affects the probability of the occurrence of event F), then the probability of the joint occurrence of E and F is given by

$$P(E \cap F) = P(E)P(F/E) = P(F)P(E/F)$$

(2) Multiplication of probabilities for independent events. If E and F are independent, (i.e., the occurrence of E does not affect the occurrence of F), then

$$P(E \cap F) = P(E)P(F)$$

(3) Addition of probabilities when events are not mutually exclusive. If E and F are events which are not mutually exclusive, (i.e., events E and F can happen together), then the probability of the occurrence of E or F is given by

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

(4) Addition of probabilities for mutually exclusive events. If two events E and F are mutually exclusive, (i.e., they cannot happen together), then

$$P(E \cup F) = P(E) + P(F)$$

(5) Complementation. Suppose E is an event, then

$$P(\overline{E}) = 1 - P(E).$$

Section III. STATISTICS AND PROBABILITY DISTRIBUTIONS

A-7. General. Statistics has sometimes been defined as the collection, analysis and presentation of numerical data. Numerical expression of reliability requires a basic understanding of certain statistical methods.

A-8. Basic descriptive statistics. a. There exist certain characteristics which may be used to describe a group or population of numerical data. Basic descriptive characteristics to be considered herein are central tendency, variability and shape of the data distribution. Central

tendency has to do with location of the data on the measurement scale. Variability pertains to the dispersion of the data values. Shape has to do with the pattern of data variability. Each of these characteristics has its own effect on reliability measurement.

b. To illustrate the meaning of these descriptive characteristics, consider the following failure times which resulted from a hypothetical life test of 100 items (figure A-6).

24	41	30	37	25	32	28	35	28	51
36	26	43	25	27	39	21	45	39	25
29	43	66	25	24	56	29	31	41	41
36	57	36	48	25	36	48	24	48	22
40	7	31	24	32	53	33	46	22	33
25	36	34	32	41	36	19	32	25	19
19	37	20	21	48	44	35	19	44	34
29	48	38	43	48	35	42	37	35	36
58	45	34	40	37	21	41	11	41	27
50	24	37	39	33	45	39	43	21	34

Figure A-6
Failure Times

c. These data have been grouped to form a frequency table (figure A-7).

Interval	Frequency	Relative Frequency	Cumulative Relative Frequency
4.5 - 9.5	1	.01	.01
9.5 - 14.5	1	.01	.02
14.5 - 19.5	4	.04	.06
19.5 - 24.5	12	.12	.18
24.5 - 29.5	15	.15	.33
29.5 - 34.5	14	.14	.47
34.5 - 39.5	21	.21	.68
39.5 - 44.5	15	.15	.83
44.5 - 49.5	10	.10	.93
49.5 - 54.5	3	.03	.96
54.5 - 59.5	3	.03	.99
59.5 - 64.5	0	0	.99
64.5 - 69.5	1	.01	1.00

Figure A-7
Frequency Table

d. The relative frequency histogram (figure A-8) provides a pictorial approach to describing the population of failure times and the way they are distributed along the measurement (time) scale. With reference to central tendency, the data appears to be clustered about the interval 34.5-39.5 hours. Inspection of the histogram provides a pictorial indication of the amount of variability in the data as well as the shape or pattern of variability.

e. For purposes of making probability statements about failure time, the vertical scale of the relative frequency histogram may be modified in such a manner that the total area of the histogram is unity. In this case, the vertical scale must be divided by 5. We shall refer to the resulting diagram as a relative frequency density histogram (figure A-9). Then the portion of the failure times falling in a particular interval is merely the area of that interval.

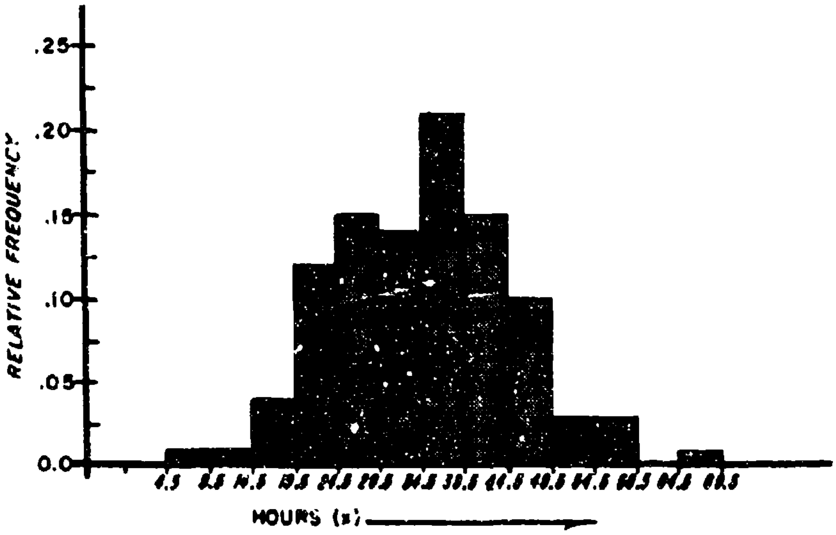


Figure A-8
Relative Frequency Histogram

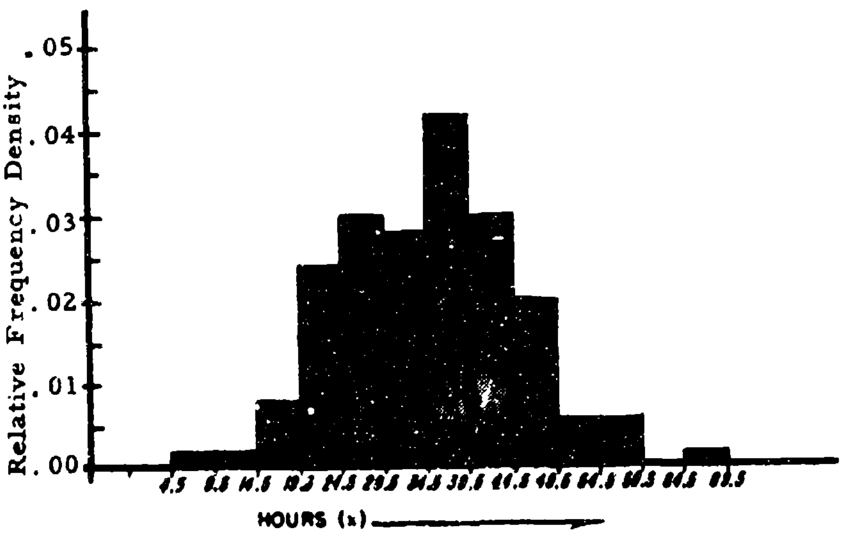


Figure A-9
Relative Frequency Density Histogram

f. Three quantitative measures of central tendency (mean, median and mode) are defined here.

(1) The mode is that value which occurs most frequently.

(2) The median is defined as the middlemost value, i.e., that value above which (and below which) 50% of the observations fall. Finding the median of a group of data involves ordering the observations from smallest to largest and counting to the middle value.

(3) The mean or arithmetic average is the measure of central tendency with which we shall be concerned herein. The mean (μ) is

$$\mu = \frac{\Sigma x}{n}$$

where n is the number of observations; Σx is the sum of the n observations. For the data given in figure A-1, the sum of the 100 observations is $\Sigma x = 3475$. Then the mean of this data is

$$\mu = \frac{\Sigma x}{n} = \frac{3475}{100} = 34.75$$

g. The measure of variability which will be most useful in reliability analysis is the standard deviation (σ).

$$\sigma = \sqrt{\frac{n\Sigma x^2 - (\Sigma x)^2}{n^2}}$$

where

n is the number of observations

Σx is the sum of the observation values

Σx^2 is the sum of squared observation values

for the data in figure A-1, $\Sigma x^2 = 131801$ and the standard deviation becomes

$$\sigma = \sqrt{\frac{100(131801) - (3475)^2}{100(100)}} = 10.51$$

h. The shape characteristic is not easily quantified. The density histogram (figure A-9) provided a visual description of the shape of data. Sometimes a mathematical equation can be identified to serve as a model for the shape or pattern of variability for a particular group of data. For example, the normal density function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

is often useful as a model. Figure A-10 shows this function (solid line) plotted along with the density histogram for the data in figure A-1. μ and σ values were 34.75 and 10.51 hours, respectively, as found in previous calculations. It seems that the normal probability density function provides a good model describing the distribution of data values along the time scale for this particular group of data.

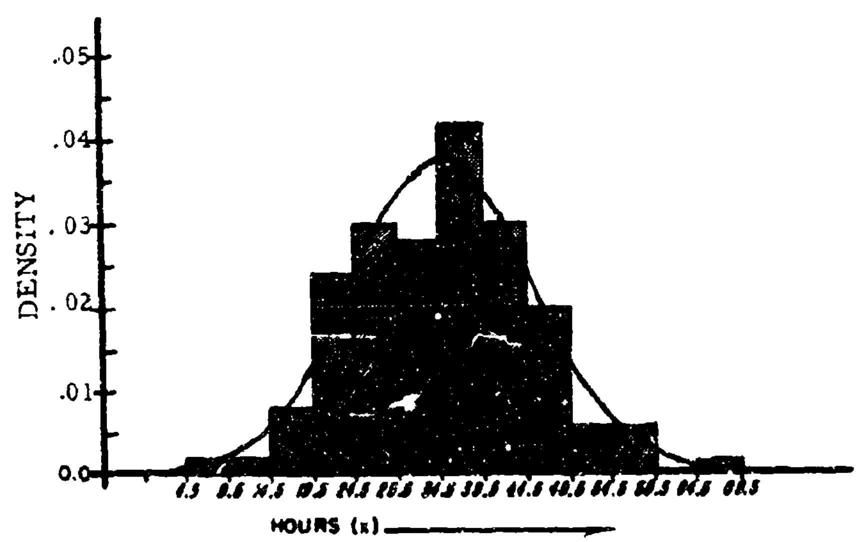


Figure A-10
Normal Density Function vs.
Relative Frequency Density Histogram

i. In general, a probability density function, denoted as f(x), serves as a model relating the outcomes of a random variable (X) to probability statements. At this time, for purposes of illustration, consider the random variable X to be the failure time for an item. For example, $P(A \leq X \leq B)$ is merely the area under the density function between the points A and B as illustrated by the shaded area in figure A-11.

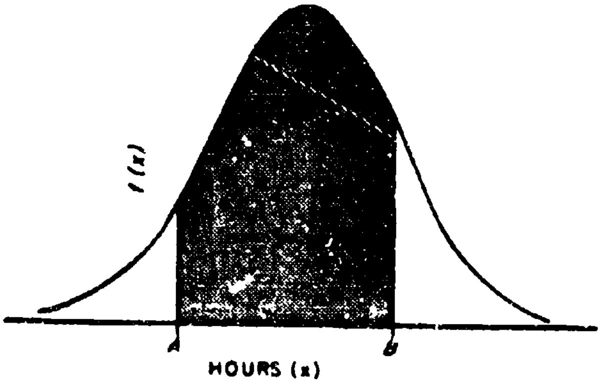


Figure A-11
Probability of a Failure Time Between A and B

j. Another function of interest in reliability analysis is the distribution function F(x) where

$$F(x) = P(X \leq x)$$

In other words, F(x) is the probability that a failure time will be less than a specified time x and is represented as the area under the density function for values less than x on the horizontal scale (figure A-12). When evaluated for all x, F(x) for the normal model fitting the data in figure A-1 becomes as shown in figure A-13 (solid line).

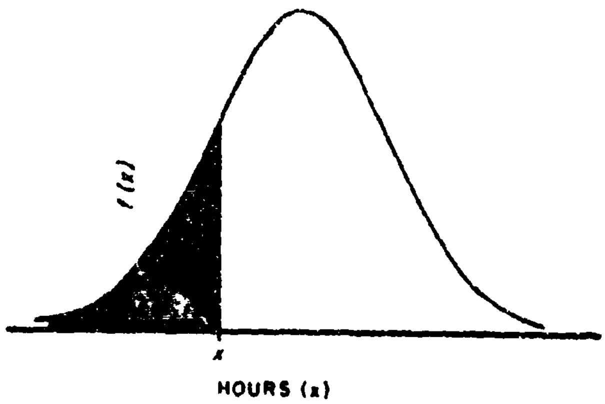


Figure A-12
Probability of a Failure Time Less Than or Equal to X

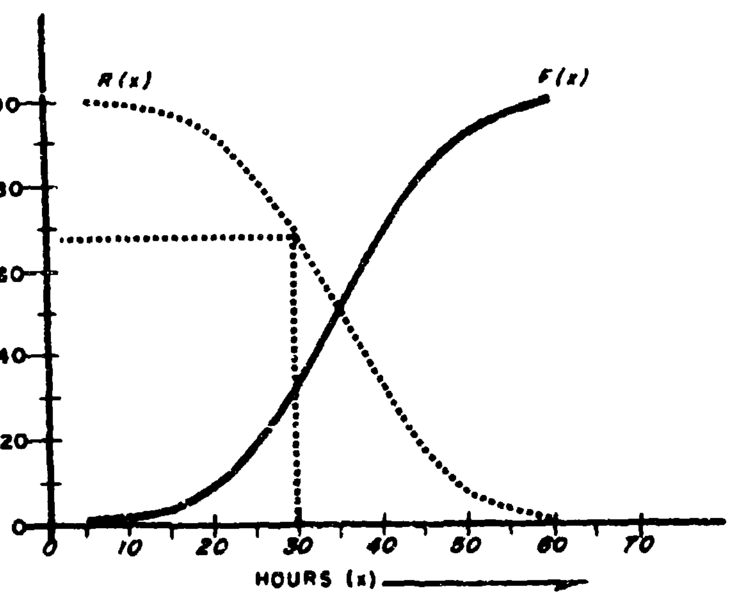


Figure A-13
Reliability and Distribution Functions

k. The reliability function is defined as the probability that an item will survive beyond x hours mission time.

$$R(x) = P(X > x) = 1 - F(x)$$

The reliability for x hours mission time may be represented as the unshaded area in figure A-12. Using the normal probability density function to describe the data in figure A-1, the reliability function is drawn as a broken line in figure A-13. To illustrate, $R(30) = 0.68$.

l. The other function to be defined at this time is the hazard function, $h(x)$, sometimes referred to as instantaneous failure rate. It can be shown that the hazard function, designated as $h(x)$, is

$$h(x) = \frac{f(x)}{R(x)}$$

failures per unit time.

A-9. Probability distributions. a. Knowledge of the distribution of failure times for a population of items provides a basis for reliability analysis. The preceding graphical illustrations pertained primarily to a particular group of failure time data. At this time, certain typical probability density functions and the related reliability functions will be summarized with appropriate mathematical notation.

b. Probability density functions describe the variability and behavior of random variables. Each random variable has its own probability distribution. A random variable may be defined as a rule for assigning a numerical value to the outcome of a random experiment. Some examples of random variables are height of an individual, sum of the upturned faces resulting from the toss of two dice, the number of aces in a poker hand, the time to failure of a piece of equipment, etc. The data in figure A-1 represents 100 observations of failure time, a continuous random variable.

(1) Consider the continuous random variable X which has a probability density function $f(x)$. The density function has the properties

that

$$f(x) \geq 0$$

and

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

(2) The density function serves as a model for probability statements about the random variable; e.g.,

$$P(A < X < B) = \int_{A}^{B} f(x)dx$$

and

$$P(x < X < x + dx) = f(x)dx$$

where

$$dx \longrightarrow 0$$

(3) The central tendency of X may be measured by the mean or expected value, E(X), of the random variable.

$$E(X) = \int_{-\infty}^{\infty} xf(x)dx$$

(4) The variance, V(X), of the random variable measures its variability and is defined as

$$V(X) = E(X^2) - \left[E(X) \right]^2$$

c. The distribution function, F(x), which in reliability analysis is often referred to as unreliability for x hours mission time is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)dt$$

and has these properties:

(1) $F(x) \geq 0$

(2) $F(x)$ is a non-decreasing function

(3) $F(-\infty) = 0$

(4) $F(\infty) = 1$

d. For a random variable, X, which represents failure times, the reliability function becomes

$$R(x) = 1 - F(x) = \int_x^\infty f(t)dt.$$

e. The hazard function, h(x), sometimes referred to as instantaneous failure rate, of a probability distribution of times to failure is often used in reliability considerations. It is defined as the conditional probability density function of time to failure, given the item has not failed prior to time x. In other words,

$$h(x)dx = P\left[(x < X \leq x + dx) \mid (X > x)\right]$$

which reduces to

$$h(x) = \frac{f(x)}{R(x)}$$

f. Figure A-11 provides a summary of the above relationships for continuous random variables.

Density Function:	$f(x)$
Distribution Function:	$F(x) = \int_{-\infty}^{x} f(t)dt$
Reliability Function:	$R(x) = 1 - F(x) = \int_{x}^{\infty} f(t)dt$
Hazard Function:	$h(x) = \frac{f(x)}{R(x)}$
Expected Value:	$E(X) = \int_{-\infty}^{\infty} xf(x)dx$
Variance:	$V(X) = E(X^2) - [E(X)]^2$

Figure A-14
Summary of Reliability Related Functions

A-10. Binomial distribution. a. The binomial distribution provides a model often useful in probability computations. It differs from the previously discussed distributions in that it operates on a discrete scale.

b. In reliability analysis, we are frequently interested in the total number of failures in a sequence of n Bernoulli trials. Bernoulli trials are defined as repeated independent trials of an experiment if there are only two possible outcomes of each trial, classified as success or failure, and the probability of failure remains constant for each and every trial. For purposes of reliability analysis, subjection of n identical items to identical use conditions may be identified as n Bernoulli trials.

c. Let the random variable, K, be the number of failures in n trials. Then the probability density function, or $P(K=k)$, is

$$f(k) = \binom{n}{k} p^k q^{n-k}, \quad k = 0, 1, \cdots, n$$

$$f(k) = 0, \text{ otherwise}$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

p = probability of failure on a single trial

q = probability of success on a single trial

$p + q = 1$

Figure A-15 shows graphically a binomial probability density function for the parameters $n = 8$, $p = 0.7$, and $q = 0.3$.

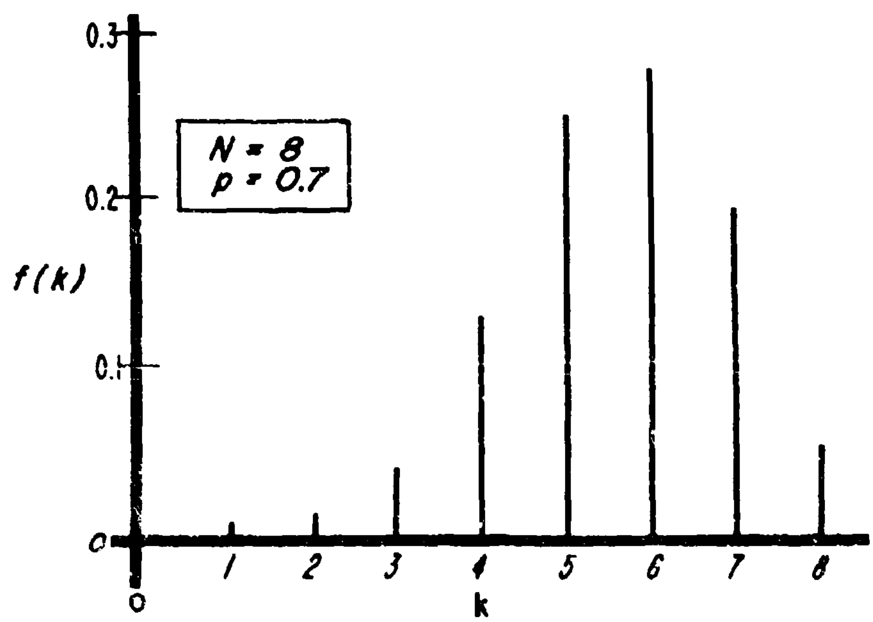


Figure A-15
Binomial Probability Density Function, f(k)

d. For the binomial distribution, the probability distribution function is defined as the probability of k or fewer failures in n identical (Bernoulli) trials, i.e.,

$$F(k) = P(K \leq k) = \sum_{i=0}^{k} \binom{n}{i} p^i q^{n-i}$$

A typical binomial distribution model is shown in figure A-16 for parameters n = 8, p = 0.7, and q = 0.3.

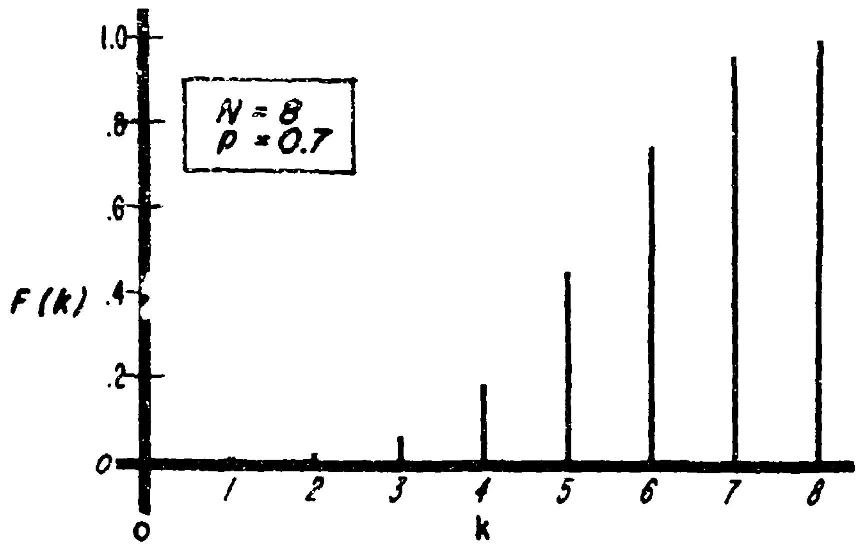


Figure A-16
Binomial Probability Distribution Function, F

e. To exemplify the use of the binomial distribution, consider a particular type of electrical fuse which has a probability of 0.1 of failing to perform properly in a circuit. If five such fuses are subjected to the circuit, what is the probability of 0 failures, 1 failure, and more than 1 failure?

$n = 5,\ p = 0.1,\ q = 0.9$

$$P(k) = \binom{n}{k} p^k q^{n-k}$$

$$P(0) = \binom{5}{0} (0.1)^0 (0.9)^5 = 0.59049$$

$$P(1) = \binom{5}{1} (0.1)^1 (0.9)^4 = 5(0.1)(0.6561) = 0.32805$$

$$P(K > 1) = 1-0.59049-0.32805 = 0.08146$$

f. To illustrate a typical reliability model based on the binomial distribution, consider a regulator on an oxygen system for high altitude flying equipment which has a probability of 0.025 of failing to provide the required oxygen flow. If four such oxygen systems are used on a mission which requires that at least three must function properly, what is the reliability of the four oxygen systems?

$n = 4$

$p = 0.025$ = probability of failure by an oxygen system

$q = 0.975$ = probability of success by an oxygen system

$k = 1$ = allowable number of failures

$$F(k=1) = P(K \leq 1) = \sum_{i=0}^{1} \binom{4}{i} (0.025)^i (0.975)^{4-i} = 0.997$$

g. The binomial distribution also is applicable to computations of reliability relative to one shot items. For example, it has been observed that a bomb fuze has a probability of 0.2 of failing to perform properly. Consider a mission involving the use of seven bombs where the mission is considered successful if at least five bombs perform properly. What is the reliability of such a mission?

n = 7

p = 0.2 = probability of failure by any one bomb

q = 0.8 = probability of success by any one bomb

k = 2 = number of allowable failures

$$F(k=2) = P(K \le 2) = \sum_{i=0}^{2} \binom{7}{i} (0.2)^i (0.8)^{7-i} = 0.852$$

A-11. Normal distribution. a. The normal distribution is sometimes useful as a model for failure times, particularly when failures are occurring because of wearout. A normal distribution of failure times is continuous and has an increasing failure rate. This model is often useful when mission times are such that failures are due to wearout. Defined below are the normal probability density function and related reliability functions.

b. The probability density function for the normally distributed random variable X is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right]$$

where $-\infty < x < \infty$, and the parameters μ and σ are the mean and standard deviation, respectively. (σ^2 is referred to as the variance.) Figure A-17 shows graphically the normal probability density function with parameters $\mu = 1$ and $\sigma^2 = 0.25$.

c. Probability tables (table H-2, appendix H) are available for the standard normal distribution. Since any normal random variable X can be transformed to the standard normal random variable Z, the tables may be applied to any normal distribution.

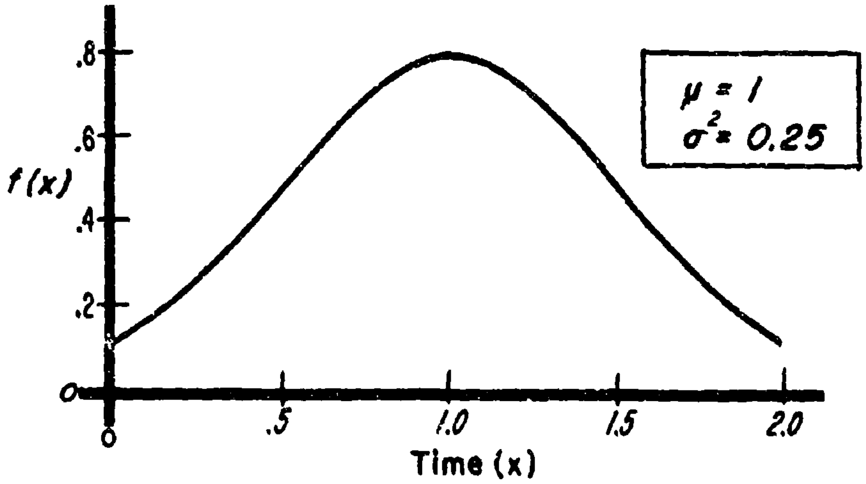


Figure A-17
Normal Probability Density Function, f(x)

(1) The transformation is

$$Z = \frac{X - \mu}{\sigma}$$

(2) The expected value and variance of the normal random variable X is:

$$E(X) = \mu$$

$$V(X) = \sigma^2$$

(3) The expected value and variance of the standard normal variable Z is:

$$E(Z) = 0$$

$$V(Z) = 1$$

d. The density function for the standard normal variable is

$$f(z) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-z^2}{2}\right)$$

where $-\infty < z < \infty$.

e. The distribution function of X may be expressed as a function of Z.

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{t-\mu}{\sigma}\right)^2\right] dt$$

$$= P\left[Z \leq \left(z = \frac{x-\mu}{\sigma}\right)\right]$$

$$= \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-t^2}{2}\right) dt$$

where

$$z = \frac{x-\mu}{\sigma}.$$

F(z) values can be obtained from table H-2, appendix H.

f. The reliability function of the normal random variable X is

$$R(x) = 1-F(x) = P(X > x) = P\left[Z > \left(\frac{x-\mu}{\sigma} = z\right)\right]$$

$$= 1-F(z)$$

The normal reliability function is illustrated in figure A-18 for parameters $\mu = 1$ and $\sigma^2 = 0.25$

g. The hazard function for the normal distribution of failure times may be found by

$$h(x) = \frac{f(x)}{R(x)}$$

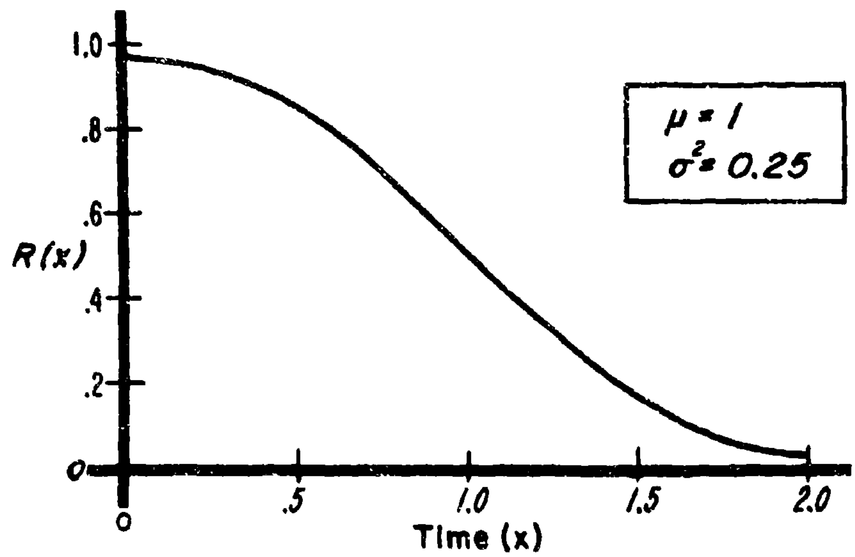


Figure A-18
Normal Distribution Reliability Function, R(x)

where

$$f(x) = \frac{f(z)}{\sigma}$$

and f(z) may be found from table H-9, appendix H. Figure A-19 contains a plot of the normal hazard function for parameters $\mu = 1$, $\sigma^2 = 0.25$.

h. To illustrate the use of the normal probability density function as a reliability model, consider a model 555 rifle which has demonstrated a normal distribution of failure times with $\mu = 100$ hours and $\sigma = 10$ hours. Find the reliability of such a rifle for a mission time of 104 hours and the hazard rate of one of these rifles at age 105 hours.

$$R(x) = P\left(Z > \frac{x-\mu}{\sigma}\right)$$

$$R(104) = P\left(Z > \frac{104-100}{10}\right) = P(Z > 0.40) = 0.34 \text{ as found in table H-2.}$$

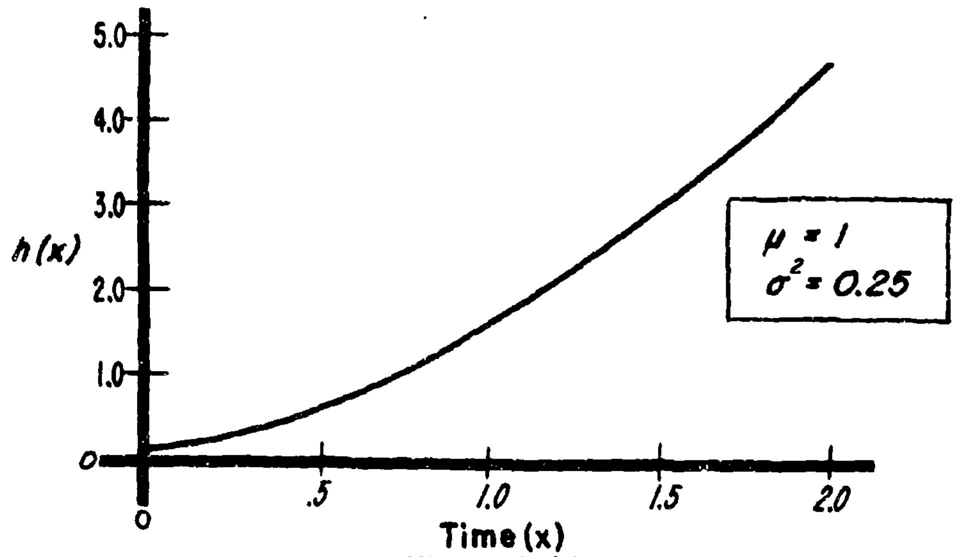


Figure A-19
Normal Distribution Hazard Function, h(x)

$$h(x) = \frac{f(x)}{R(x)}$$

$$f(x) = \frac{f(z)}{\sigma}$$

$$f(x = 105) = 0.10\, f\left(z = \frac{105-100}{10}\right) = 0.10\, f(z = 0.5) =$$

$$0.10(0.35) = 0.035$$

where $f(z = 0.5)$ was found from table H-9.

$$R(105) = P\left(Z > \frac{105-100}{10}\right) = P(Z > 0.50) = 0.31$$

$$h(105) = \frac{f(105)}{R(105)} = \frac{0.035}{0.31} = 0.11 \text{ failures per hour}$$

i. The example which follows pertains to an electronic item. Failure times of a Type GLN microwave tube have been observed to follow a normal distribution with μ = 5000 hours and σ = 1500 hours. Find the reliability of such a tube for a mission time of 4100 hours and the hazard rate of one of these tubes at age 4400 hours.

$$R(x) = P\left(Z > \frac{x-\mu}{\sigma}\right)$$

$$R(4100) = P\left(Z > \frac{4100-5000}{1500}\right) = P(Z > -0.6) = 1-P(Z > 0.6)$$

$$= 0.73$$

as found from table H-2.

$$h(x) = \frac{f(x)}{R(x)}$$

$$f(x) = \frac{f(z)}{\sigma}$$

$$f(x = 4400) = \left(\frac{1}{1500}\right) f\left(z = \frac{4400-5000}{1500}\right) = \left(\frac{1}{1500}\right) f(z=0.4)$$

$$= 0.00067)(0.37) = 0.00025$$

where $f(z = 0.4)$ was found from table H-9.

$$R(4400) = P\left(Z > \frac{4400-5000}{1500}\right) = P(Z > 0.4) = 0.66$$

$$h(4400) = \frac{f(4400)}{R(4400)} = \frac{0.00025}{0.66} = 0.00038 \text{ failures per hour.}$$

A-12. <u>Lognormal Distribution.</u> a. Another model which is sometimes useful as a failure model is the lognormal distribution. It is summarized at this time because of its relationship to the normal distribution. Consider

the random variable X as failure time. If ln X is a normally distributed random variable, X is said to be distributed in accordance with a log-normal distribution. A summary of this distribution follows.

b. The density function is

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right], \quad x > 0$$

$$f(x) = 0, \quad x \leq 0.$$

Figure A-20 shows this density function for $\mu = 2$ and $\sigma = 0.5$. The expectations are

$$E(X) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

$$V(X) = \left[\exp\left(2\mu + \sigma^2\right)\right]\left[\exp\left(\sigma^2\right) - 1\right]$$

where

$$\mu = E(\ln X)$$

$$\sigma^2 = V(\ln X)$$

c. The distribution function is

$$F(x) = P(X \leq x) = P\left(Z \leq \frac{\ln x - \mu}{\sigma}\right)$$

d. The reliability function is

$$R(x) = P(X > x) = P\left(Z > \frac{\ln x - \mu}{\sigma}\right)$$

as shown in figure A-21 for $\mu = 2$ and $\sigma = 0.5$.

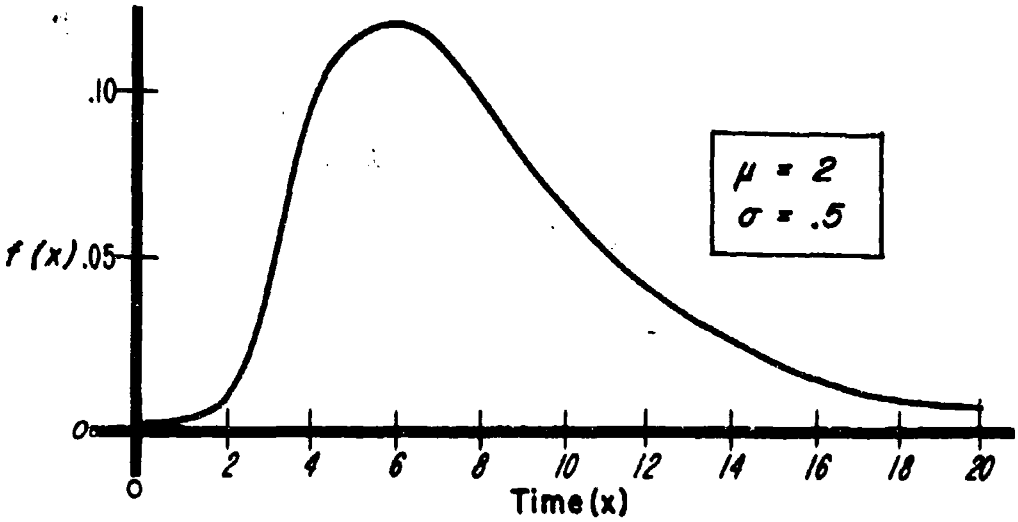


Figure A-20
Lognormal Probability Density Function, f(x)

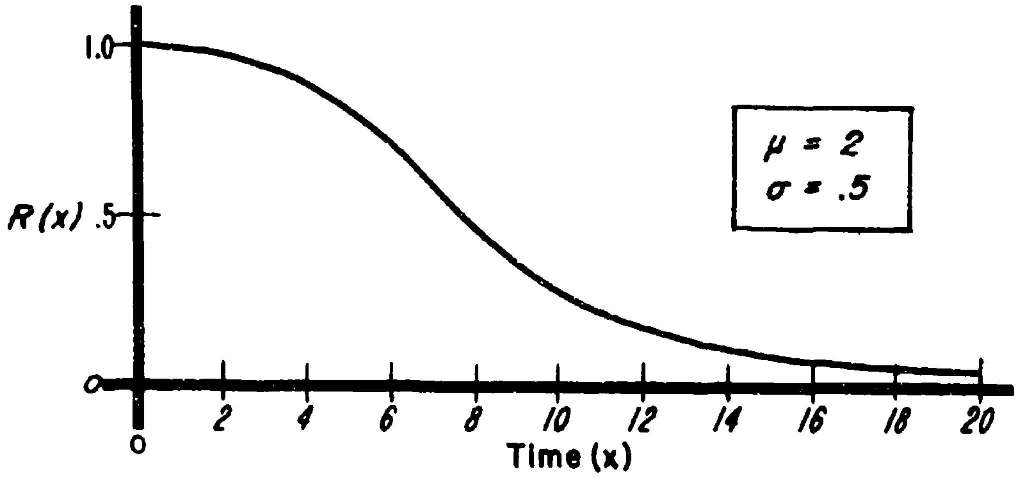


Figure A-21
Lognormal Distribution Reliability Function, R(x)

e. The hazard function is

$$h(x) = \frac{f(x)}{R(x)} = \frac{f(z)}{\sigma x R(x)} \quad \text{where } z = \frac{\ln x - \mu}{\sigma}$$

as shown by figure A-22 for $\mu = 2$ and $\sigma = 0.5$.

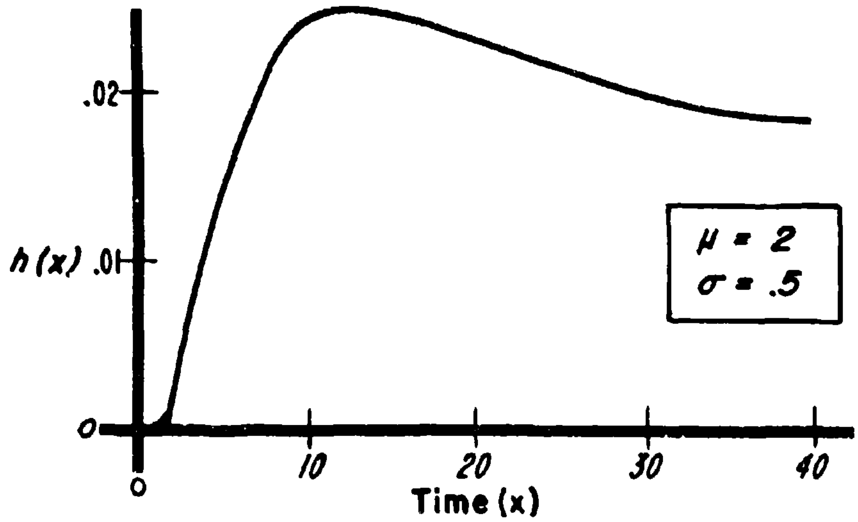


Figure A-22
Lognormal Hazard Function, h(x)

f. To exemplify the lognormal distribution as a reliability model, consider a voltage regulator which has a lognormal distribution with $\mu = 6.8$ and $\sigma = 1$. Find the reliability for a 200-hour mission and the hazard rate at 200 hours.

$$R(x) = P\left(Z > \frac{\ln x - \mu}{\sigma}\right)$$

$$R(200) = P\left(Z > \frac{\ln 200 - 6.8}{1}\right)$$

$$= P(Z > -1.50) = 0.93$$

$$h(x) = \frac{f(x)}{R(x)} = \frac{f(z)}{\sigma x R(x)}$$

$$h(200) = \frac{f(200)}{R(200)} = \frac{0.0006475}{0.93} = 0.00070 \text{ failures per hour.}$$

g. The following example relates the lognormal reliability model to a mission length expressed in units other than time. Suppose it has been observed that gun tube failures occur according to lognormal distribution due to metal fatigue with parameters $\mu = 7.0$ and $\sigma = 2.0$. Find the reliability for a 1000-round mission and the hazard rate at 800 rounds.

$$R(x) = P\left(Z > \frac{\ln x - \mu}{\sigma}\right)$$

$$R(1000) = P\left(Z > \frac{\ln 1000 - 7.0}{2.0}\right)$$

$$= P(Z > -0.045) = 0.52$$

$$h(x) = \frac{f(x)}{R(x)}$$

$$h(800) = \frac{f(800)}{R(800)} = \frac{f\left(z = \frac{6.68-7}{2}\right)}{2(800)\, P\left(Z > \frac{6.68-7}{2}\right)}$$

$$= \frac{0.3939}{2(800)(0.5636)} = 0.0004 \text{ failures per round.}$$

A-13. Poisson distribution. a. A brief investigation of the Poisson process will provide an intuitive basis for evaluating the usefulness of the Poisson distribution as a reliability model.

b. Consider a probabilistic (stochastic) process which is subject to the occurrence of events, all of which are of the same kind, and we are interested in the number of events that occur. Each event occurrence may be represented as a point on a time scale. For purposes of reliability analysis, an event will be defined as a failure. Such a process having the following characteristics is called a Poisson process.

(1) The probability that a given number of failures is contained in a time interval depends only on the length of the interval, (and not on where the interval is located or on the past history of the system).

(2) If P(h) is the probability of 2 or more failures in an interval of length h, then

$$\lim_{h \to 0} \frac{P(h)}{h} = 0.$$

Essentially, this implies that failures do not occur simultaneously.

(3) If $P_1(h)$ is the probability of 1 failure in an interval of length h, then

$$\lim_{h \to 0} \frac{P_1(h)}{h} = \lambda.$$

Essentially, this implies that failure rate does not depend on item age; i.e. failure rate is constant.

c. If these properties are satisfied, the Poisson probability density function may be used as a model for the number of failures in a time interval of length, x. If the random variable K is the number of failures in a time interval of length x, the Poisson density function

is

$$f(k) = P(K = k) = \frac{(\lambda\ x)^k \exp(-\lambda\ x)}{k!},\ k = 0,\ 1,\ 2,\ \cdots,$$

$$= 0,\ \text{otherwise}$$

where

λ = constant failure rate

x = time interval considered.

Figure A-23 portrays graphically the Poisson density function for parameter, $\lambda\ x = 4$.

(1) The expected value of K is $E(K) = \lambda\ x$

(2) The variance of K is $V(K) = \lambda\ x$.

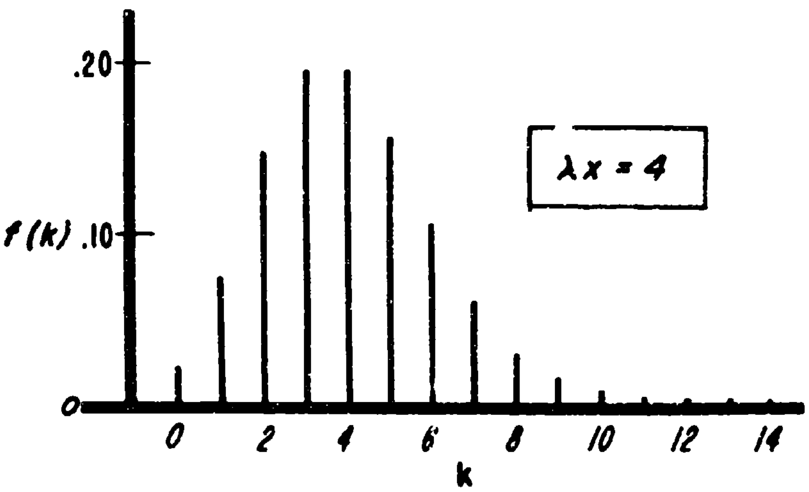


Figure A-23
Poisson Density Function, f(k)

d. The probability distribution function is defined herein as

$$F(k) = P(K \leq k) = \sum_{i=0}^{k} \frac{(\lambda x)^i \exp - \lambda x}{i!}$$

which is graphed in figure A-24 for parameter $\lambda x = 4$. This function may be graphically evaluated using figure H-12, appendix H.

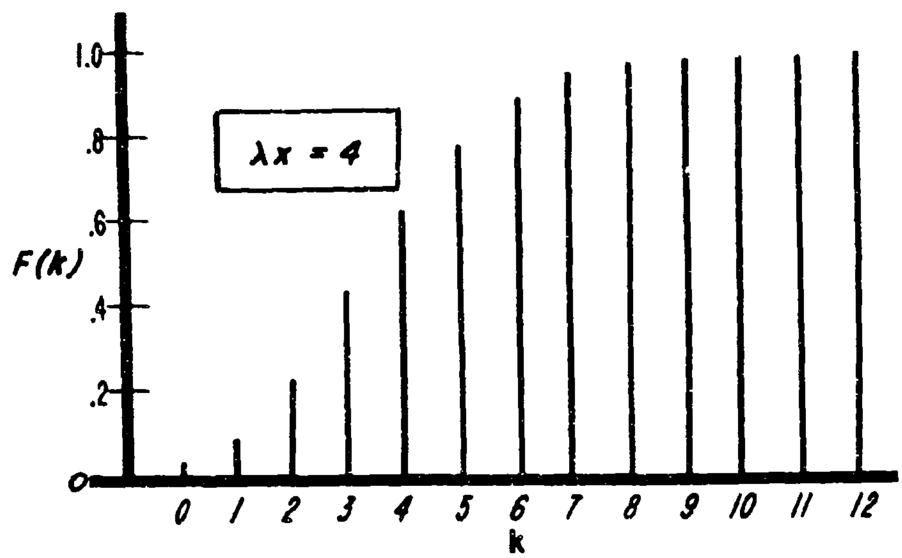


Figure A-24
Poisson Probability Distribution Function, F

e. To exemplify use of the Poisson distribution, consider the following example. A Minuteman launch console averages 0.001 lamp failures per hour. What is the reliability for a 500-hour mission if no more than 2 failures can be tolerated?

$k = 2$ failures

$x = 500$ hours

$\lambda = 0.001$ failures per hour

$\lambda x = 0.50$

$$F(k = 2) = P(K \leq 2) = \sum_{i=0}^{2} \frac{(0.50)^i \exp - 0.50}{i!} = 0.986$$

f. A second Poisson example considers failure of a mechanical item. During the first year of operation, a 1/4-ton truck experiences failures in the drive train due to defects and workmanship. Failures occur in accordance with a Poisson process with a mean time between failures of 400 hours. What is the reliability of such a truck if no failures are allowed for a mission of 40 hours?

$x = 40$

$k = 0$

$\lambda = 1/400 = 0.0025$ failures per hour

$$P(K=k) = \frac{(\lambda x)^k \exp(-\lambda x)}{k!}$$

$$F(k=0) = P(K=0) = \frac{[(0.0025)(40)]^0 \exp[(-0.0025)(40)]}{0!}$$

$$= \exp(-0.10) = 0.90$$

g. The following example illustrates a use for the Poisson distribution when x is not a time interval. The number of rocket-bomb hits within a specified small portion of a comparatively large area under prolonged bombardment has been observed to follow a Poisson distribution. The rocket-bombs average 0.02 target misses per bomb. What is the reliability of a 50-shot bombardment if no more than 2 misses are allowed?

$k = 2$ failures

$x = 50$ rounds

$\lambda = 0.02$ failures per round

$\lambda x = 1.00$

$$F(k=2) = P(K \leq 2) = \sum_{i=0}^{2} \frac{(1.00)^i \exp -1.00}{i!} = 0.920$$

A-14. Exponential distribution. a. The exponential distribution is a popular model for failure times. Some particular applications of this model include:

(1) Items whose failure rate does not change with age.

(2) Complex items which do not include excessive redundancy of components and/or subsystems.

(3) Items for which early failures have been eliminated, e. g., vacuum tubes which have survived a burn-in period.

b. The exponential density function may be obtained directly from the Poisson density function. Consider the continuous random variable X as the time to failure (or time between failures). Then

$$f(x) = \lambda \exp(-\lambda x), \; x \geq 0$$

$$f(x) = 0, \; x < 0$$

c. The resulting distribution is called an exponential distribution. It describes the random variable denoting the time to first occurrence in a Poisson process. Since the Poisson process is temporally homogeneous, the time between successive occurrences (failures) has the same distribution. Figure A-25 portrays graphically the exponential distribution where the x scale is expressed in multiples of the mean time to failure (θ). The expected value and variance of X are:

$$E(X) = \int_0^{\infty} x\lambda \exp(-\lambda x)\, dx = \frac{1}{\lambda} = \theta$$

where θ is mean time to failure and may be used in lieu of $\frac{1}{\lambda}$ in these expressions.

$$V(X) = E(X^2) - [E(X)]^2 = \left(\frac{1}{\lambda}\right)^2 = \theta^2$$

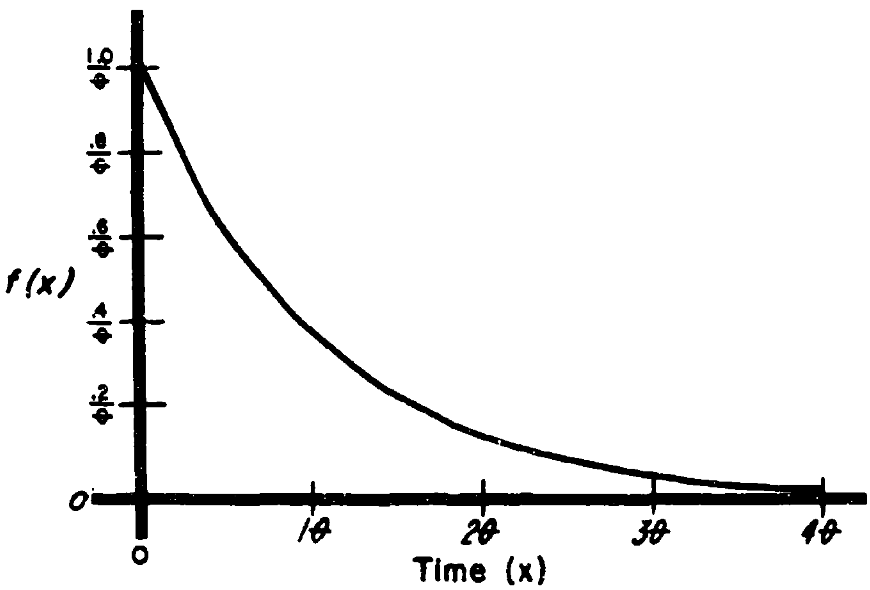


Figure A-25
Exponential Probability Density Function, f(x)

d. The reliability function for exponential failure times becomes:

$$R(x) = \exp(-\lambda x)$$

and is expressed graphically in figure A-26.

e. The distribution function is

$$F(x) = 1 - \exp(-\lambda x)$$

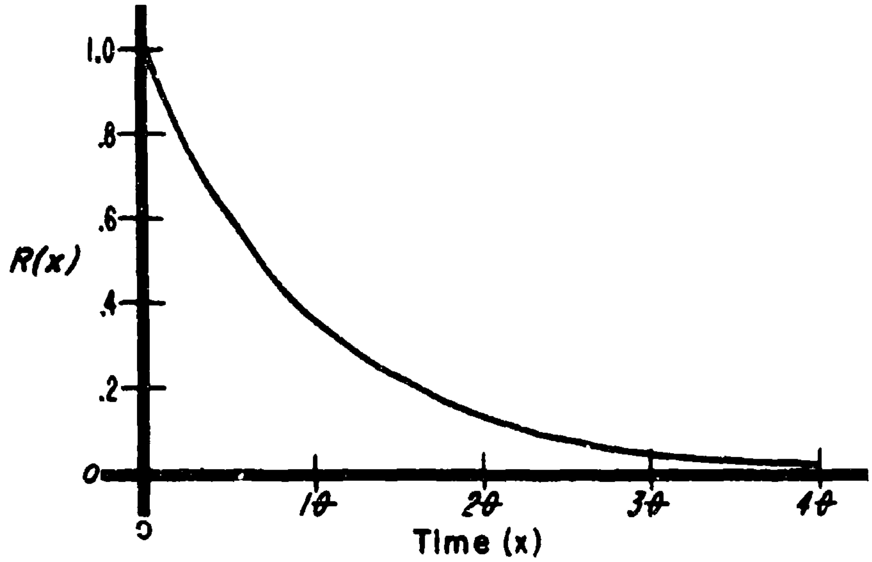


Figure A-26
Exponential Reliability Function, R(x)

f. The hazard rate of the exponential is

$$h(x) = \frac{f(x)}{R(x)} = \frac{\lambda \exp(-\lambda x)}{\exp(-\lambda x)} = \lambda$$

which indicates the distribution applies only when the failure rate remains constant with age. This is expressed graphically by figure A-27.

g. To illustrate the use of the exponential distribution, consider a computer which has a constant error rate of 1 error every 17 days of continuous operation. What is the reliability associated with the computer to correctly solve a problem that requires 5 hours time; 25 hours time? In addition, find the hazard rate after 5 hours of use.

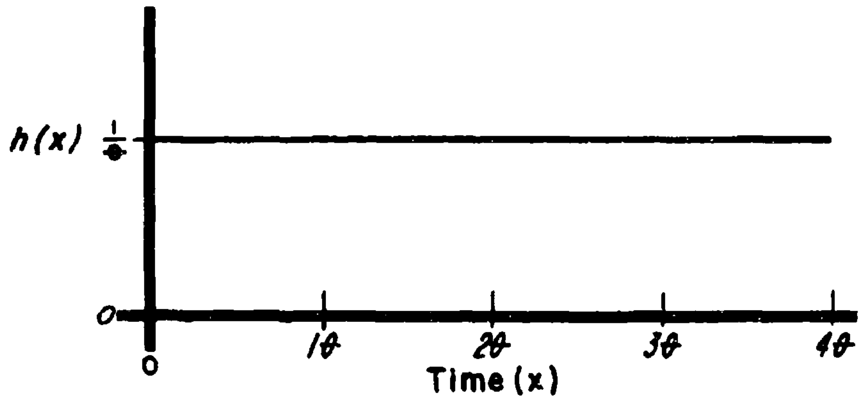


Figure A-27
Exponential Hazard Function, h(x)

$$\theta = 408 \text{ hours}$$

$$\lambda = \frac{1}{408} = 0.0024 \text{ failures per hour}$$

$$R(x) = \exp(-\lambda x) = \exp\left(-\frac{x}{\theta}\right)$$

$$R(5) = \exp[(-0.0024)(5)] = \exp(-0.012) = 0.99$$

$$R(25) = \exp[(-0.0024)(25)] = \exp(-0.06) = 0.94$$

$$h(x) = \frac{f(x)}{R(x)} = \frac{\lambda \exp(-\lambda x)}{\exp(-\lambda x)} = \lambda$$

$$h(5) = 0.0024 \text{ failures per hour}$$

h. A second example of the exponential distribution as a reliability model considers a hydraulic assembly on a LAU-1967 aircraft which has exhibited an exponential distribution of failure times with a mean time to failure of 800 hours. Find the reliability of this assembly for a mission time of 50 hours and the hazard rate at an age of 100 hours.

$$\theta = 800 \text{ hours}$$

$$\lambda = \frac{1}{800} = 0.00125 \text{ failures per hour}$$

$$R(x) = \exp(-\lambda x) = \exp\left(-\frac{x}{\theta}\right)$$

$$R(50) = \exp[(-0.00125)(50)] = \exp(-0.0625) = 0.94$$

$$h(x) = \frac{f(x)}{R(x)} = \frac{\lambda \exp(-\lambda x)}{\exp(-\lambda x)} = \lambda = 0.00125 \text{ failures per hour}$$

for all x.

A-15. Weibull distribution. a. The exponential distribution is applicable as a model for failure times only if the failure rate is constant over time. In reality, failure rates which change with time are sometimes encountered. The normal distribution is a realistic model only if an increasing failure rate is encountered. The Weibull distribution is continuous and can account for a decreasing failure rate, an increasing failure rate, or a constant failure rate; but the failure rate must be monotone.

b. The Weibull density function for the random variable X is:

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right], \quad x \geq 0$$

$$f(x) = 0, \quad x < 0$$

where

$\beta, \eta > 0$

β = shape parameter

η = scale parameter

Figure A-28 shows Weibull density functions for various values of β and $\eta = 1$.

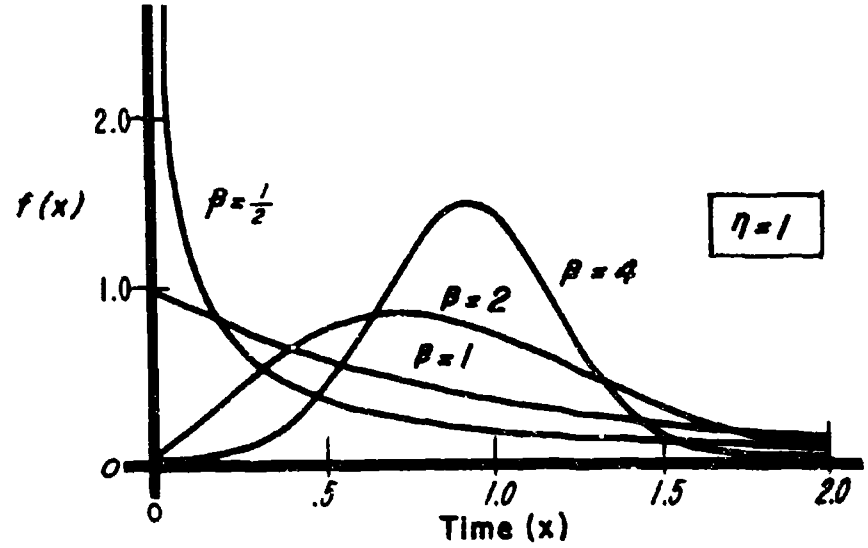


Figure A-28
Weibull Density Function, f(x)

c. The expected value and variance of the Weibull distribution is:

$$E(X) = \eta\,\Gamma\left(\frac{1}{\beta} + 1\right)$$

$$V(X) = \eta^2 \left[\Gamma\left(\frac{2}{\beta} + 1\right) \right] - \left[\eta\, \Gamma\left(\frac{1}{\beta} + 1\right) \right]^2$$

d. The distribution function is

$$F(x) = \int_0^x \frac{\beta}{\eta} \left(\frac{t}{\eta}\right)^{\beta-1} \exp\left[- \left(\frac{t}{\eta}\right)^{\beta} \right] dt = 1 - \exp\left[- \left(\frac{x}{\eta}\right)^{\beta} \right].$$

e. The reliability function is

$$R(x) = 1 - F(x) = \exp\left[- \left(\frac{x}{\eta}\right)^{\beta} \right].$$

Figure A-29 contains graphs of reliability functions for various values of β and $\eta = 1$.

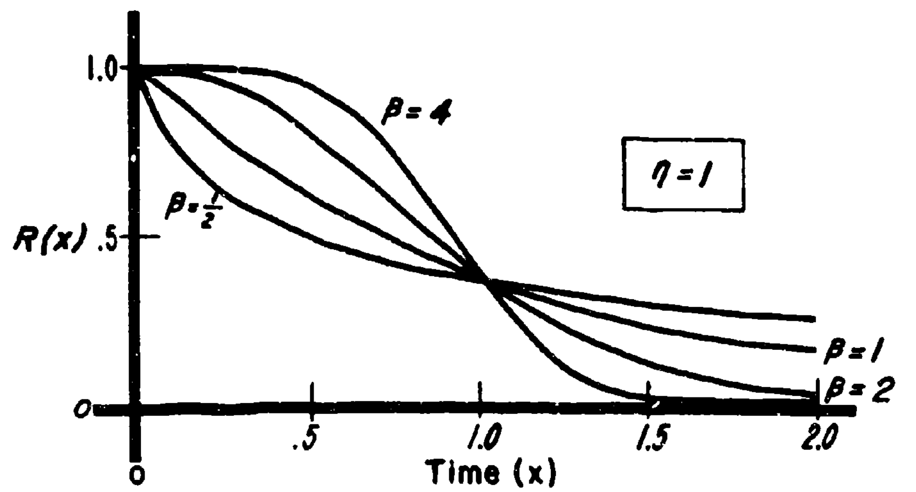


Figure A-29
Weibull Reliability Function, R(x)

f. The hazard function is

$$h(x) = \frac{f(x)}{R(x)} = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$$

Weibull hazard functions are portrayed by figure A-30 for various β values and for $\eta = 1$.

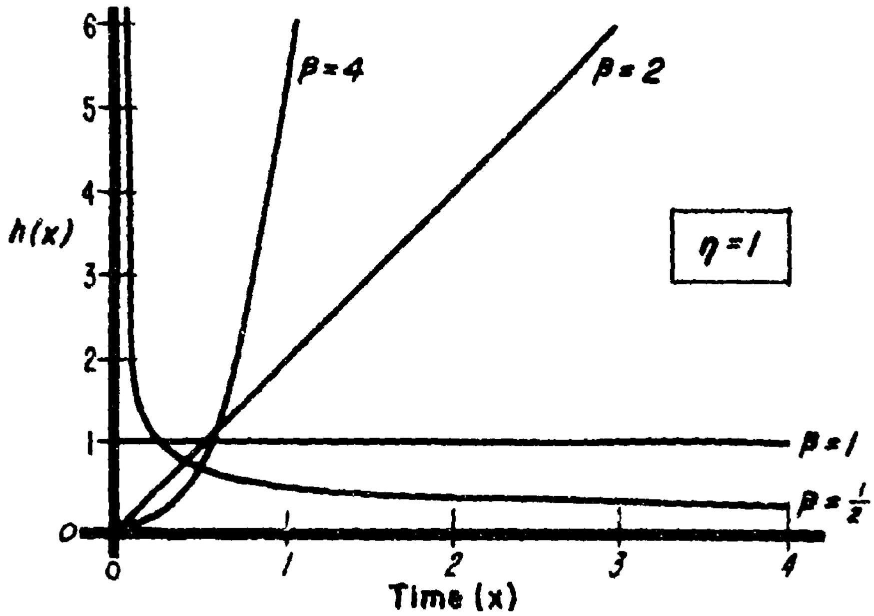


Figure A-30
Weibull Hazard Function

g. To illustrate, consider the failure times of JP29M transmitting tubes which are Weibull distributed with $\beta = 2$ and $\eta = 1000$ hours. Find the reliability of one of these tubes for a mission time of 100 hours and the hazard rate associated with one that has operated successfully for 100 hours.

$$R(x) = \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right]$$

$$R(100) = \exp\left[-\left(\frac{100}{1000}\right)^{2}\right] = \exp\left[-(0.1)^{2}\right]$$

$$= \exp(-0.01) = 0.99$$

$$h(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$$

$$h(100) = 2/1000\left(\frac{100}{1000}\right)^{2-1} = 0.0002 \text{ failures per hour.}$$

h. The following example further exemplifies the use of the Weibull distribution as a reliability model. An aircraft fuel system has failure times which follow a Weibull distribution with $\beta = 3$ and $\eta = 40$ hours. Find the reliability of this fuel system for a mission of 10 hours and a hazard rate after 10 hours of usage.

$$R(x) = \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right]$$

$$R(10) = \exp\left[-\left(\frac{10}{40}\right)^{3}\right] = \exp\left[-(0.25)^{3}\right] = 0.985$$

$$h(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1}$$

$$h(10) = 3/40\left(\frac{10}{40}\right)^{3-1} = 0.005 \text{ failures per hour.}$$

A-16. Gamma distribution. a. Another continuous distribution which is sometimes useful as a failure model is the Gamma distribution. This distribution is a two-parameter distribution. Consider the random variable X which is distributed in accordance with a Gamma distribution. A summary of this distribution follows.

b. The density function is

$$f(x) = \frac{\lambda^{\alpha} x^{\alpha-1} \exp(-\lambda x)}{\Gamma(\alpha)}, \quad x \geq 0$$

$$f(x) = 0, \quad x < 0$$

where

$\alpha > 0$

$\lambda > 0$

α is a shape parameter

λ is a scale parameter

$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} \exp(-x)\, dx$ and can be evaluated from table H-10, appendix H. The Gamma density function is displayed for various α values and $\lambda = 1$ in figure A-31.

c. The expected value is

$$E(X) = \frac{\alpha}{\lambda}$$

and the variance is

$$V(X) = \frac{\alpha}{\lambda^2}.$$

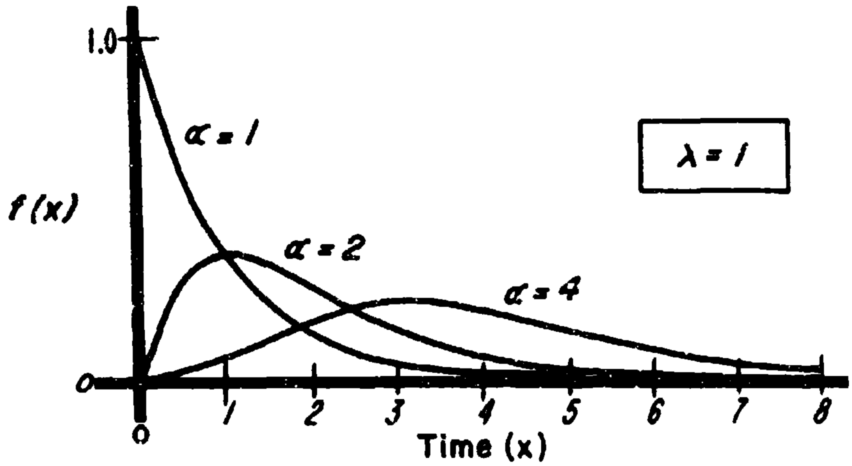


Figure A-31
The Gamma Density Function, f(x)

d. The distribution function is

$$F(x) = P(X \leq x) = \int_0^x \frac{\lambda^\alpha t^{\alpha-1} \exp(-\lambda t)}{\Gamma(\alpha)} dt$$

Special tables, Table of Incomplete Gamma Function, are required to evaluate F(x). However, if α is an integer,

$$F(x) = \sum_{k=\alpha}^{\infty} \frac{(\lambda x)^k \exp(-\lambda x)}{k!}$$

which may be evaluated from a Poisson table.

e. The reliability function is

$$R(x) = 1 - F(x).$$

If α is an integer

$$R(x) = \sum_{k=0}^{\alpha-1} \frac{(\lambda x)^k \exp(-\lambda x)}{k!}$$

The Gamma reliability function is displayed for various values of α and $\lambda = 1$ in figure A-32.

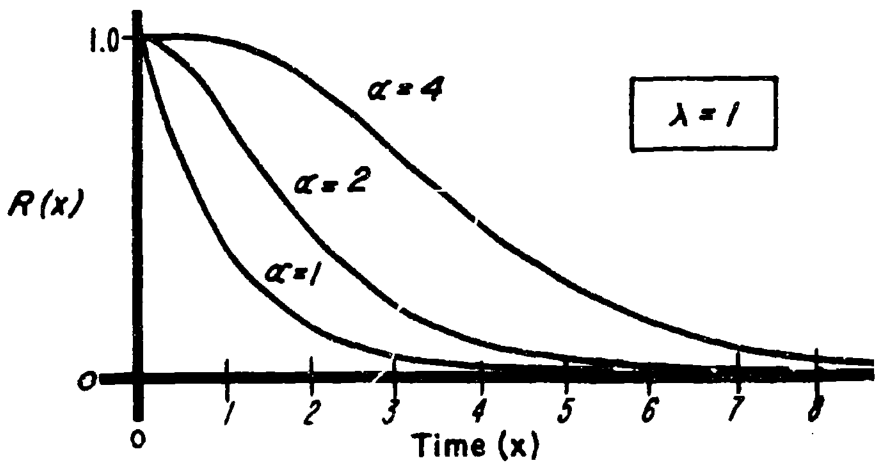


Figure A-32
The Gamma Reliability Function

f. The hazard function is

$$h(x) = \frac{f(x)}{R(x)}$$

is displayed for various α values and $\lambda = 1$ in figure A-33.

g. In addition to its potential use as a failure distribution, the Gamma distribution serves as a model for the time to the α^{th} failure if the underlying failure distribution is exponential. For this purpose, the random variable X is the time to the α^{th} failure and α can assume only positive integer values.

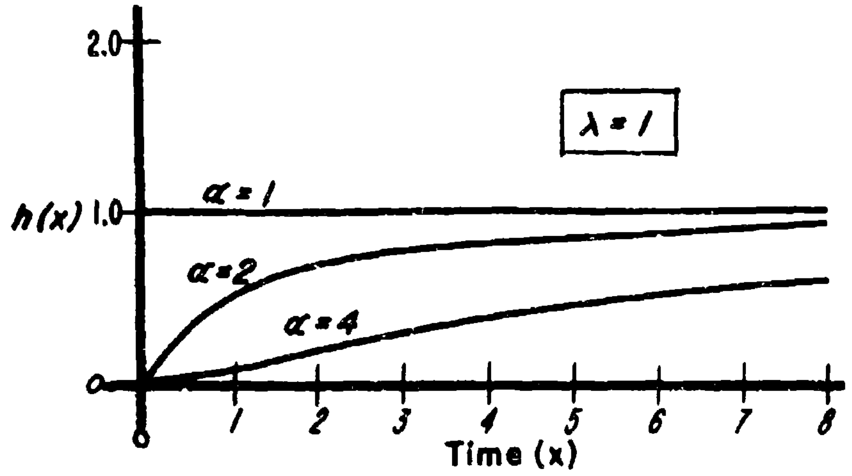


Figure A-33
The Gamma Hazard Function

h. The following example illustrates the use of the Gamma distribution as a reliability model. A Nark missile system has demonstrated a failure distribution which fits a gamma distribution with $\alpha = 3$ and $\lambda = 0.05$. Determine the reliability for a 24 hour mission time and the hazard rate at time 24 hours.

$$R(x) = 1 - F(x) = 1 - \sum_{k=\alpha}^{\infty} \frac{(\lambda x)^k \exp(-\lambda x)}{k!}$$

$$R(24) = 1 - \sum_{k=3}^{\infty} \frac{(1.2)^k \exp(-1.2)}{k!}$$

$$= 1 - \sum_{k-3}^{\infty} \frac{(.301)(1.2)^k}{k!} = 0.88$$

from Poisson curves, figure H-12, appendix H.

$$h(x) = \frac{f(x)}{R(x)}$$

$$f(x) = \frac{\lambda^{\alpha} x^{\alpha-1} \exp(-\lambda x)}{\Gamma(\alpha)}$$

$$f(24) = \frac{(0.05)^3 (24)^2 \exp(-1.20)}{\Gamma(3)} = \frac{(0.000125)(576)(0.301)}{2} = 0.011$$

$$h(24) = \frac{f(24)}{R(24)} = \frac{0.011}{0.88} = 0.012 \text{ failures per hour.}$$

Section IV. STRESS-STRENGTH ANALYSIS

A-17. Introduction. a. In previous reliability discussions, reliability has been defined as a function of mission length. Failure of certain materiel items such as one-shot items, mechanical items, etc. is not necessarily dependent upon time of usage. In some cases, failures may be more directly traceable to some other stress variable. Then reliability of an item may be determined by comparing its strength to the stress to which it will be subjected. Reliability may be defined as the probability that strength exceeds stress.

b. Neither item strength nor the stress to which it is subjected are constant values, but both are random variables each with its own probability density function. If these distributions are known, reliability may be determined analytically. Since data is generally limited to sample information, the goodness-of fit methods of appendix F may be used in an attempt to identify the appropriate underlying distributions.

A-18. Normal stress and strength distributions. a. Assuming both the stress and strength distributions to be normal, reliability can be determined as follows:

(1) Item strength (S) is a random variable with normal probability density function

$$f(S) = \frac{1}{\sigma_S \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{S - \mu_S}{\sigma_S}\right)^2\right]$$

where

μ_S = mean strength

σ_S = strength standard deviation

(2) Stress (s) is a random variable with normal probability density function

$$f(s) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{s - \mu_s}{\sigma_s}\right)^2\right]$$

where

μ_s = mean stress level

σ_s = stress standard deviation

(3) Then the difference D = S-s is a random variable with normal probability density function

$$f(D) = \frac{1}{\sigma_D \sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{D - \mu_D}{\sigma_D}\right)^2\right]$$

where

$$\mu_D = \mu_S - \mu_s = \text{mean difference}$$

$$\sigma_D = \sqrt{\sigma_S^2 + \sigma_s^2} = \text{standard deviation of difference}$$

(4) Then reliability may be defined as

$$R = P(D > 0) = P\left(Z > \frac{0 - \mu_D}{\sigma_D}\right) = P\left(Z > \frac{-\mu_D}{\sigma_D}\right)$$

which may be determined from table H-2, appendix H.

b. To exemplify, the material strength (ultimate shear stress) of a lug shear is a normally distributed random variable with a mean of 104,300 psi and a standard deviation of 3,600 psi. The stress to which the lug is subjected is a normally distributed random variable with a mean of 95,160 psi and a standard deviation of 2,070 psi. What is the reliability of the lug in such an environment?

$$\mu_s = 95,160 \text{ psi} \qquad \mu_S = 104,300 \text{ psi}$$

$$\sigma_s = 2,070 \text{ psi} \qquad \sigma_S = 3,600 \text{ psi}$$

$$\mu_D = \mu_S - \mu_s = 9,140 \text{ psi}$$

$$\sigma_D = \sqrt{\sigma_S^2 + \sigma_s^2} = 4,153 \text{ psi}$$

$$\frac{\mu_D}{\sigma_D} = \frac{9140}{4153} = 2.20$$

$R = P(Z > -2.20) = 1 - P(Z > 2.20) = 0.986$ as found in table H-2.

A-19. General stress-strength distributions. a. Determination of reliability based on stress-strength analysis requires that both the stress and strength probability density functions [f(s) and f(S) respectively] be known. As shown above, analytic determination is quite straightforward if both distributions are normal. Although analytic determination is difficult for other distributions, reliability can be determined from

$$R = \int_0^\infty f(s) \left[\int_s^\infty f(S)dS \right] ds$$

which lends itself to numerical methods easily adapted for use by digital computers.

b. Graphical determination. The following technique, using transformations, provides a graphical reliability determination which may be applied to any distribution. It may also be applied to sample data when the underlying distribution cannot be identified. The following elements are involved.

(1) Let $G = \int_s^\infty f(S)dS$

(2) Let $H = \int_0^s f(s)ds$ which implies

$$dH = f(s)ds$$

(3) Then $R = \int_0^1 G\,dH$ which may be evaluated by plotting G as a function of H and finding the area between the function and G = 0 and between H = 0 and H = 1. Figure A-34 indicates this graphically for a hypothetical function.

c. Example - Known mathematical distribution. (1) The above procedure may be applied to density functions either in mathematical or empirical form. The example which immediately follows

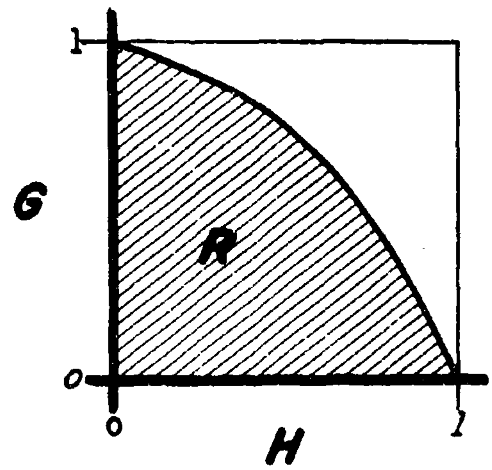


Figure A-34
Hypothetical Plot of G as a Function of H

applies the technique to the situation where both distributions are known in mathematical form. The bursting strength of a given class of rocket motors is known to be exponentially distributed with mean strength of 20,000 psi, i.e., the strength density function is

$$f(S) = \frac{1}{20,000} \exp\left(\frac{-S}{20,000}\right)$$

The pressure exerted by a given propellant charge is distributed Weibull with parameters $\beta = 2$ and $\eta = 18,000$ psi, i.e., the stress density function is

$$f(s) = \frac{2}{18,000}\left(\frac{s}{18,000}\right) \exp\left[-\left(\frac{s}{18,000}\right)^2\right]$$

Find the reliability of this class of rocket motors when propelled by the above type charge.

Define

$$G = \int_s^{\infty} f(S)dS = 1-F(S=s) = \exp\left(\frac{-s}{20,000}\right)$$

and

$$H = \int_0^s f(s)ds = F(s) = 1 - \exp\left[-\left(\frac{s}{18,000}\right)^2\right]$$

Figure A-35 is a table of (H, G) coordinates calculated from several different stress values. These coordinates are to be plotted to determine the reliability graphically.

s	H	G
0	0	1
5,000	0.07	0.78
7,000	0.14	0.70
10,000	0.27	0.61
12,000	0.36	0.55
15,000	0.50	0.47
18,000	0.63	0.41
20,000	0.71	0.37
25,000	0.85	0.29
27,000	0.91	0.26
∞	1	0

Figure A-35
Calculated (H, G) Coordinates

(2) Figure A-36 is a plot of these (H, G) coordinates and the resultant reliability is represented by the shaded area and is numerically equal to 0.49. (This was obtained by measuring the shaded area.)

d. Example - Empirically determined distributions. (1) This example is the same as the preceding example except that f(s) and f(S) are not known and reliability must be estimated using observed sample data for both stress and strength. Bursting strengths (psi) for a sample of ten rocket motors have been observed. These values listed in ascending order, are shown in figure A-37. Also shown are $\hat{F}$ (S) values which represent an estimate of the unknown distribution function F(S)

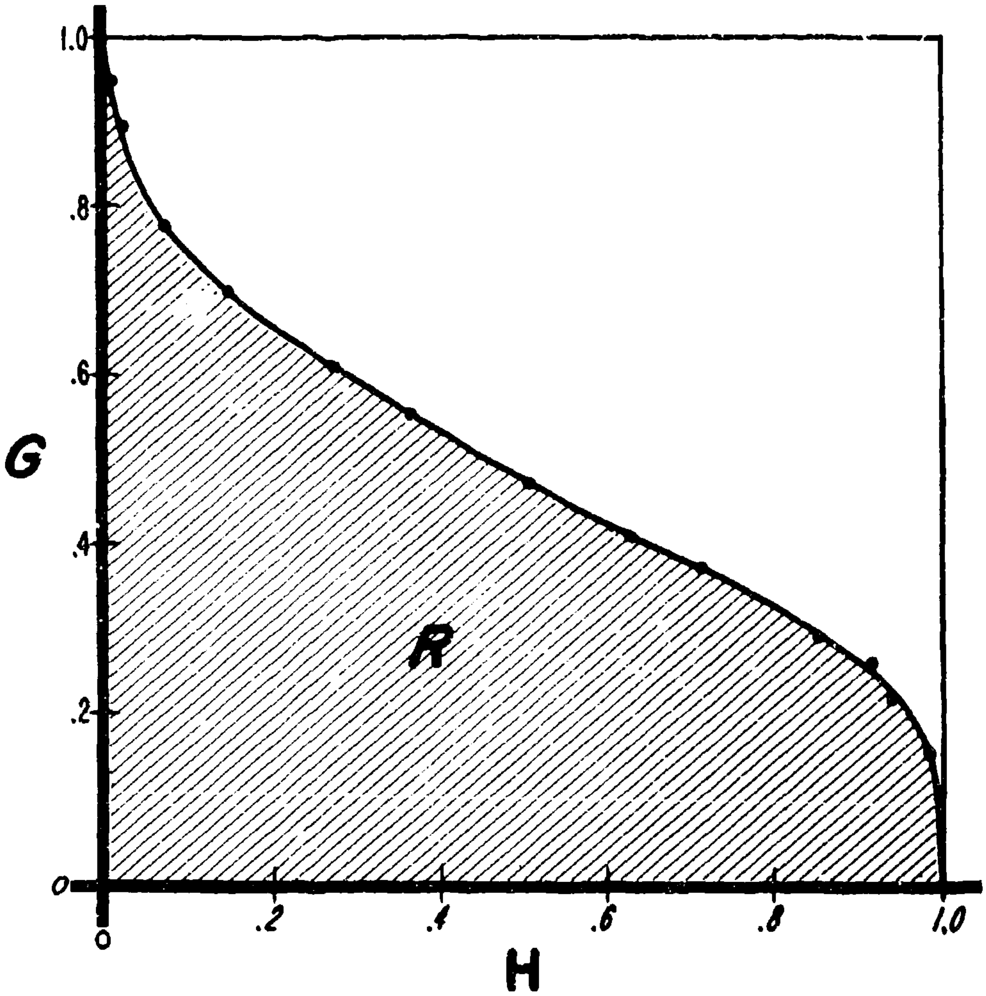


Figure A-36
Plot of (H, G) Coordinates

bursting strength corresponding to each observed strength value. ($\hat{F}(S)$ is the relative frequency of sample values which are less than or equal to S.)

S	$\hat{F}(S)$
14, 100	0.10
15, 200	0.20
16, 300	0.30
16, 600	0.40
17, 700	0.50
17, 700	0.60
18, 800	0.70
19, 000	0.80
20, 800	0.90
25, 100	1.00

Figure A-37
Observed Bursting Strengths (psi)

(2) Figure A-38 shows a plot of these coordinates and a smooth curve is drawn to fit the trend of the points. This curve, $\hat{F}(S)$, is used as an estimator of the F(S) function.

(3) The exerted pressures (psi) observed from a sample of twenty propellant charges of a given type are shown, in ascending order, by figure A-39. Also shown are estimates, $\hat{F}(s)$, of F(s) for each observed stress value.

(4) Figure A-40 shows a plot of these coordinates and a curve drawn through the trend of the points. This curve, $\hat{F}(s)$, is an estimate of the F(s) function.

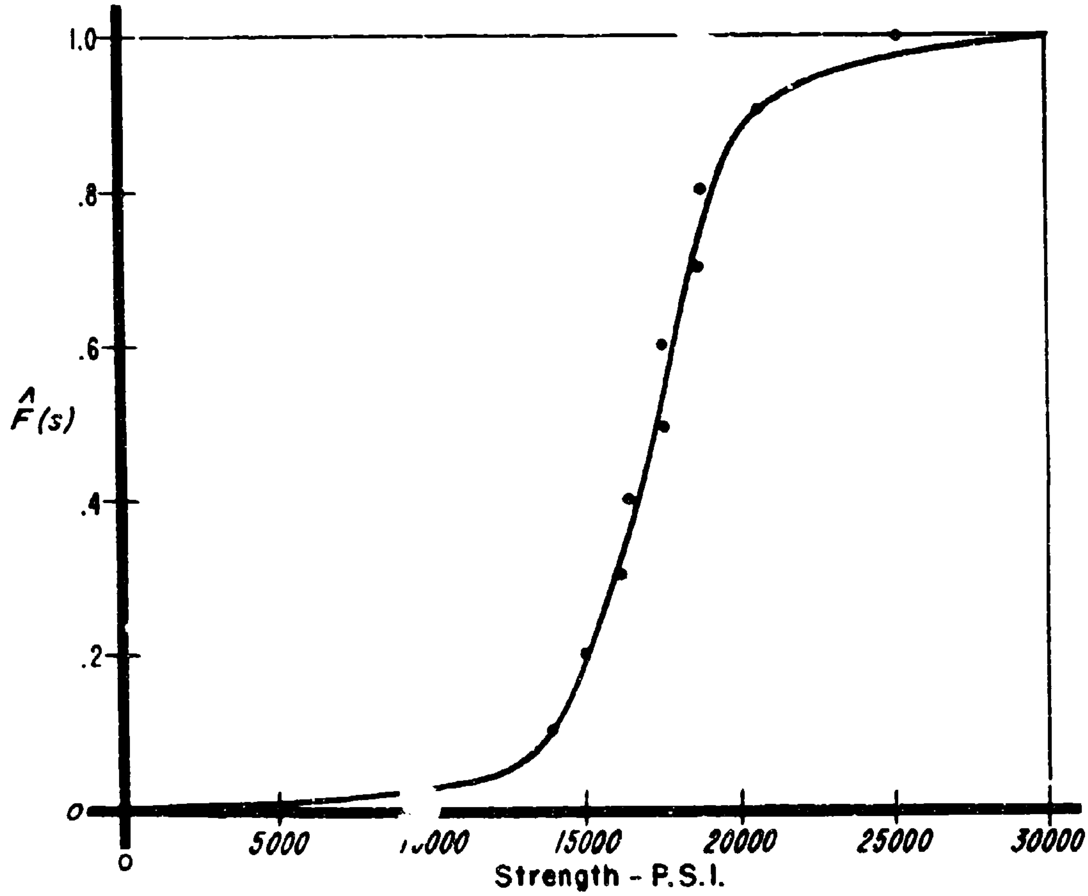


Figure A-38
Estimate of the Strength Distribution Function

(5) Using the estimated stress and strength distribution functions, (H, G) coordinates have been found for assorted stress values and listed in figure A-41. The values were found as follows:

$$H = \hat{F}(s)$$

$$G = 1 - \hat{F}(S = s)$$

s	$\hat{F}(s)$
9,200	0.05
10,100	0.10
10,800	0.15
11,800	0.20
12,100	0.25
12,200	0.30
12,300	0.35
14,100	0.40
14,800	0.45
15,000	0.50
15,400	0.55
16,200	0.60
16,800	0.65
17,200	0.70
17,200	0.75
17,800	0.80
18,300	0.85
18,500	0.90
18,700	0.95
19,100	1.00

Figure A-39
Observed Stresses (psi)

(6) Figure A-42 is a plot of these (H, G) coordinates and the resultant reliability is represented by the shaded area and is numerically equal to 0.71. (This was obtained by measuring the shaded area.)

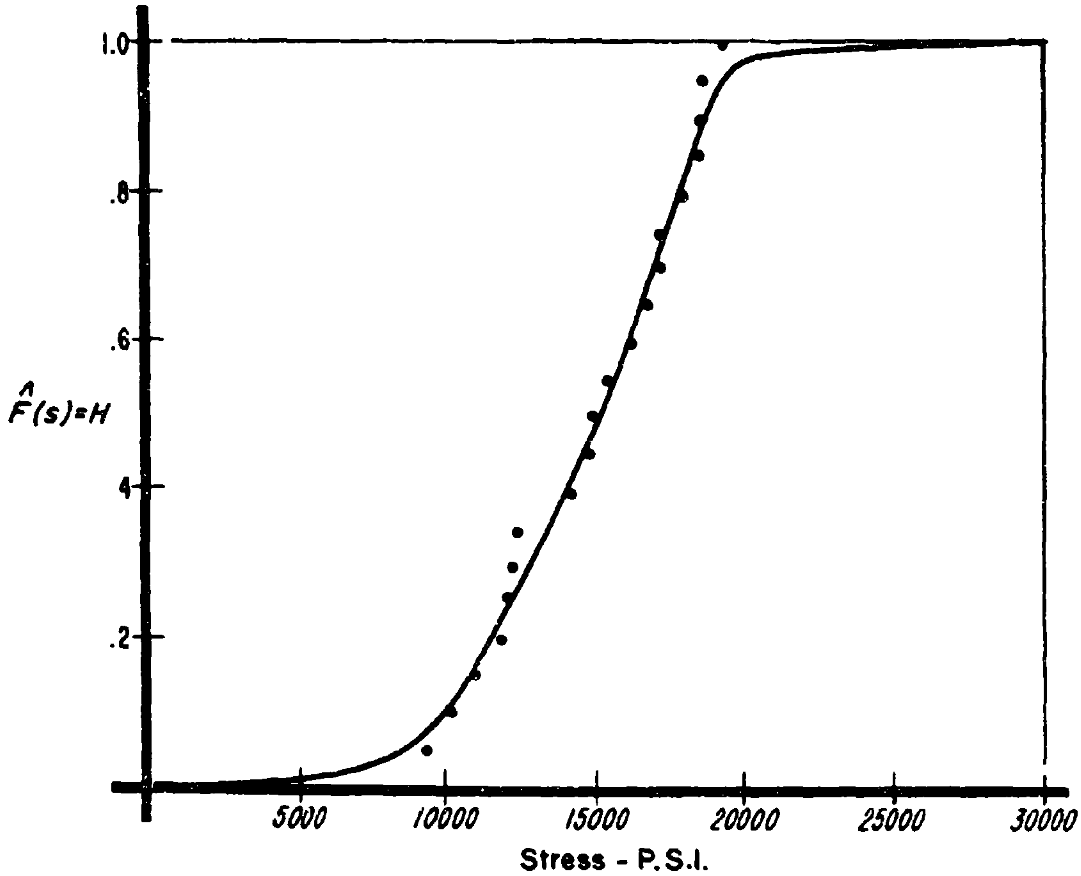


Figure A-40
Estimate of the Stress Distribution Function

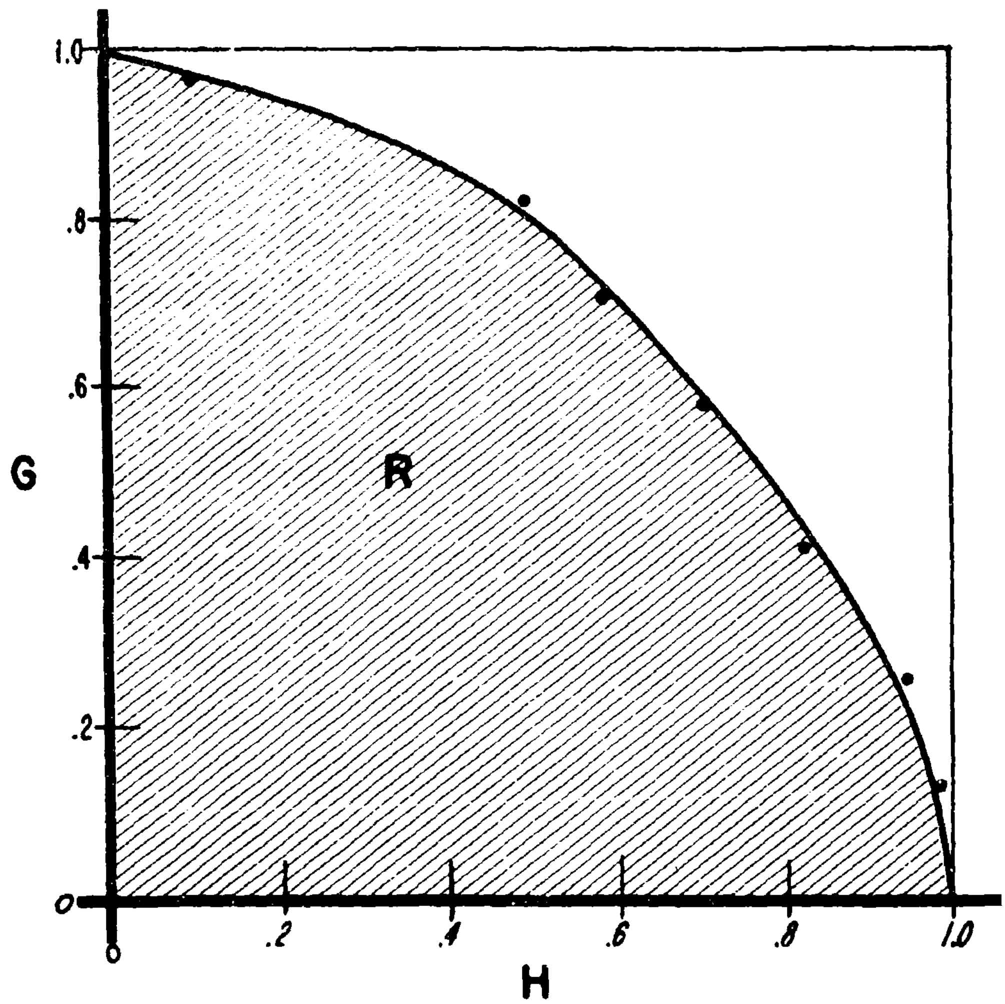


Figure A-41
Plot of (H, G) Coordinates

s	H	G
0	0	1.00
5,000	0.01	0.99
10,000	1.10	0.97
15,000	0.49	0.82
16,000	0.59	0.71
17,000	0.71	0.58
18,000	0.83	0.41
19,000	0.94	0.25
20,000	0.99	0.03
∞	1.00	0

Figure A-42
Calculated (H, G) Coordinates

Section V. CONCEPT OF SYSTEM EFFECTIVENESS

A-20. System effectiveness evaluation. a. General. The concept of system effectiveness was defined in chapter 1 as the function of three groups of variables --- i.e., those pertaining to availability, dependability and capability. The example which follows is hypothetical and is a gross oversimplification, but it is intended to emphasize the reliability considerations associated with the concept of system effectiveness.

b. Problem statement. The system to be considered is that comprised of the XXX helicopter and its communication equipment. It is to operate in a limited warfare environment where rapid movement of supplies upon request is important. The mission of the system is that of transporting, upon random call, supplies from a central supply base to troop activities within a radius of 1/2 hour flying time. Once the helicopter has reached the target area, proper functioning of the communication equipment enhances the chances of a successful delivery of the supplies in terms of safe landing area, location of enemy troops, etc. Some major assumptions which are inherent in this example are:

(1) A call for supplies is directed to a single helicopter which is located at the base. If this craft is not in flyable condition (i.e., it is in process of maintenance) the mission will not be started. A flyable craft is defined as one which is in condition to take off and fly with a standard supply load.

(2) The flight time required to reach the target area is 1/2 hour.

(3) The communication equipment cannot be maintained or repaired in flight.

(4) A loaded helicopter which goes down while on route to or does not reach the target area has no delivery value, i.e., assume that supplies lost in route cannot be effectively recovered by ground troops.

c. Model determination. (1) For purposes of model formulation, the system condition is divided into 3 states, namely:

(a) State 1 - Helicopter flyable, communication equipment operable.

(b) State 2 - Helicopter flyable, communication equipment non-operable.

(c) State 3 - Helicopter non-flyable.

(2) The effectiveness model is defined as

$$E = ADC$$

where A, D and C are defined as follows:

(a) The availability vector is a three element row vector, i.e.

$$A = (a_1, a_2, a_3)$$

where a_i is the probability that the helicopter will be in state i at the time of call.

(b) The dependability matrix is a 3 x 3 square matrix, i.e.,

$$D = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}$$

where d_{ij} is the probability that, if the helicopter is in state i at the time of call, it will complete the mission in state j.

(c) The capability vector is a three element column vector i.e.,

$$C = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

where c_i is the probability that, if the helicopter is in state i at the time of arrival at the target area, the supplies can be successfully delivered. (For multicapability items, C would be a multicolumn matrix.)

d. Determination of model elements. (1) Past records indicate that the average time between maintenance activities (including preventive and failure initiated maintenance) for this type helicopter is 100 hours and the average duration (including such variables as maintenance difficulty, parts availability, manpower, etc.) of a maintenance activity is 10 hours. Comparable data for the communication equipment shows an average time between maintenance activities of 500 hours and an average duration of a maintenance activity of 5 hours.

(2) From the preceding data the elements of A can be determined.

a_1 = P(helicopter flyable)P(communication equipment operable)

$$= \left(\frac{100}{100+10}\right)\left(\frac{500}{500+5}\right) = 0.900$$

a_2 = P(helicopter flyable)P(communication equipment not operable)

$$= \left(\frac{100}{100+10}\right)\left(\frac{5}{500+5}\right) = 0.009$$

a_3 = P(helicopter not flyable) $= \dfrac{10}{100+10} = 0.091$

(3) Data available from past records indicate that the time between failures of the communication equipment during flight are exponentially distributed with a mean of 500 hours. Also the probability that a helicopter in flight will not survive the 1/2 hour flight to its destination is .05 (includes probability of being shot down, mechanical failures, etc.). Then the elements of the D matrix may be calculated as follows:

(a) If the system begins in state 1:

d_{11} = P(helicopter will survive flight)P(communication equipment will remain operable)

$$= (1-0.05)\left[\exp\left(-\frac{1/2}{500}\right)\right] = 0.94905$$

d_{12} = P(helicopter will survive flight)P(communication equipment will fail during flight)

$$= (1-.05)\left[1-\exp\left(-\frac{1/2}{500}\right)\right] = 0.00095$$

d_{13} = P(helicopter will not survive the flight) = 0.05000

(b) If the system begins in state 2:

d_{21} = 0 because the communication equipment cannot be repaired in flight.

d_{22} = P(helicopter will survive the flight) = 0.95000

d_{23} = P(helicopter will not survive the flight)

= 0.05000

(c) If the system begins in state 3:

$d_{31} = d_{32} = 0$ because the mission will not start.

d_{33} = 1, i.e., if the helicopter isn't flyable, it will remain non-flyable with reference to a particular mission.

(4) Experience and technical judgment have determined the probability of successful delivery of supplies to be c_i if the system is in state i at the time of arrival in the target area, where

$c_1 = 0.95$

$c_2 = 0.80$

$c_3 = 0$

e. Determination of effectiveness. (1) The effectiveness of the subject system becomes

$$E = (0.900 \quad 0.009 \quad 0.091) \begin{pmatrix} 0.94905 & 0.00095 & 0.05000 \\ 0 & 0.95000 & 0.05000 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0.95 \\ 0.80 \\ 0 \end{pmatrix}$$

$$= 0.82 = (0.900)(0.94905)(0.95) + [(0.900)(0.00095) + (0.009)(0.95000)](0.80) + [(0.900)(0.05) + (0.009)(0.05) + (.091)(1)](0)$$

which means that the system has a probability of 0.82 of successful delivery of supplies upon random request.

(2) The effectiveness value attained provides a basis for deciding whether improvement is needed. The model provides the basis for evaluating the effectiveness of alternative systems considered.

f. The preceding example was not intended to emphasize the technical approach to system effectiveness evaluation, but to point out that reliability is a factor of effectiveness and may be considered at more than one point in system life. For example, reliability of the system (and subsystems) must be considered for the environment encountered while awaiting mission call (such as storage, temporary use, etc.) i.e., for determining the availability elements. Reliability also comes into the picture during the mission, i.e., in determining the dependability elements. In addition, reliability must be considered relative to each of the system states.

Section VI. RELIABILITY DESCRIPTORS

A-21. Some meaningful parameters. Any of several different descriptors may be used for measuring or specifying the reliability of a product. A useful descriptor must be meaningful in terms of the definition of reliability. Commonly used reliability descriptors include the probability of success in x hours mission time, mean time between failures (MTBF), hazard function, and probability of success where time isn't of primary importance. Each of these is dependent upon a specified intended function and use environment.

a. Probability of success for x hours mission time. (1) This descriptor is actually the basic definition of reliability and is meaningful whether or not the underlying failure distribution is known. It is composed of a probability statement as well as the mission duration time to which the probability applies. It represents a single point on the reliability function curve as defined earlier in this appendix.

(2) This descriptor may be related to other reliability indices if the underlying failure distribution is known.

b. Mean time between failures. (1) MTBF is a very popular measure of reliability. However, care must be exercised in its

use since the reliability level associated with MTBF is dependent upon the failure distribution. In other words, MTBF is a meaningful measure of reliability only if the failure distribution is known.

(2) To illustrate the different reliability levels associated with a mission time equal to MTBF, consider the exponential and normal failure models.

(a) For the exponential failure distribution,

$$R(\text{MTBF}) = \exp\left(-\frac{\text{MTBF}}{\text{MTBF}}\right) \approx 0.37$$

(b) For the normal failure distribution,

$$R(\text{MTBF}) = P(Z > 0) = 0.50$$

c. Hazard function. (1) The hazard function is sometimes used as a measure of reliability. Its use requires both an instantaneous failure rate and the equipment age at which this failure rate is in effect. The hazard function can be related to reliability level only if the underlying failure distribution is known.

(2) The special case of exponential failures reduces the hazard function to a constant failure rate which is independent of equipment age.

d. Probability of success. For items whose failure is not dependent on time, reliability may be expressed as the probability of success under specified stress conditions. This descriptor is especially useful when considering one-shot items.

APPENDIX B

RELIABILITY PROGRAM REQUIREMENTS PLANNING AND MANAGEMENT GUIDE

Section I. INTRODUCTION

B-1. General. This appendix is designed to supplement chapter 2. It provides guidance information for planning and management of a reliability program. This guidance takes two forms: first, a discussion of some fundamentals of network diagramming and second, a general network to use as an example when developing a plan for reliability management of a specific project.

Section II. FUNDAMENTALS OF NETWORK DIAGRAMMING

B-2. General. Several forms of this technique exist, the details vary somewhat, although the underlying principles are the same. The technique is assigned to assist program managers in planning and controlling a variety of interrelated projects. None of the techniques are discussed in detail herein. It is the purpose of this appendix to show the sequence of events and identify the interfaces, relationships, and constraints of a reliability program by means of an activity diagram. The reader must remember that networks such as this are not devices for measuring the reliability of a product.

B-3. Definitions of terms used in network diagramming.

a. Network.

(1) A network is a flow diagram consisting of the activities and events which must be accomplished to reach the project objectives (total life cycle) showing the planned sequences of accomplishment, interdependencies and interrelationships. It is used as a tool in the project management decision making process for planning activities to be performed, as well as progress reporting and corrective action. The network includes all action involved and is not limited to work activities. Time consuming actions, such as lead time for procurement of purchased parts, shipment of materiel from one location to another, and management action, are shown on the network.

(2) The network serves as the basis for scheduling work. The sequence of activities and their relationships, elapsed time estimates, and directed completion dates of key milestones are basic information for establishing work schedules.

(3) The network can also identify problem areas which require management emphasis. Critical activities which could delay the accomplishment of the project objectives are highlighted so that management attention is continously focused on such activities. A program change or anticipated schedule slippage of one activity can quickly be analyzed to show the effect on other activities and the entire project.

(4) The network, in addition, serves as a communication device for all levels of management. It is a common language which operating and management personnel can interpret easily and accurately.

b. Event.

(1) An event is a specific definable accomplishment in a program plan, recognizable at a particular instant in time. Events do not consume resources nor take time to complete. They simply reflect a state of being, i.e., something is developed, tested, started, completed, etc. Words which express a state of being should be used to describe an event.

(2) Events may represent points of decision, the accomplishment or beginning of a significant phase of the total job, the transfer of responsibility from one organization to another, or the completion or start of one or more activities. Thus, events by themselves cannot always specify all of the activities which are connected to them.

(3) The usual procedure for representing events is to use circles or boxes, although a number of various shaped enclosures can be used to represent different types of events. An example of two events is shown in figure B-1.

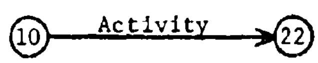


Figure B-1. Event-Activity Relationship

(4) Events numbered 10 and 22 are related and unless event 10 is completed, the activity between them cannot be started. Furthermore, it will be impossible to reach the point in time represented by event 22 until the activity represented by the arrow is completed.

c. Activity.

(1) An activity is the work effort of a program. It is that which must be done, characterized by people using facilities, materials, and equipment over some period of time to accomplish a stated objective. Activities imply doing such things as researching, building, negotiating, testing, etc. Activities are the flow of a network, and it is this flow of human effort, materials, use of facilities, investment, and expense that can be controlled by the manager.

(2) In order to avoid confusion in the milestone list, and generally to improve the clarity of presentation, considerable attention should be given to the names of events and activities. Since an activity implies doing or acting, it should be expressed as a verb form (develop, test) which will not be confused with the beginning or completion point of the activity. Each description of an event or an activity should be concise so that personnel with different backgrounds and points of view will interpret it in the same manner.

(3) An activity is represented on a network by an arrow with the head of the arrow pointing in the direction of the time flow as shown in figure B-1. The activity connecting the two events cannot begin until event number 10 is completed nor can event number 22 take place unless the activity is completed. When several activities lead to an event, all activities must be completed before the event comes into existence. For instance, the network activities numbered 7-10, 9-10, and 6-10 in figure B-2 must all be completed before event number 10 can occur.

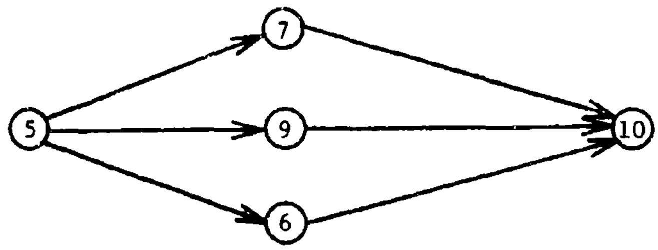


Figure B-2 Network Schematic

(4) A simple example of the proceding schematic diagram illustrated in figure B-3.

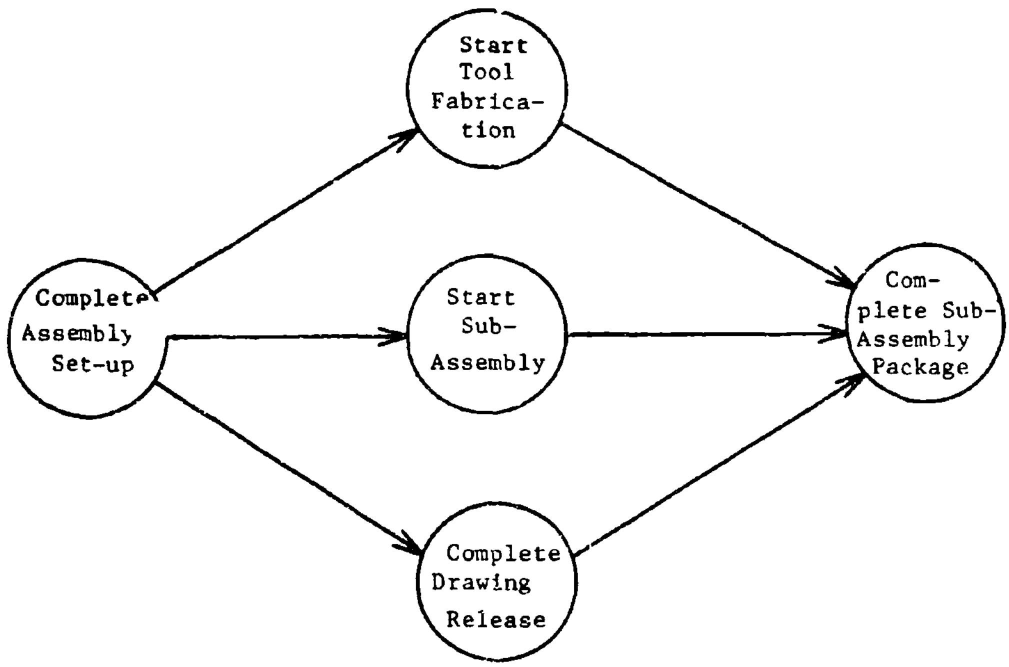


Figure B-3 Network Diagram

d. Dummy activities.

(1) A dummy activity or zero time activity is one which neither consumes resources nor takes time to complete. The dummy activity is useful because it constrains the beginning of a following activity, or the completion of the event to which it leads by requiring that the event from which it proceeds be completed first. It is often used to tie the completion of several activities to the beginning of a single activity, or vice versa.

(2) The restraint may also be used in cases where it is desired to indicate by separate events the ending of one activity and the beginning of the following one. This may be desirable in cases where it is necessary to be quite sure that the following activity begins immediately as planned. Figure B-4 shows a dummy (zero time) activity (B-C) between two events to indicate the completion of one activity and the beginning of another activity.

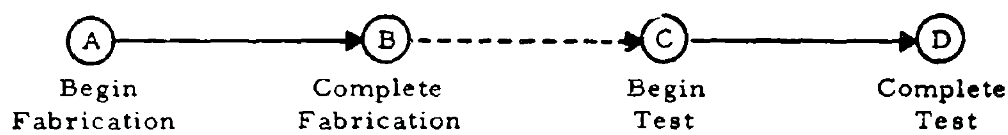


Figure B-4 Dummy Activity

B-4. <u>Network organization.</u> a. The typical weapon (or equipment) system development project requires a hierarchy of networks in order to meet the needs of different levels of management. Each of the various networks is similar in concept, but differs in level of detail in accordance with the user's responsibility. Since the project manager has total project responsibility, his emphasis would be upon key milestones and gross project activities portrayed on the top level network. The operating engineer's network, on the other hand, contains checkpoints related to his particular portion of the total task, interconnected by precisely defined activities. All networks in the hierarchy are related to one another via common events, whereby activity time estimates from the detailed networks may be summarized into forecasted achievement dates for major project milestones.

b. The appropriate organization of the hierarchy of networks for any project must be determined by the project manager to meet the requirements of his project. Many factors influence the network organization, such as the size and complexity of the project, the organizational structure of the project manager's office and performing organizations, and the computer programs when applicable.

Section III. RELIABILITY NETWORK DIAGRAM AND MILESTONE CHART

B-5. General description. a. To obtain a reliable system, a reliability management program must be a continuing process throughout the life cycle of an item. This section contains a major milestone network identifying those events generally necessary for effective monitoring of the reliability program throughout a system's life cycle.

b. The network is not all inclusive and should not be interpreted as the panacea in reliability management. It is, however, considered an excellent guide in the major areas of reliability management. Not every milestone is the responsibility of a reliability manager, but the interface of general milestones to reliability milestones is required to grasp the overall relationship in the life cycle.

c. Both general managers and reliability managers must be aware of the interrelationship and impact each has on the other's discipline. The attainment of a reliable system is not the responsibility of any one organization or individual, but represents an integrated effort of all management throughout the item's life cycle.

d. The network in this appendix is divided into top management and top work-level milestones.

(1) Top Management. Those responsible for decisions and policies, i.e., personnel levels 1, 2 and 3 in figure B-5.

(2) Top Work Level. Those responsible for implementing policies and securing necessary reliability data, i.e., levels 4 and 5 in figure B-5.

Personnel Chart

Personnel Level*	Responsible Agency		
	DA & CD[illegible]	AMC	COMMODITY COMMAND
#1.	I, IV, XXVI, XLIII	V, XXI, XXVI	XXI
#2.	II, III, XXIII, XXVII	VII, IX, XI, XII [illegible]XXI, XXII, XXV XX[illegible] XLII, XLVIII	XIV, XXII, XXIV, XXX[illegible] XXXIII, XXXV, XXXVI XXXVIII, XL, XLIV, XLVII
#3.	L, LI	VII[illegible], X, L[illegible]	XV, XVI, XVII, XVIII, XIX XX, XXIX, XXX, XXXII, XXXIV, XXXVII, XLI, XLIII, XLV, XLVI, XLIX
#4.	1-2	3, 4, 5, 6, 7, 8, 9, 19, 20 40, 44	4[illegible]
#5.			20-39, [illegible], 43, 45-59

*Personnel Level - A descending order of management:

1. Top Management, or Policy Level.
2. Representatives of top management.
3. Systems or directorate level.
4. Project or division level.
5. Specialist or branch level.

Figure B-5

Reliability Management Milestones

e. A sample personnel chart (figure B-5) is included to generally relate the milestones to a particular command and action level. The acceptance and assignment of responsibilities should filter from the top level downward.

f. The milestone definitions included are event oriented. The explanation is a brief description of activities leading into or from the event. Thus, it is possible for each manager to expand the events within his functional area of responsibility to cover all aspects of reliability. Top management milestones are identified by Roman numerals; top work-level milestones by Arabic numerals, with AMC interface milestones from AMCR 11-27 shown parenthetically following the milestone title.

g. The milestone chart is divided into the concept and definition phases (figure B-6), the development phase (figure B-7), and the production, operation and disposal phases (figure B-8).

B-6. Definitions of management milestones for reliability.

a. Concept Phase (see figure B-6).

I. Guidance Documents Initiated. The Army planning documents BASE, ASP, AFDP, et al, which s[illegible] guidelines, objectives, and priority operational requirements, are initiated. These documents forecast the needs and technological advances, thereby forming the framework for the Army's total mission responsibility. They also initiate the materiel life cycle and provide guidance during the concept, definition, and development phases. The above documents are reviewed and updated annually.

II. User Requirements Established. Studies, which culminate in new or improved doctrine, organizational concepts, operational capability objectives, logistical concepts, QMDO's, and QMR's, are completed. These studies establish user requirements for new items and materiel to satisfy future tactical concepts. A desirable, but realistic, reliability goal is established as a result of these studies. Activities are planned and conducted within the framework provided by the guidance documents.

III. Technological Forecasting Completed. All basic research projects, such as parametric analysis, applied research, technological forecast, system synthesis, technical estimates, etc., are completed. These analytical studies provide data and a theoretical basis for predicting the expected scientific and technical gains to be achieved by the anticipated production date. By determining what and where fundamental knowledge is lacking, development efforts are precluded from less productive areas until additional research work is accomplished. The predictions, capabilities, objectives, concepts, and estimates resulting from these projects provide information for QMDO requirements, such as the proposed reliability goal.

IV. QMDO Approved and Assigned (0100). A Qualitative Materiel Development Objective (QMDO) is an Army-approved statement of military need for the development of new materiel, the feasibility or specific definition of which cannot be determined sufficiently to permit the establishment of a Qualitative Materiel Requirement (QMR). Approval of the QMDO provides the first formal requirements document that the Army uses in the research and development cycle. This document should contain a proposed reliability goal for the item under consideration. The QMDO is assigned by AMC to one of the subordinate commands or laboratories on a sole, prime, or attendant basis. The designated development agency will prepare and coordinate, with other developing agencies that have competence and responsibilities in the area concerned, a brief but specific plan that outlines all the research and development work that is required to prove out the feasibility of the QMDO. The plan summarizes existing research projects and tasks, states an estimate of the additional projects and tasks required to achieve the technological capability, and gives an estimate of the research risks involved in each approach. Cost estimates are also included.

1. Total Feasibility Study Completed (0600). The total feasibility study, which includes consideration of technical feasibility, cost effectiveness, system effectiveness, availability of all funds throughout the life cycle, qualitative and quantitative personnel implications, operational and organizational concepts, logistical support implications, and impact upon inventory, is completed. This study states determinations that the Department of Army is capable of supporting the desired new item of materiel throughout its life cycle. Completed when AMC technical data to support the item or system is submitted to Department of Army.

2. QMA Established. The Qualitative Materiel Approach (QMA), based on review of research feasibility and standard equipment capability for the expected state of the art, is established. The QMA describes recommended technical approach(es) or solution(s) to a materiel requirement. It will include trade-offs in approaches, time to develop, size and weight, cost of operation (manpower and funds), technological risks, estimated development and procurement costs, and comparison with existing items, if applicable.

V. TDP Initiated. The preliminary Technical Development Plan (TDP) is used for programming purposes in order to obtain program approval on a timely basis. Information included in the preliminary plan should include initial concepts, schedules, and funding estimates. In addition, the plan is expected to outline those plans for development and provide guidance, goals, and specific direction necessary to assure that operational effectiveness will be achieved. The inclusion of statements delineating performance, reliability, and maintainability are aimed at this goal. This document is to be prepared by the responsible Army developing agency.

VI. QMR Initiated. The initial draft proposed Qualitative Materiel Requirement (QMR) document is initiated. Contents of the QMR are based upon national defense objective, intelligence estimates, and concept or feasibility studies which determine the requirements for a new capability and the need for a new item. The QMR expresses Department of Army requirements for new equipment or for major innovations or improvements related to research and development as developed from new concepts. During preparation of the QMR, the CDC will coordinate with AMC to assure that proposed requirements reflect current state of the art and best integration of competitive characteristics. This milestone taks place when AMC begins preparation of the technical data to complete the QMR or SDR.

VII. Reliability Documentation Initiated. Documentation of the reliability requirements to be placed in the QMR and TDP is initiated.

3. QMR Reliability Requirements Established. Documentation of the basic reliability requirements to be placed on the item is completed. This is a clear and concise set of statements which includes the object or mission, the quantitative reliabiltiy requirements--essential and desired--and the definition of what constitutes a failure.

VIII. Draft Proposed QMR Submitted (0400). The draft proposed QMR, consisting of information required in milestones VII, VIII and 3, is reviewed and submitted to CDC.

4. TDP Reliability Requirements Established. A summation of reliability requirements covering system and subsystem characteristics, performance requirements, and the reliability review point schedule are established for the preliminary TDP document.

IX. TDP Reliability Documentation Completed. Integration of reliability requirements (milestone 4) and other information pertinent to the preliminary TDP (milestone VI) is completed.

X. Preliminary TDP Approved. The preliminary TDP containing reliability documentation is reviewed and approved.

b. Definition Phase (see figure B-6).

XI. RFP Initiated. The Request for Proposal (RFP) documentation is initiated by the consumer complete with the system mission and requests for contractor schedules and methods for development. The information to be included must consist of both technical and managerial aspects of the proposed engineering development. It should be information that will be meaningful to the potential contractors in preparing their respective proposals.

5. Prediction Requirement Established. The requirement for contractor prediction of item reliability is established and documented in the RFP. This requirement should include consideration of quantitative predictions and a schedule for updating predictions.

6. Apportionment Requirement Established. The requirement for contractor apportionment of item reliability is established and documented into the RFP. The requirement shall be clearly stated so as to assure achievement of item reliability. Any previous experiences with these systems shall be referenced in the RFP.

7. Record Keeping Requirement Established. The requirement for contractor record keeping of item reliability data is established and documented in the RFP. This requirement should call for an effective record keeping system which specifies the type of information to be recorded, such as reliability test results, technical problems, and progress reports on item reliability. It should also require a schedule for status reporting.

8. Testing Requirement Established. The requirement for the contractor to adequately demonstrate achievement of item reliability is established and documented in the RFP. This will include any definite tests required, methods of testing, conditions for testing, test schedule, use of test results, and other test requirements whenever applicable.

9. Trade-off Policy Requirement Established. The requirement for contractor establishment of a trade-off policy, covering the interrelationships of cost, support,performance, and safety as related to reliability, is documented into the RFP.

XII. RDP Approved. The RDP is reviewed for compliance with original reliability requirements. After compliance has been verified and approved, the RDP is used as a communication link between the Army and prospective contractors.

XIII. Contract Definition Contracts Awarded. The findings and recommendations resulting from evaluation of the initial proposals have been provided by the appropriate agency to Department of Army for contract award to potential development contractors.

XIV. Proposals Received and Review Initiated. As the contractor's proposals are received, they are thoroughly reviewed for compliance with reliability requirements as requested in the RDP.

XV. Organization Review Completed. Reviews of the prospective contractors' managerial acticities, including organization structures, training programs, and personnel technical capabilities. Results of this review should indicate whether or not the prospective contractor gives proper emphasis to reliability.

XVI. Testing Program Review Competed. The testing programs proposed by the interested contractors to demonstrate reliability are reviewed. These reviews encompass test environments, equipment, data feedback programs, analysis techniques, and use of test results to answer that the RFP requirements are included and understood.

XVII. Detailed Reliability Review Initiated. After all the proposals have been submitted, a comprehensive review of reliability provisions is initiated. The results of this review should plan an important role in deciding upon the prospective development contractor. The purpose of this review is to evaluate proposals for apportionment, prediciton, and trade-off policy with respect to feasibility, completeness, and clarity.

10. Block Diagram Review Completed. The reliability block diagram for the overall system is reviewed to assure all aspects of the system have been given proper consideration. This includes such things as determining if the diagram is sufficiently descriptive of the proposed system from both an operational and technical viewpoint. This diagram is the basis for a reliability apportionment model.

11. Standard Subsystems Review Completed. Proposed subsystems utilizing standardized parts and assemblies are evaluated for acceptability of reliability apportionment. Also, it should be assured by this review that standard subsystems have been employed when applicable in order to minimize cost.

12. Developmental Subsystem Review Completed. Proposed subsystems requiring development of new items are reviewed with regard to reliability apportionment. This review covers the non-standard subsystems that are to be employed in the various proposals. The proposals should provide adequate empirical data to determine that the required reliability of these new subsystems can be achieved. Also, math models should be provided in the proposals to illustrate how the reliabilities apportioned to the subsystems are to be achieved.

13. Reliability Design Techniques Evaluation Completed. The developmental subsystems are reviewed for reliability design techniques. This includes the proper use of partial redudancy, mixed models, standby redundancy, etc. The review should concentrate on

both the possible use of such techniques where they are applicable but not considered in the proposals, and the use of such techniques in situations where their use is not warranted because of already acceptable reliability, nonfeasibility, excessive costs, etc.

XVIII. Reliability Apportionment Evaluation Completed. Evaluation of the reliability apportionment section of the proposal is completed. This apportionment represents a possible means of achieving the required system reliability. It should be ascertained whether or not the apportionment has been done optimally; i.e., to minimize cost while maintaining overall system reliability.

14. History Check Evaluation Completed. The contractor's use of failure data concerning standard subsystems utilized in the proposals is checked for accuracy, applicability, and completeness. This evaluation should verify that the data used to support the use of any such subsystem was obtained from conditions similar to those which the subsystem will be subjected to in the new system. This evaluation establishes the validity of the data for reliability prediction purposes.

15. State of the Art Review Completed. Projections for the state of the art throughout development and production are reviewed and compared to the projection from the total feasibility study conducted earlier in the CDP. The purpose of such a review is to assure that the progress for development of such equipment is consistent with the anticipated schedule established earlier in the CDP. Any differences should be noted in order that potential problems can be identified. It is important that such problems be pointed out in this review because they may cause delay in project completion or even prove the project infeasible for completion.

16. Minimum Reliability Evaluation Completed. The minimum reliability predictions are evaluated to assure that the system requirement is met. If the system requirement is not met, the results should point out possible courses of action to correct such deficiencies.

17. Optimum Reliability Evaluation Completed. Predictions of desirable reliability are evaluated to assure that the predictions are realistic. Careful attention should be given to the contractor's underlying assumptions for meeting the reliability values quoted. Any

invalid assumptions or unrealistic reliability values should be called to his attention.

XIX. Reliability Prediction Accepted. Evaluation of the reliability prediction section of the proposals, as a suitable outcome for system reliability, is completed. The results of this evaluation are to be used in deciding which of the prospective contractors will receive the contract.

18. Trade-Off Policy Reliability Evaluation Completed. Any contractor proposed change in estimated development cost, time, performance, support, safety, human engineering, etc., made at the expense of reliability will be carefully evaluated as trade-off considerations.

XX. Trade-Off Policy Accepted. After the trade-off considerations are analyzed, the selected proposed trade-off policy is accepted.

XXI. Proposal Selection Completed. The most probable proposal(s) to succeed in development and to meet the technical requirements is(are) selected. Major reliability considerations, which the contractor expresses in his proposal, should be given proper emphasis in proposal selection.

XXII. Coordinated Test Program Approved (1400). The Coordinated Test Program (CTP) is an all-inclusive materiel test plan which specifies test objectives, number of prototypes to be available, environmental testing required, testing schedule, funding requirements, test support requirements, and whether an integrated, concurrent, or sequential testing program will be conducted. This milestone is completed on acceptance of the test program by major commands, project managers, or agencies involved. After acceptance, the program is included in the TDP. It is reviewed and updated at the technical characteristics, engineering concept, and design characteristics in-process review.

19. QMR Updated. Based on review and evaluation of the proposals, the QMR may need to be revised and updated. Changes affecting reliability will be worded so as not to lose the intent of the reliability requirement.

XXII. QMR Approved (0700). The updated QMR is reviewed with proper emphasis placed on reliability. After receiving a satisfactory review, the QMR is approved by Department of Army.

XXIV. Technical Characteristic IPR Completed (1500). This review is held upon receipt of the approved QMR and prior to finalizing the technical characteristics. The primary purpose of the technical characteristics in-process review (IPR) is to insure that the developer understands the requirement and has properly stated it in terms of technical characteristics. The technical characteristics, once approved by the proper authority, are used for full-scale development, either in-house or by contract.

20. TDP Revised. A re-evaluation and revision of the TDP incorporating inputs from the CTP, selected proposal(s), and the technical characteristic IPR, is completed. This TDP presents in detail an illustrative format and a comprehensive plan for development of a system. Factors considered on a continuing basis are the management plan, configuration management plan, test and evaluation plan, personnel and personnel training plan, logistics support plan, facilities, foreign technology, planned production, and technical documentation.

XXV. Updated TDP Approved (0900). The updated TDP is reviewed, with proper emphasis being placed on the reliability section, and approved. It is the prime basis for approval, disapproval, or modification of the project. It describes and is the approved plan for execution by the Army. The TDP is to be updated and changed as required throughout the program.

XXVI. Development Decision Made. A decision is made on whether or not to enter the development phase. The decision sh-uld be based on the results of effectiveness studies, as well as whether or not the proposals show that the reliability requirements can be met.

c. Development Phase (see figure B-7)

XXVII. Development Contract Awarded (2000). The primary contract is executed by the contractor and contracting officer for development of the item.

XXVIII. Reliability Specialist Assigned to Development Program. When an agency receives developmental responsibility for an item, an individual(s) knowledgeable in the technical aspects of reliability is(are) assigned the responsibility for monitoring for reliability throughout the development program. This is to provide assurance to the Army that reliability is being given proper emphasis in the development activities.

XXIX. Predevelopment Conference Completed. A predevelopment conference to clarify the contractor's responsibilities for meeting contractual requirements, including those relating to reliability, is completed.

XXX. Start Preliminary Design Review. Review during preliminary design, as performed by the contractor, is to include reliability considerations. Such reviews will facilitate early evaluation and identification of potential design trouble areas.

21. Math Model Evaluation Completed. The math model, including the block diagram, is evaluated for total consideration and adequacy. This provides assurance that all functional and failure modes have been properly considered and that the model provides an adequate representation of the relationship between item reliability and the reliability of subsystems, parts, and components.

22. Reliability Prediction Review Completed. Basic contractor reliability predictions, which support preliminary design, are reviewed to assure that predictions, including any revised predictions, are suitable and are based on sound rationale. Important considerations include standard parts, derating, functional complexity, redundancy, and failure definition.

23. Reliability Apportionment Review Completed. The reliability apportionment of the preliminary design is reviewed against the proposed values of the definition phase. Major deviations should be investigated. Of major importance is the compatibility of apportioned goals with the cost and schedule for fulfilling these goals.

24. Potential Problem Action Evaluation Completed. Evaluation of the corrective action program details as proposed by the contractor to handle potential reliability problems is completed. The program should provide for action which is both timely and technically adequate.

25. Review and Recommendation Completed. The preliminary development design is reviewed for reliability aspects and relevant trade-off recommendations are presented to decision makers.

XXXI. Engineering Concept In-Process REview Completed (1700). The contractor's preliminary design is reviewed and decisions are reached on all recommendations. These activities may become a part of the engineering concept review (ECR). Rigid attention to reliability in this IPR assures that the design concept is not beyond the state of the art and that all feasible engineering approaches have been considered.

XXXII. Initate Engineering Design Review. Reliability analysis of the system as development proceeds from preliminary design into detailed engineering development is initiated. Analyses should be continuously updated to reflect the most recent information available.

26. First Hardware Assessment Completed. The reliability assessment of experimental and standard component test results and of the contractor's failure mode and effects analysis (FMEA) is completed. Data is obtained from tests conducted by the contractor. Analysis of these data will provide an assessment of reliability growth and may provide an identification of additional potential problem areas.

27. Corrective Action Verification Completed. Verification that the corrective actions which have been taken by the contractor will improve reliability and are in accordance with established trade-off policy.

28. GFE Document Completed. A document stating the predicted reliability and potential reliability problems of proposed government-furnished equipment (GFE) is completed. This report will be based upon prior field use data and, when necessary, due to lack of sufficient data on the capabilities of GFE, tests of these items shall be performed. When the apportioned reliability requirement is not met, a trade-off recommendation is established and placed in the document.

29. Math Model Review Completed. The math model is reviewed for adequacy based on the latest reliability information. Requests for updating are made as appropriate. An updated model will provide an assessment of item reliability, reflecting latest test information and latest block diagram changes which have resulted from corrective action.

30. Reliability Prediction Review Completed. Any revised predictions made by the contractor are reviewed for adequacy and rationale. The predictions should reflect the updated block diagram and math model.

31. Reliability Apportionment Review Completed. The updated reliability apportionment is reviewed to insure that all changes are acceptable to the government and that they do not conflict with other performance parameters. The revised apportionment goals should be compatible with economic and schedule constraints.

32. Maintenance Support Plan Inputs Completed (2200). As the initial maintenance support plan is prepared, reliability personnel have the responsibility of supplying data. This data should contain as a minimum: failure mode; failure rate; 50% failure life; and the failure impact. The purpose of this plan is to assure timely availability of all elements required for support of the item in the field.

33. Recommendations for Trade-Offs Submitted. The contractor's suggested trade-offs are assessed and recommendations are submitted for decision at the design characteristics in-process review.

XXXIII. Design Characteristics In-Process Review Completed (2700). Results of the engineering design are reviewed and decisions reached on all recommendations to avoid development of hardware that does not satisfy the requirement.

XXXIV. Detailed Design Review Initiated. Review of all detailed design efforts in terms of reliability is initiated. This review will normally be carried down to part level. Major effort should concentrate in areas such as unique design and new or unproven parts.

34. Engineering Design Testing Completed (290m). Engineering design tests are a series of tests conducted on a system or item by components or subassemblies as soon as they are fabricated and delivered by the contractor. These tests are conducted by or under the control of the design agency for determining inherent reliability. The series includes statistical laboratory-controlled tests to determine the effects of functional and environmental stresses on the subsystems. The purpose of such testing is to collect design data, confirm preliminary concepts and calculations, and determine the compatibility of components. The activity preceding this event is completed when the developing agency is ready to release the prototype for ET/ST testing or to procure ET/ST models.

35. Engineering Design Testing Results Analysis Completed. The results of the engineering design tests are analyzed to pinpoint areas of low reliability. The analysis will cover mechanical, electrical, and chemical properties, as well as the functional interfaces, of the subsystems. GFE items are not to be passed over during the analysis.

36. Probable Failure Modes Review Completed. Probable failure modes of the system are updated by the contractor and reviewed by the government for critical modes and frequency of failure.

37. Proposed Corrective Actions Review Completed. The contractor's suggested corrective actions are reviewed to determine the effects on reliability and trade-off policy.

38. Final Data Check Completed. The final data check is completed. This involves the review and evaluation of the final updated math model, system apportionment, and reliability prediction based on test results.

39. Initiate Trade-off Study. Initiate trade-off study based on results of detailed design efforts.

40. Formulation of Trade-Off Recommendations Completed. To preclude potential problems, trade-off recommendations are formulated in consonance with trade-off policy established during the definition phase and submitted to decision level.

XXXV. Prototype System In-Process Review Completed (3300). This IPR is held after completion of the engineering design tests and prior to prototype testing to insure compliance with the QMR. The data from the detailed design phase of development is reviewed and decisions reached on all recommendations.

XXXVI. Advance Production Engineering (APE) Initiated (3400). APE activities are usually initiated after the prototype system IPR. However, APE may be initiated at any time during the development phase when research and development design is sufficiently stable and funding is available. APE includes advising design and development engineers on materiel and assemblies, economy of materiel, and process limitations of various materials for simplification, developing technical data for production material, including conversion of technical data which results from development and preparation of specifications, standards, purchase descriptions, packaging data, and quality test and inspection procedures; development of special quality assurance production equipment (tools, fixtures, and jigs; fabrication of prototypes or special production equipment; establishment, operation, and support of pilot production lines to prove capability to produce; fabrication of preproduction prototypes to prove quantity producibility, prove out the technical data package and quality assurance procedures for equipment, and prove that production engineering material meets requirements; and the initiation of studies or measures to devise improved or new manufacturing methods and techniques to reduce production costs, reduce item leadtime, improve process reliability, or assure materiel producibility. APE activities initiate the start of the production phase and overlap the development phase, and as such are considered to be a major decision point.

41. Technical Data Package Initiated. Preparation of a package, consisting of drawings, specifications, and sometimes descriptions of the processes needed for manufacture, is started. The technical data package has as its aim the production of the system using conventional practices within the capability of industry. It is used as the basis for invitation for bid (IFB) of production contracts.

XXXVII. Prototype Testing and Demonstration Initiated. Prototype tests to evaluate performance and demonstrate reliability are initiated. Test results will be used to assess design reliability and pinpoint system problem areas prior to type classification.

42. R&D Acceptance Test Completed (3600). The R&D acceptance test is conducted by or under the control of the responsible government agency of an item or system designed and developed by the contractor to insure that the specification of the development contract has been fulfilled. The test is completed when the responsible government agency has approved test results and determines the system acceptable for further testing. The reliability analysis of the results should be used for guidance in the next test series.

43. Integrated ET/ST Completed (5100). The integrated engineering and service tests conducted by the government are completed. The objective of the engineering test is to determine the safety and technical performance characteristics of the prototypes. The service test has as its objective the determination of whether the system satisfies the QMR and is suitable for use by the Army. A reliability analysis is conducted and recommendation formulated for the IPR.

XXXVIII. Integrated ET/ST In-Process Review Completed (5200). This review will be held upon completion of the integrated ET/ST to arrive at a recommendation concerning adoption of the item or system as standard or planned action to remedy inadequacies and deficiencies found in the test.

44. Check Test Completed (5900). The check test to prove out corrective actions taken by the contractor on deficiencies from ET/ST is completed. Test results should be assessed to assure that corrective actions meet reliability requirements and that the system is now suitable for type classification. This milestone is completed when the final check test report, is such a test is required, is forwarded to AMC. These reports indicate that the item and all elements of maintenance support have been found suitable or unsuitable for Army use.

XXXIX. Type Classification Completed (6000). Standard classification designates the items that have been adopted as suitable for Army use, are acceptable as assets to meet operational requirements, are authorized for inclusion in equipment authorization documents, and are described in published adopted items lists Completed upon approval of AMC Technical Committee action by Headquarters, Department of Army.

45. Technical Data Package Preparation Completed. A technical data package, including inspection tables and AQL's to assure that the reliability demonstrated in development is kept during production, is prepared for the IFB.

XL. Invitation for Production Bid Released. The Invitation for Bid (FB) or Request for Proposal (RFP) for production of the system is released.

d. Production Phase (see figure B-8).

XLI. Contractor's Capability Review Completed. After receipt of bids, a production preaward survey board completes assessment of contractor's capability to perform production functions essential to upholding the reliability designed into the item. Primary emphasis should be given to the completeness of each prospective contractor's production testing program. These tests are required to assure that the product from production meets the user reliability requirements and is at least as good as the reliability requirements expressed in the QMR or the type classified item. Such tests primarily serve to prevent production of unsatisfactory products.

XLII. Production Contract Awarded (6500). This event is the date of award of a production contract by the contracting officer after securing required award approvals or clearances.

XLIII. Reliability Specialist Assigned to Production Program. After award of the production contract, a reliability specialist(s) from the procuring agency, knowledgeable in the technical aspects of reliability, is(are) assigned the responsibility for reliability through the production phase. This assignment includes assuring that production does in no way reduce the reliability designed into the item.

XLIV. Preproduction Conference Completed. The preproduction conference to clarify the contractor's responsibility for meeting contractual requirements, including those relating to reliability, is completed. Primary emphasis should be given to the contractor's manner for carrying out production tests. It should be pointed out by the procuring agency that a production testing sequence is required for each producer and will usually be repeated whenever there is a lengthy delay

or interruption of production, or where major changes (engineering change orders) during production are effected. The frequency and extent of production testing will vary from contract to contract.

46. Component Test Results Review Completed. Components fabricated by production techniques are tested and the results reviewed to assure reliability is not abridged. Testing is normally conducted by the contractor.

47. Verification of AQL's and Inspection Tables Completed. A re-inspection of accepted lots to verify that AQL's and sampling tables are providing lots capable of meeting the required reliability is completed.

48. Engineering Change Test Review Completed. Engineering changes are tested as appropriate and the results reviewed to determine the effect on system reliability. Testing is conducted by the government.

49. Records Review Initiated. After the contractor makes available all reliability records required by the contract, an in-depth review is initiated.

50. Contractor's Records Review Completed. The reliability records of the contractor are reviewed against government records to assure potential problems have been identified and steps taken to preclude them.

51. Analysis of Preproduction Test Results Completed. Results of the preproduction test are analyzed. Reliability problems are identified and trade-off recommendations established if necessary.

XLV. Trade-Off Decision Completed. Trade-off decisions affecting reliability and based upon component and preproduction test results are completed. This decision should include what reliability will be traded with so that the produced item still will be effective.

52. IPT Evaluation Completed. Initial production test (IPT) results are evaluated to determine system reliability and degree of conformance to type classified system.

53. Confirmatory Test (Type I) Completed (7500). A confirmatory type I test of a system is conducted to insure that modification required by corrective actions and trade-offs are acceptable. The results are to be assessed for their impact on system reliability.

54. Confirmatory Test (Type II) Completed (7700). A confirmatory type II test of a system is used to preclude time-consuming retrofit programs by early and intensive use in the field by using table of organization and equipment (TOE) type units to conduct the test. The results are to be assessed for their impact on system reliability.

55. In-House Statistical Analysis Completed. A thorough pre-release data analysis of the produced system, covering all aspects of the production contractual documents, is completed. This analysis is based on all data from production records and testing, the results of which are to be used for determining item suitability for release to user. Carrying out of this task is the responsibility of the procuring agency.

XLVI. Statistical Analysis Review Completed. The in-house statistical analysis is reviewed to determine if the system meets the minimum requirements and is suitable for release to the user.

XLVII. Release Request Initiated. Release of the production item to the user if requested. This is the date when the first item has been accepted by the procuring agency and is available for delivery to an operational or training activity with the assurance that proper support requirements are available.

e. Operation and Disposal Phases (see figure B-8).

XLVIII. Release Completed (7900). All necessary action to release the item to the user is completed. This event is completed upon first shipment of end item to the overseas command or CONUS installation scheduled to receive the end item in accordance with the approved distribution plan.

56. ICT Evaluation Completed. Inspection comparison test (ICT) results are evaluated to assure production reliability does not decrease below limits of the QMR. This evaluation is the responsibility of the production reliability specialist.

XLIX. Reliability Specialist Assigned to Operation Phase. After release of the system to the user, a reliability specialist, knowledgeable in the technical aspects of reliability, is assigned the responsibility for reliability through the operation phase.

57. Field Reliability Determined. Based on data feedback from the field use of the item, field reliability level is determined. This information may be useful for future procurement of this item or for establishment of requirements associated with development of future generation items.

58. Storage Reliability Determined. The data from depot storage inspection and depot maintenance reports are used to determine storage reliability level of the item. The information may also be used as input to future procurement and development of next generation items.

59. Rebuilt Equipment Reliability Determined. Field data and reconditioning test results are used to establish the reliability of rebuilt equipment.

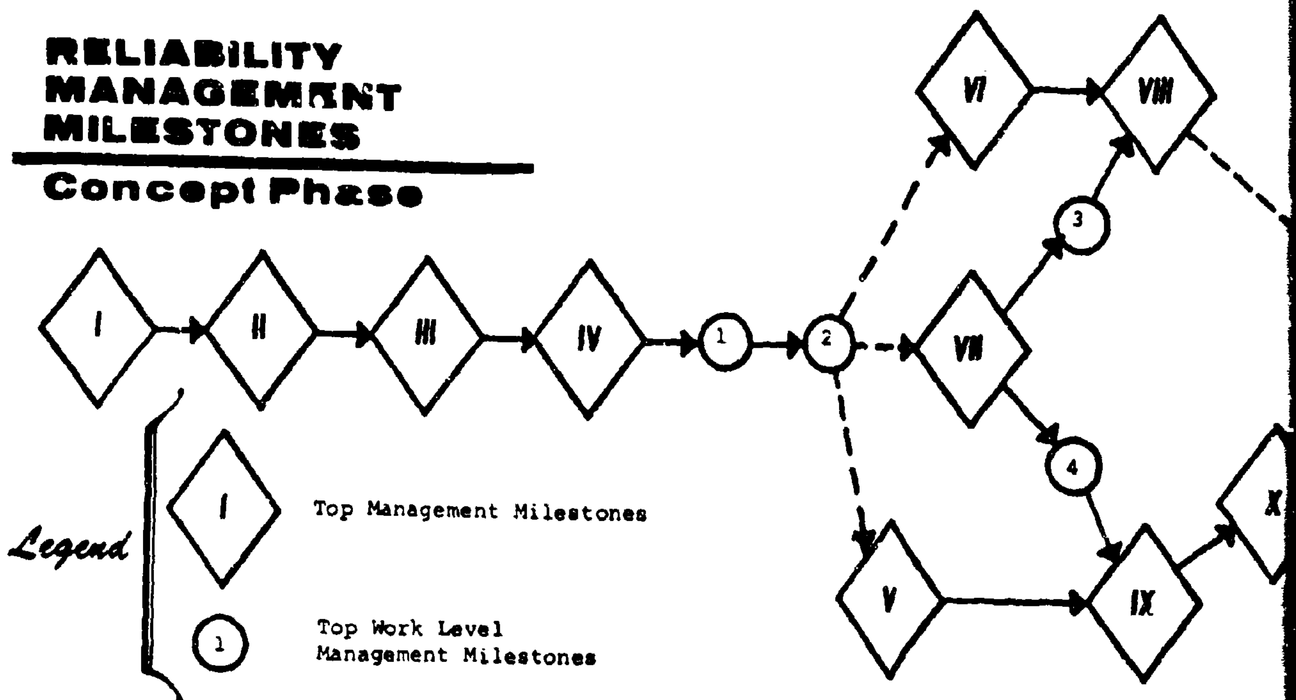
60. Parts Reliability Determined. Data from field, storage, and rebuild are cumulated and analyzed to determine parts reliability for history banks.

L. Statistical History Review Completed. Prior to a second buy or disposal action, the statistical history of the system is reviewed. The operational reliability demonstrated in the field is checked for parametric deviation from production test results.

LI. Equipment Disposal Completed (8700). The point in time when disposal and/or demilitarization of the system is completed.

RELIABILITY MANAGEMENT MILESTONES

Concept Phase



I. Guidance Documents Initiated

II. User Requirements Established

III. Technological Forecasting Completed

IV. QMDO Approved and Assigned (0100)

1. Total Feasibility Study Completed (0600)

2. QMA Established

V. TDP Initiated

VI. QMR Initiated

VII. Reliability Documentation Initiated

3. QMR Reliability Requirements Established

NOTE: Completion of these milestones does not require formalized scheduling other than that required by AMCR 11-27.

VIII. Draft Propo

4. TD

IX. TDP Reliabi

X. Preliminary

XI. RFP Initiate

5. Pr

6. App

7. Re

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9. Tr

XII. RFP Approv

XIII. Award of Co

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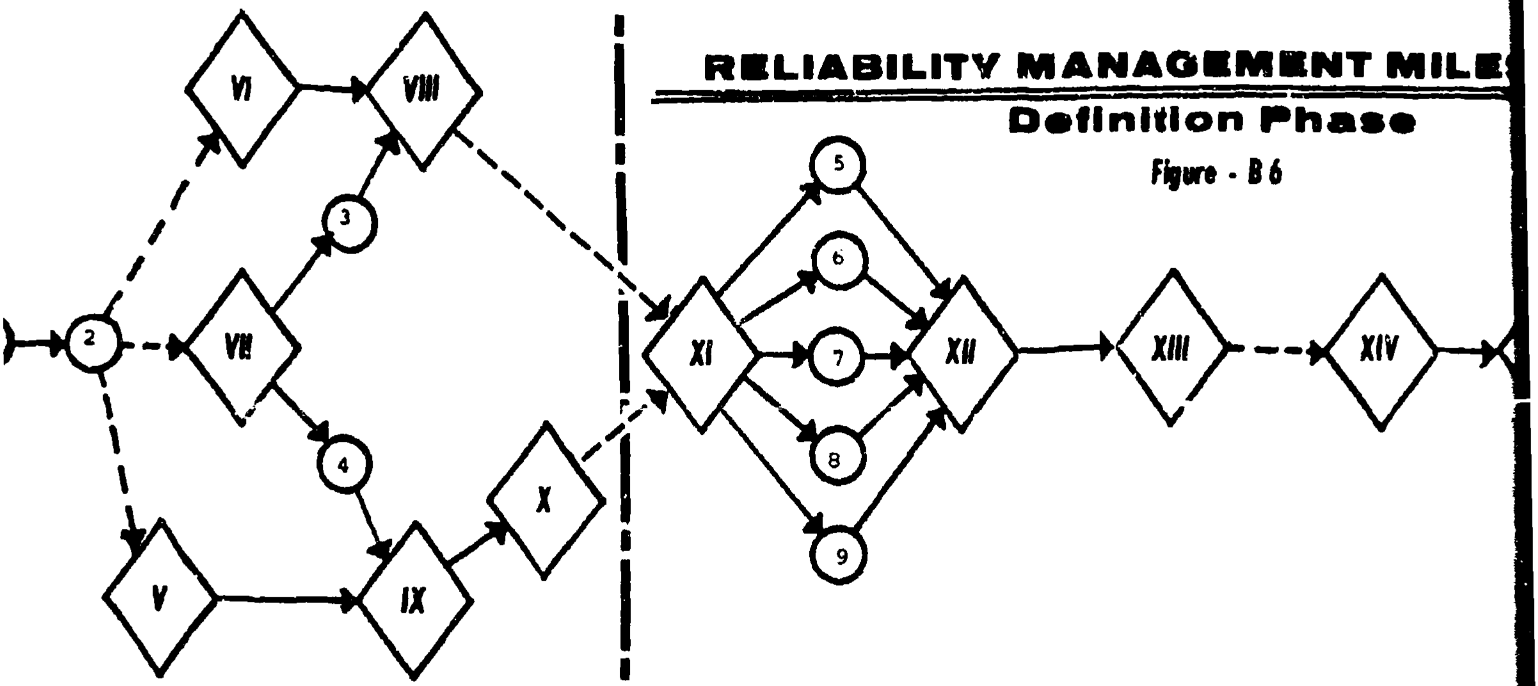
RELIABILITY MANAGEMENT MILE

Definition Phase

Figure - B 6



malized

VIII. Draft Proposed QMR Submitted (0400)

4. TDP Reliability Requirements Established

IX. TDP Reliability Documentation Complete

X. Preliminary TDP Approved

XI. RFP Initiated

5. Prediction Requirement Established

6. Apportionment Requirement Established

7. Record Keeping Requirement Established

8. Testing Requirement Established

9. Trade-Off Policy Requirement Established

XII. RFP Approved

XIII. Award of Contract Definition Contracts

XIV. Pro

XV. Orga

XVI. Test

XVII. Deta

1

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XVIII. Reli

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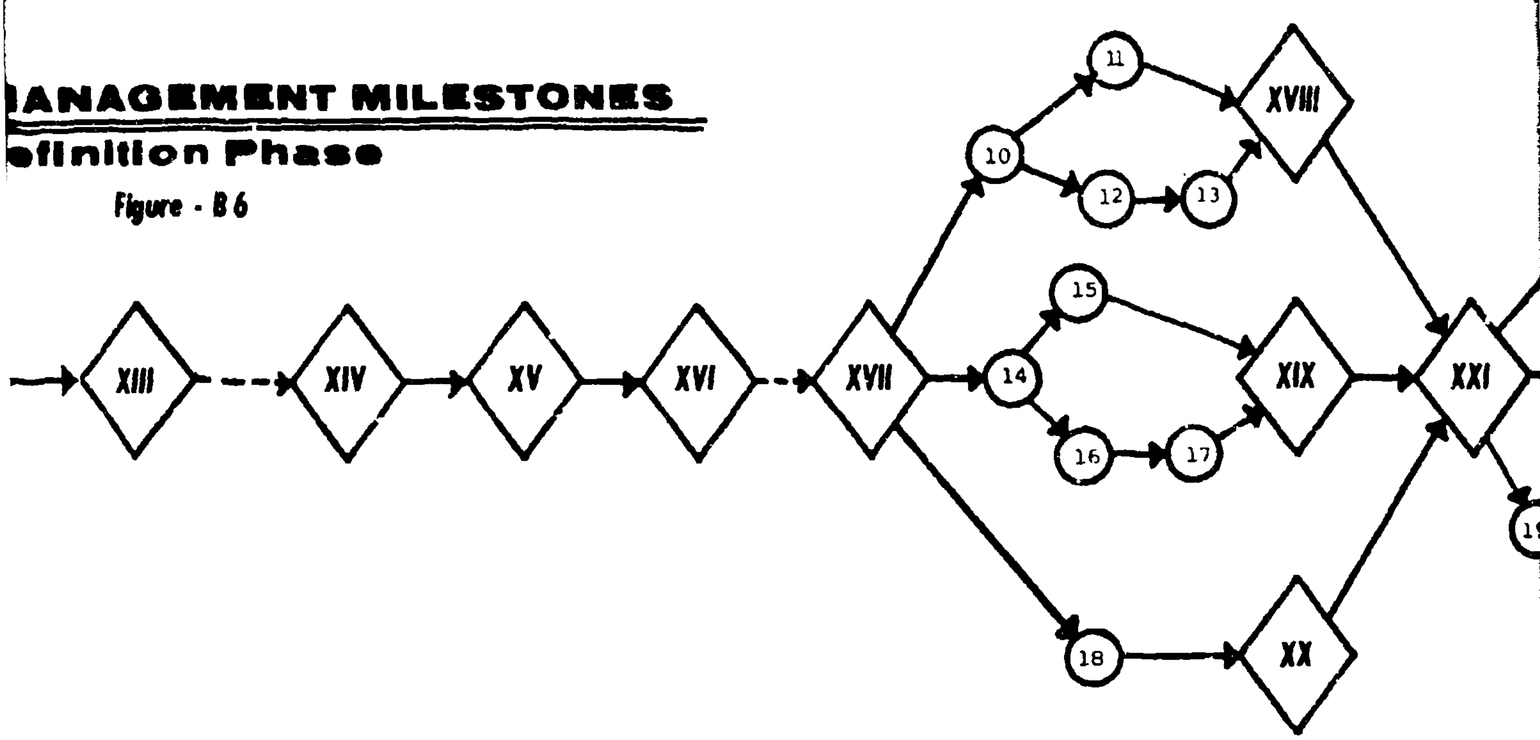
ANAGEMENT MILESTONES
efinition Phase

Figure - B6



XIV. Proposals Received and Review Initiated

XV. Organization Review Completed

XVI. Testing Program Review Completed

XVII. Detailed Reliability Review Initiated

  10. Block Diagram Review Completed

  11. Standard Subsystems Review Completed

  12. Developmental Subsystems Review Completed

  13. Reliability Design Techniques Evaluation Completed

XVIII. Reliability Apportionment Evaluation Completed

  14. History Check Evaluation Completed

  15. State of the Art Review Completed

  16. Minimum Reliability Evaluation Completed

  17. Optimum Reliability Evaluation Completed

XIX. Rel

XX. Tra

XXI. Pro

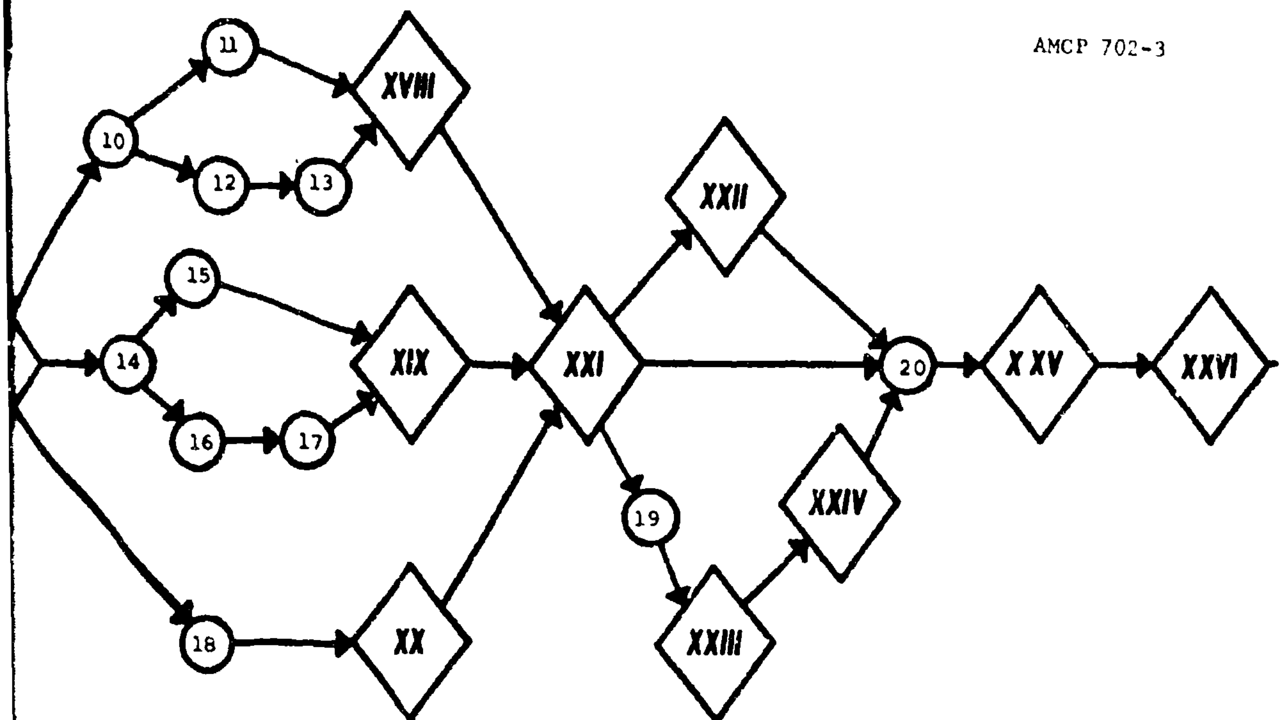
XXII. Coo

XXIII. QM

XXIV. Tec

XXV. Upd

XXVI. Dev



ated

pleted

w Completed

Review Completed

es Evaluation Completed

Completed

ompleted

pleted

tion Completed

tion Completed

XIX. Reliability Prediction Accepted

18. Trade-Off Policy Reliability Evaluation Completed

XX. Trade-Off Policy Accepted

XXI. Proposal Selection Completed

XXII. Coordinated Test Program Approved (1400)

19. QMR Updated

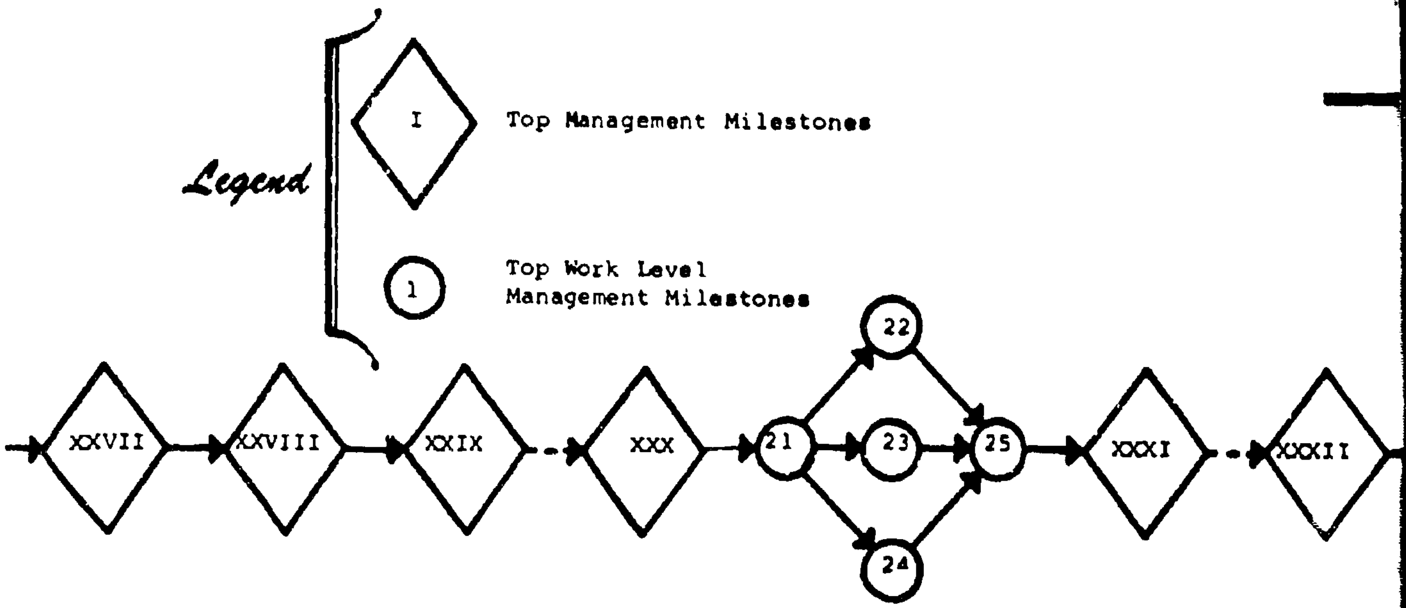
XXIII. QMR Approved (0700)

XXIV. Technical Characteristic IPR Completed (1500)

20. TDP Revised

XXV. Updated TDP Approved (0900)

XXVI. Development Decision Made



XXVII. Development Contract Awarded (2000)

XXVIII. Reliability Specialist Assigned to Development Program

XXIX. Predevelopment Conference Completed

XXX. Initiate Preliminary Design Review

- 21. Math Model Evaluation Completed
- 22. Reliability Prediction Review Completed
- 23. Reliability Apportionment Review Completed
- 24. Potential Problem Action Evaluation Completed
- 25. Review and Recommendation Completed

XXXI. Engineering Concept In-Process Review Completed (1700)

XXXII. Initiate Enginee

- 26. First Hard
- 27. Correctiv
- 28. GFE Docu
- 29. Math Mod
- 30. Reliability
- 31. Reliability
- 32. Maintenan
- 33. Recommen

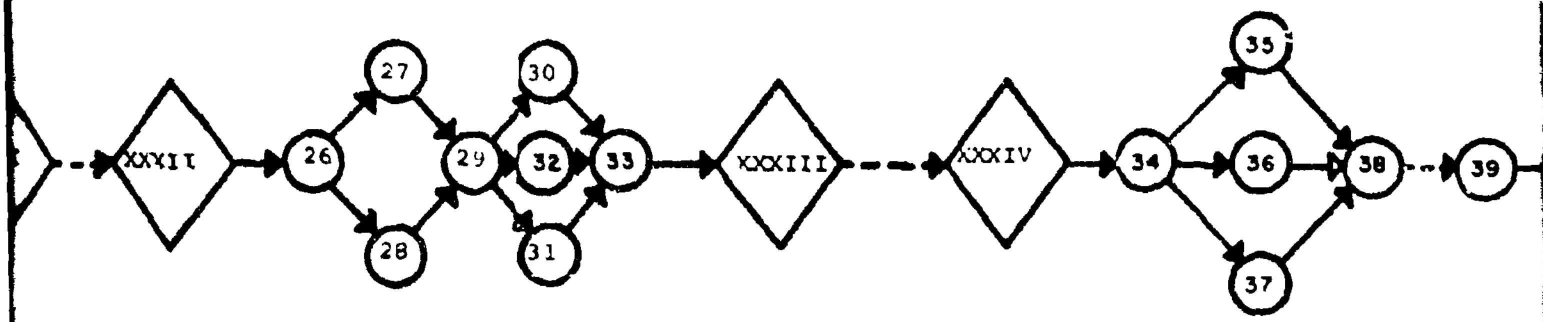
XXXIII. Design Characteri

NOTE: Completion of these milestones does not require formalized scheduling other than that required by AMCR 11-27.

RELIABILITY MANAGEMENT MILESTONES

Development Phase

Figure B-7



Initiate Engineering Design Review

26. First Hardware Assessment Completed
27. Corrective Action Verification Completed
28. GFE Document Completed
29. Math Model Review Completed
30. Reliability Prediction Review Completed
31. Reliability Apportionment Review Completed
32. Maintenance Support Plan Inputs Completed (2200)
33. Recommendations for Trade-Offs Submitted

esign Characteristics In-Process Review Completed (2700)

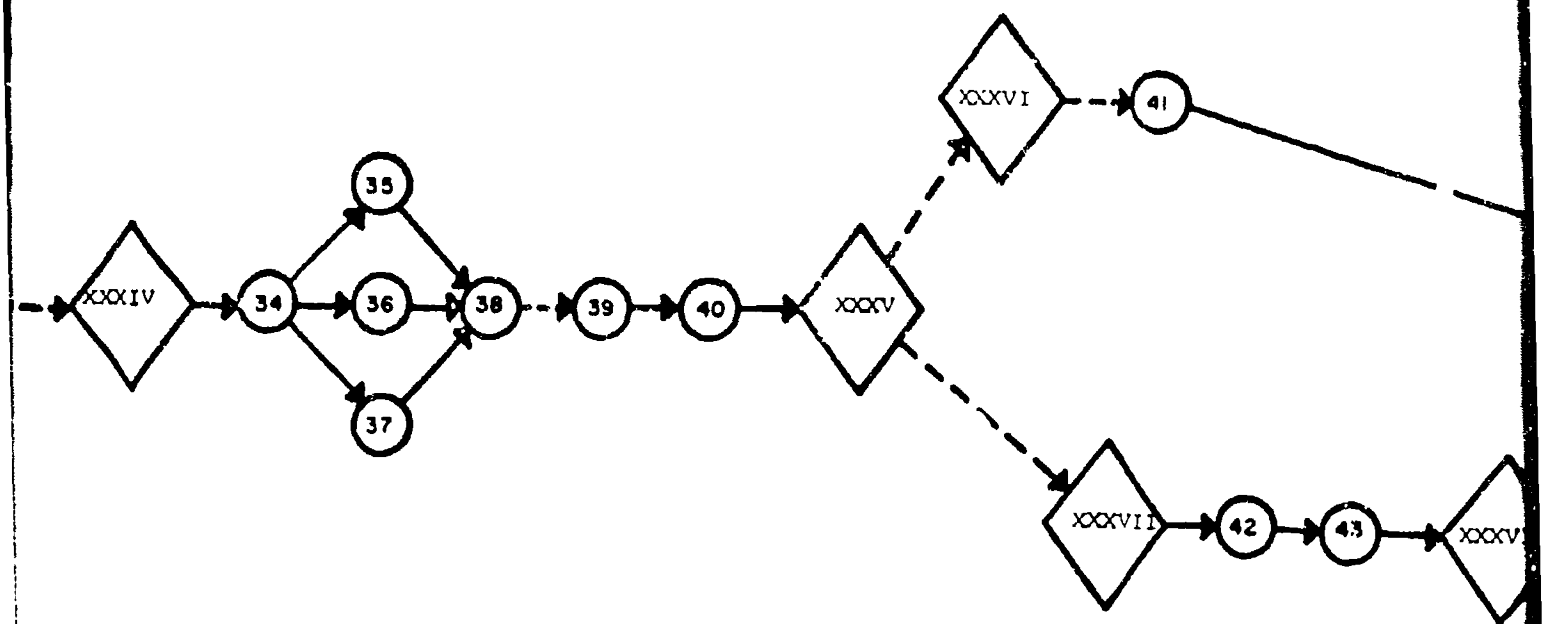
XXXIV. Detailed Design Review Initiated

34. Engineering Design Testing
35. Engineering Design Testing
36. Probable Failure Modes Re
37. Proposed Corrective Action
38. Final Data Check Completed
39. Initiate Trade-Off Study
40. Formulation of Trade-Off R

XXXV. Prototype System In-Process Review

ENT MILESTONES

Phase



XXXIV. Detailed Design Review Initiated

34. Engineering Design Testing Completed (2900)

35. Engineering Design Testing Results Analysis Completed

36. Probable Failure Modes Review Completed

37. Proposed Corrective Actions Review Completed

38. Final Data Check Completed

39. Initiate Trade-Off Study

40. Formulation of Trade-Off Recommendations Completed

XXXV. Prototype System In-Process Review Completed (3300)

XXXVI. Advance Production Engin

41. Technical Data

XXXVII. Prototype Testing Initiate

42. R&D Acceptance

43. Integrated ET/ST

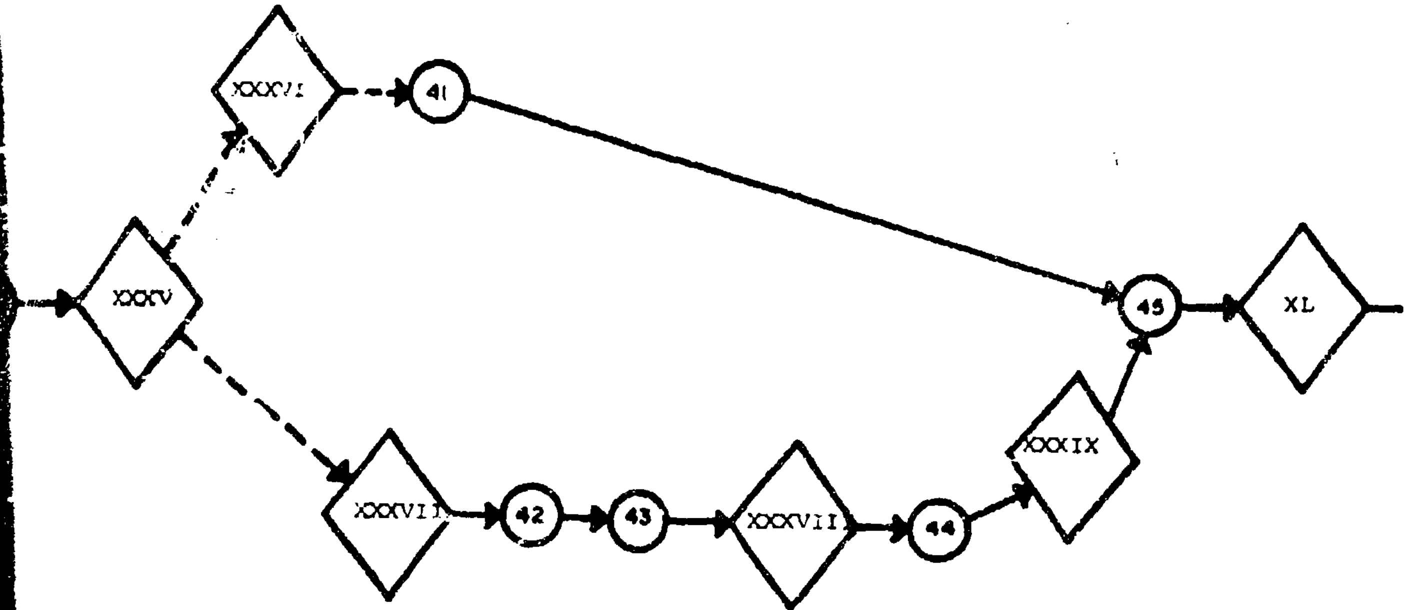
XXXVIII. Integrated ET/ST In-Proc

44. Check Test Comp

XXXIX. Type Classification Compl

45. Technical Data Pa

XL. Invitation for Production B



XXXVI. Advance Production Engineering (APE) Initiated (3400)

...leted (2900) 41. Technical Data Package Initiated

...ts Analysis Completed XXXVII. Prototype Testing Initiated

...ompleted 42. R&D Acceptance Test Completed (3600)

...iew Completed 43. Integrated ET/ST Completed (5100)

XXXVIII. Integrated ET/ST In-Process Review Completed (5200)

44. Check Test Completed (5900)

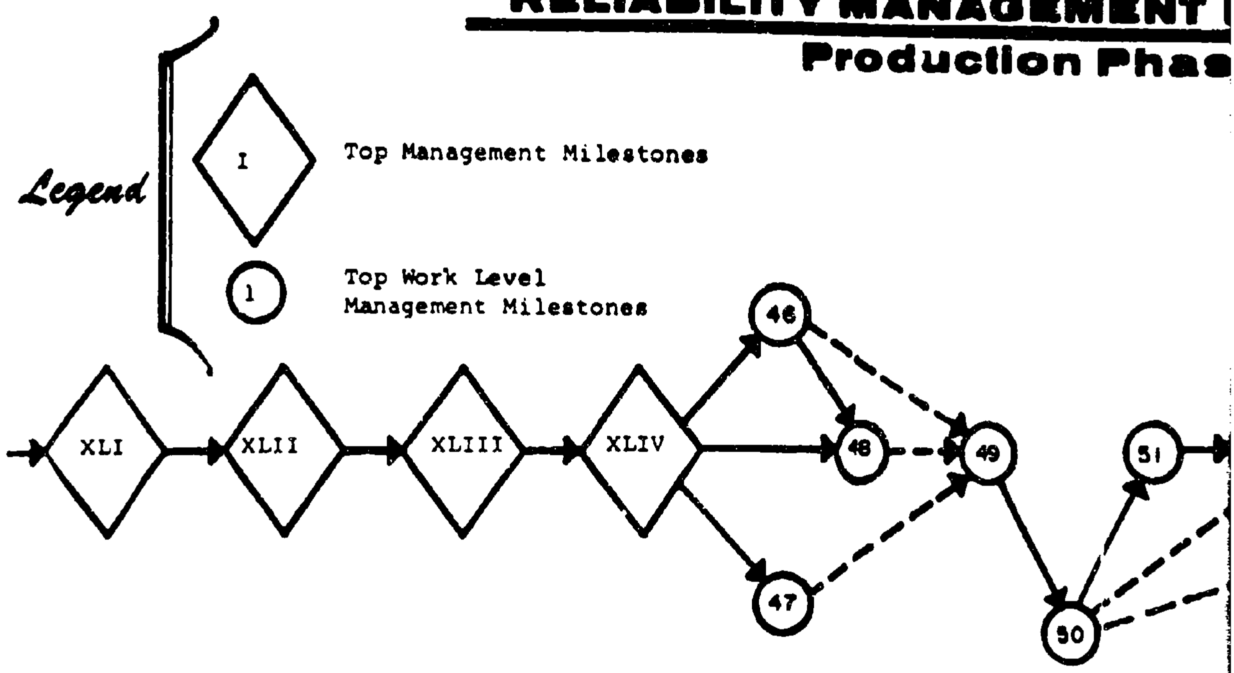
...nendations Completed XXXIX. Type Classification Completed (6000)

...pleted (3300) 45. Technical Data Package Preparation Completed

XL. Invitation for Production Bid Released

RELIABILITY MANAGEMENT

Production Phas



XLI. Contractor's Capability Review Completed

XLII. Production Contract Awarded (6500)

XLIII. Reliability Specialist Assigned to Production Program

XLIV. Preproduction Conference Completed

46. Component Test Results Review Completed
47. Verification of AQL's and Inspection Tables Completed
48. Engineering Change Test Review Completed
49. Records Review Initiated
50. Contractor's Records Review Completed
51. Analysis of Preproduction Test Results Completed

XLV.

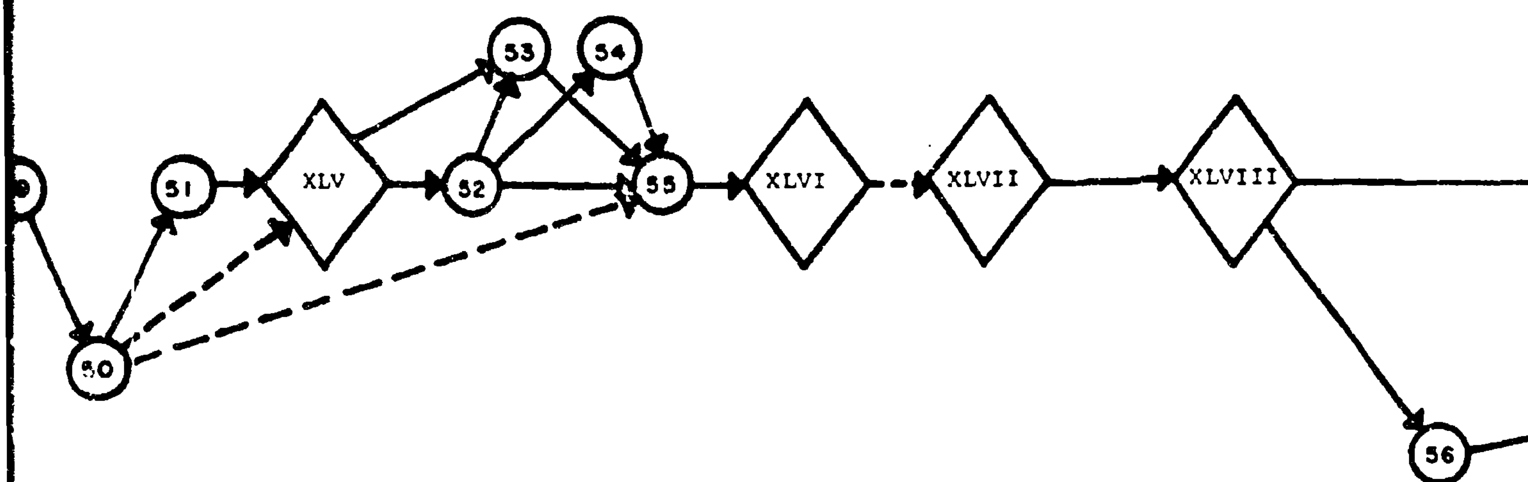
XLVI.

XLVII

GEMENT MILESTONES

ion Phase

Figure B-8



npleted

ompleted

XLV. Trade-Off Decision Completed

52. IPT Evaluation Completed

53. Confirmatory Test (Type I) Completed (7500)

54. Confirmatory Test (Type II) Completed (7700)

55. In-House Statistical Analysis Completed

XLVI. Statistical Analysis Review Completed

XLVII. Release Request Initiated

XLVIII. Release Com

56. ICT

XLIX. Reliability S

57. Fie

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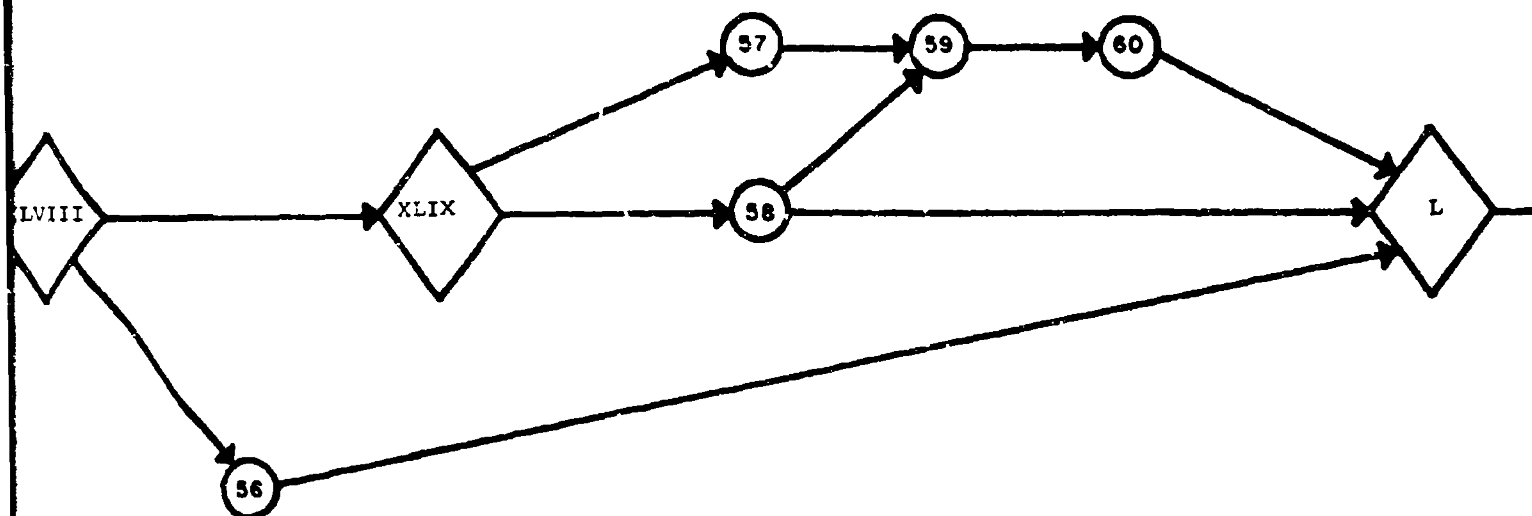
RELIABILITY MANAGEMENT MILESTONES
Operation & Disposal Phases



XLVIII. Release Completed (7900)

56. ICT Evaluation Completed

XLIX. Reliability Specialist Assigned to Operation Phase

57. Field Reliability Determined

58. Storage Reliability Determined

59. Rebuilt Equipment Reliability Determined

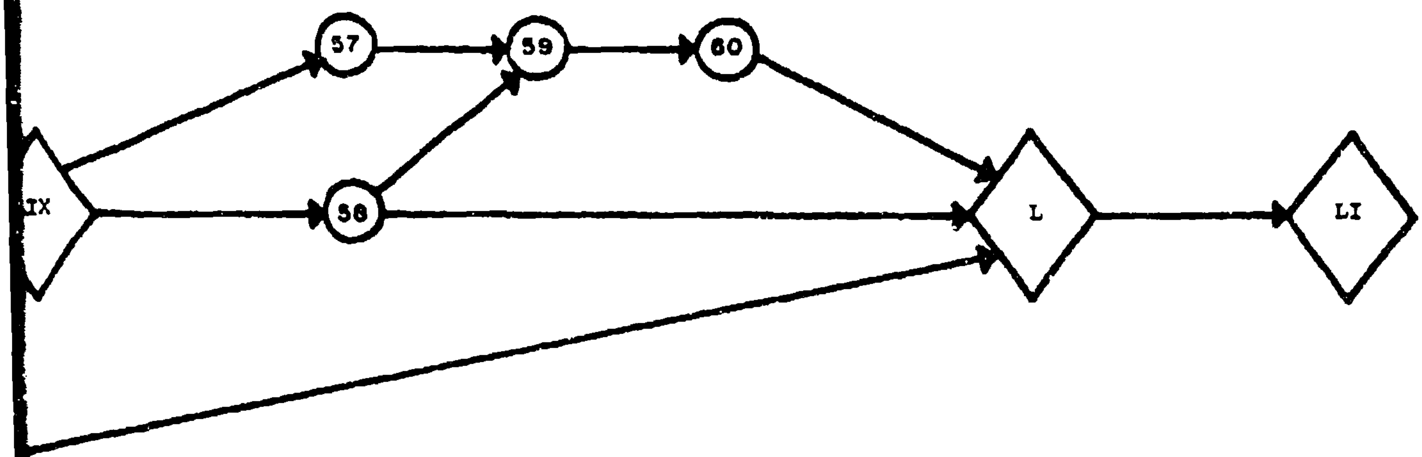
60. Part Reliability Determined

L. Statistical History Review Co

LI. Equipment Disposal Complet

NOTE: Completion of these milestones does no scheduling other than that required by A

LIABILITY MANAGEMENT MILESTONES
Operation & Disposal Phases



(7900)

tion Completed

t Assigned to Operation Phase

iability Determined

iability Determined

quipment Reliability Determined

ility Determined

L. Statistical History Review Completed

LI. Equipment Disposal Completed (8700)

NOTE: Completion of these milestones does not require formalized scheduling other than that required by AMCR 11-27.

APPENDIX C

TECHNICAL REQUIREMENTS ANALYSIS AND DOCUMENTATION OF RELIABILITY REQUIREMENTS

Section I. INTRODUCTION

C-1. General. The contents of this appendix are intended to provide the reader with guidance for implementation of the discussions in chapter 3. In addition to examples, it provides information explaining how reliability requirements should be stated in QMR's/SDR's. A discussion of the relationship between reliability and incentive contracting is also included.

Section II. COMMENTS ON TYPICAL FAULTS IN STATING RELIABILITY REQUIREMENTS IN QMR's/SDR's

C-2. Examples. The following examples represent typical cases as to how reliability requirements have been stated in QMR's and SDR's. Following each example are comments which critique the example and state ways of improving the requirement. When writing reliability requirements in QMR's/SDR's, the following may be used as guidance:

a. Draft Proposed QMR for Automatic Test Equipment for Communications Electronic Material:

(1) Requirement: The mission duration for ATE/C-E at all maintenance levels will be eight hours of operation. Assuming an exponential failure distribution, a mission reliability for a mean time between failures (MTBF) of 100 hours would be 92.3%.

(2) Comment: Weakness is exhibited in three elements of this requirements paragraph.

(a) The requirement, as stated, is not a requirement but merely a statement of mathematical fact under the assumption of an 8-hour mission duration and an exponential distribution of failure times.

(b) There are two possible interpretations of the word failure. First, a failure could be defined as the inability of the tester to detect the fault when a fault exists. On the other hand, it could be defined as a false detection of a nonexistent fault. It could also be both. This should be explicitly defined in order for the reliability requirement to be meaningful.

(c) At this point of the life cycle, mission reliability should be expressed as a probability and not in terms of MTBF; i.e., it is premature to assume the distribution of failure times to be exponential or any other distribution.

b. Draft Proposed SDR for a Short Range Mission Radio Set:

(1) Requirement: Mission reliability: 100% capable of performing a mission over a twelve-hour period without changing batteries when transmitting one minute out of every ten.

(2) Comment: Reliability may be considered to be probability of successful operation for the time period indicated. In this context, the requirement as stated is impossible to comply with. All items have some distinct possibility of failing during any given period of operating time, and this possibility has been ruled out by the statement above.

c. Draft Proposed SDR for Cold Water Detergents:

(1) Requirement:

(a) Mission reliability: 99%.

(b) Availability (Combat Ready Rate): 99%.

(c) Reliability after storage: 97% after two years' storage.

(2) Comment: The quantitative requirements pertaining to mission reliability, availability, and reliability after storage should be deleted in each case. These parameters are not applicable in the frame of reference intended by AR 705-50.

d. Draft Proposed SDR for Forward Area Air Defense Alert Radar (FAAR):

(1) Requirement: The inherent equipment reliability shall provide as a minimum a reliability of ____% for a critical time of ____ hours and mean time between failures (MTBF) of ____ hours.

(2) Comment: Only one of the parameters required to be included in QMR/SDR by AR 705-50 was specifically identified in this SDR, and this requirement (reliability) is stated in such a manner that it would be impossible to comply with; i.e., the mission duration, reliability requirement, and MTBF were mutually incompatible.

C-3. Review of reliability requirements in QMR's and SDR's. The following should be considered when reviewing reliability requirements in QMR's and SDR's to determine their technical adequacy and feasibility of attainment. Maximum coordination with counterparts at USACDC is encouraged in connection with making this determination.

a. The document should include a statement relative to what constitutes a failure. Since the word reliability normally implies probability of successful operation (nonfailures) for a specified period of time, it can only take on true meaning if failure is defined.

b. The mission duration should be defined. Often QMR's and SDR's include a vague statement, such as the mission duration will vary depending upon a number of influencing factors. Yet, many of these same documents include a quantitative, minimum reliability requirement identified as being essential. Again, since reliability normally means probability of successful operation for a specified period of time, it is relatively meaningless to include a quantitative reliability requirement unless it is associated with a definite period of time (mission duration), with the obvious exception of nontime dependent devices such as one-shot devices, (e.g., munitions).

c. There should not be a statement associating a quantitative level of statistical confidence with the reliability requirement, such as: "the minimum acceptable reliability shall be 95% to be demonstrated at a 90% confidence level." Degree of confidence is determined when designing the test plan and, as such, is influenced by factors such as cost, time, and required precision of test results.

To specify it in a QMR or SDR as a requirement that must be met in association with a reliability requirement is premature. It should suffice that a quantitative reliability requirement be specified in the QMR/SDR.

d. Reliability requirements should neither be too lax nor too stringent. They are seldom too lax, but often too stringent and in some cases, literally impossible to attain and verify. A critical analysis should therefore be made on a case-by-case basis to determine the feasibility of attaining and verifying the achievement of the reliability requirements within the current or projected state of the art and within time and cost constraints.

e. Analysis of the reliability requirements in QMR's and SDR's should include a determination whether all of the appropriate requirements of AR 705-50 have been considered. Two of the most repetitive faults of QMR's and SDR's are the exclusion of these requirements when they are applicable and the inclusion of these requirements if they are not applicable. Consequently, an analysis should be made to determine whether the requirements set forth by AR 705-50 are applicable for the particular item described by the documents and, if so, whether or not they have been included.

f. If a quantitative requirement is included, that is, identified as being the durability requirement for the item, the definition of durability as it applies to the item should be included. The reason for this is that a great deal of confusion exists relative to what durability means. AR 705-50 does not require that a durability requirement be included in QMR's and SDR's, and the DoD standard for definitions of reliability terms (MIL-STD-721B) does not include a definition of durability, yet a durability requirement sometimes appears in QMR's and SDR's, particularly in the "priority of characteristics" section contained therein. It should either be omitted or be precisely defined in the document.

g. The requirements should be quantitatively expressed in terms that are mutually understandable to both the user and developer. Reliability requirements are generally expressed in terms of probability of successful operation for the duration of the mission by means of percentages or equivalent decimal fractions. However, there may

be a few situations where it is considered more feasible to express the requirement in other terms, such as mean time between failures (MTBF), or probability of correct functioning for one-shot devices, etc. It should be noted, however, that it is not considered appropriate to express a given requirement as both a probability and an MTBF, such as: "The mission reliability will be no less than 85% for a 10-hour mission with an MTBF of 62 hours." For this example, if the failure rate were determined to be approximately constant (and this cannot be verified until items are built and operated a sufficient length of time to determine the distribution of failure times), the MTBF and percentage value are redundant. If the failure rate is not constant, the percentage value and the MTBF are incompatible.

Section III. STATING THE RELIABILITY CLAUSE IN CONTRACTS

C-4. General. Presently there is much concern over the general inadequacy of contractual provisions for the assurance of equipment reliability. The DoD has emphasized the use of incentive-type contracts whenever possible in lieu of the traditional cost-plus-fixed-fee (CPFF) contracts for procurement of equipment. The purpose of this discussion, then, is twofold: (1) to provide the procuring agency with guidance on the inclusion of certain contractual provisions which emphasize contractor obligations associated with attainment of specified equipment reliability; and (2) to provide general guidelines for stipulating reliability incentive requirements and identify certain technical problems that arise in the process of doing so.

C-5. Reliability clause for development contracts. a. AR 705-50 requires that all equipment contracts will contain minimum numerical reliability requirements (minimum acceptable reliability levels). Additionally, it is required that contractual provisions include the requirement for demonstration of stated quantitative requirements at selected program milestones.

b. In addition, Military Standard 785 and AR 705-50 state that reliability tests, evaluations, or measurements will be conducted under conditions specified by the proposal or any subsequent test plan approved by competent authority. If contractual reliability requirements are not met during demonstration tests, the deficient portions of the system must be redesigned and demonstration tests continued or repeated to verify that acceptance reliability has been achieved.

c. Insomuch as Military Standard 785 and AR 705-50 require of procuring activities that appropriate reliability requirements be included in contractual documentation, it remains for the procuring agency to assure that these requirements, in fact, are included. This requires that numerical requirements and demonstration of these requirements are clearly stipulated; it requires that alternatives are provided which specify courses of action upon noncompliance, (e.g., rejections and retest).

C-6. Reliability requirements in production contracts. a. The following information is to provide the reader with an example as to how reliability should be stated as part of the production contract requirements.

b. Example.

(1) Section ___, Requirements. Paragraph ___, Reliability. Machine guns shall be capable of passing a 30,000 round reliability test as specified in paragraphs ___ and ___ with no serviceable parts and not more than a total of two machine gun malfunctions for the entire test. In addition, each machine gun shall meet the acceleration time, steady state rate, and stopping time requirements specified in table I.

Table I. Performance Requirements

	Machine gun torque at minimum steady state rate of fire.	Acceleration time (from the time of application of current to the drive motor to the time minimum steady state rate of fire is reached).	Steady state rate of fire (average rounds per minute fired).	Stopping time (from the time current is removed from the drive motor until the machine gun comes to a complete stop).
High rate	180 pound-inches (max.)	0.4 seconds (max.)	6,000 to 6,400	0.2 seconds to 0.5 seconds
Low rate	105 pound-inches (max.)	0.4 seconds (max.)	2,000 to 2,500	0.2 seconds to 0.5 seconds

(2) Section ___, Quality Assurance Provisions. Paragraph ___, Reliability Testing.

Paragraph ___, Lot Size. The initial reliability test lot size shall consist of the initial month's production; subsequent reliability test lot sizes shall consist of 100 machine guns. When five successive lots meet the requirements, the reliability test lot size shall be increased to 200 machine guns.

Paragraph ___, Procedure. Machine guns selected by the government representative from each reliability lot shall be tested by the contractor for reliability using the test method specified in paragraph ___. If the reliability requirements are not met, the represented lot shall be rejected subject to retest or reconditioning and further test as a reconditioned lot. A reliability retest of ___ machine guns from the same lot shall be made without reconditioning the represented lot, unless in the opinion of the government representative the failure indicates serious defects in the item, in which case retest shall be made only if authorized by the procuring agency. Failure of the machine guns in the retest to meet the requirements shall cause rejection of the represented lot subject to reconditioning and further testing as a reconditioned lot. Prior to submission of a lot of machine guns as a reconditioned lot, the cause of failure shall be determined and contractor correction shall be effected on all machine guns in the lot. Sample size and test methods for reconditioned lots shall be the same as for retest.

Paragraph ___, Reliability Test.

Paragraph ___. Testing of machine guns for reliability requirements (see section ___) shall be accomplished with the machine gun held in a mount using the ammunition feed system, calibrated drive (for high rate), power supply, and in instrumentation in accordance with description.

Paragraph ___. Testing shall be accomplished using ball cartridges in belts of 1500 rounds each. The machine gun shall be fired using ten bursts of 150 round each at a steady state rate of 6000 to 6400 rounds

per minute. Acceleration and stopping time, torque, and steady state rate of fire shall be measured and recorded on the last burst of each 1500-round belt. These recorded values shall be forwarded in accordance with paragraph ___ .

Paragraph ___. After each 150-round burst, the machine gun shall be allowed to cool (without cooling aids) for 5 to 15 seconds. After each 1500-round belt, the machine gun shall be cooled to within 25° F. of ambient room temperature (using cooling aids other than water). After each 3000 rounds, the machine gun shall be examined and lubricated as necessary. Parts removed from the machine gun during examination shall be replaced in original firing position from which they were removed. Headspace and breech lock gap shall be examined at the start of the test and at the end of the test on the cleaned and unlubricated machine gun. At the end of each day's firing, the machine gun shall be protected against corrosion.

Paragraph ___ . No alteration or replacement of parts shall be made unless the parts are either broken or worn to the extent that they are unserviceable. An unserviceable part is one that causes malfunctions or impairs the safety of the machine gun. A complete record shall be kept for each reliability test, showing each malfunction and all parts replaced including the number of the machine gun round at which they occurred.

Paragraph ___ . The contractor shall investigate causes of malfunctions and unserviceable parts and indicate corrective action taken.

Paragraph ___ . Reliability tested machine guns and parts shall be disposed of as specified in the procurement documents.

Section IV. INCENTIVE CONTRACTING

C-7. General. a. Increased emphasis is being placed on the use of incentive contracts, especially in lieu of cost-plus-fixed-fee contracts. DoD Incentive Contracting Guide (FM 38-34) provides guidance for the use of incentive contracting. While most of what follows applies to incentive contracting in general, much of it is applicable to reliability incentives in particular.

b. A rapidly changing technology and expanding requirements for more complex systems has radically altered the character and function of military materiel. No longer is military research and development typically the low cost predecessor of large, multi-year production runs of items with relatively stable designs. In many cases, the hardware evolved during an R&D effort becomes the system specified for operational deployment. No longer, for many military systems, is the injunction, "Make it like the drawing shows," adequate guidance for obtaining satisfactory system performance and life during the production phase. No longer can military systems be maintained satisfactorily by the using troops; highly trained and skilled maintenance personnel are required. These changes have made the contractors' trade-off decisions important to the overall cost to the Army of military systems.

c. One effective means of motivating a contractor to make these trade-off decisions amongst cost, schedules, and the various performance characteristics (including reliability) which the Army desires, is the use of properly structured incentive contracts. That is, a properly structured incentive contract, through the use of rewards and penalties, will motivate the contractor to make the same trade-off decisions that the Army would if it were doing the work itself.

C-8. <u>Incentive contracting principles</u>. If an incentive contract is to be effective and to yield an acceptable product, (e.g., technical data package, hardware, services), several principles should be observed. First, the contract terms must be clear, complete and unambiguous; these conditions are especially important in stating the incentive provisions. Second, nothing in the incentive provisions should permit the contractor to meet his contractual obligations unless the performance meets all of the minimum requirements of the contract. Third, incentive payments for changes in levels of performance should be offered only if the Army can be expected to benefit from such changes and the contractor has the ability and authority to cause such changes; similarly, incentive deductions for changes in levels of performance should be required only if the Army can be expected to be adversely affected by such changes and the contractor has the ability and authority to prevent such changes. Fourth, the magnitudes of the maximum possible incentive payment and of the maximum possible incentive deduction

should reflect both the risk taken by the contractor and the magnitudes of the benefit to the Army of optimum contract performance and of the loss to the Army of minimum acceptable performance as compared to target level performance. Fifth, the number of performance characteristics covered by the incentive provisions should be kept small; each additional characteristic considered dilutes the impact of the incentive and lessens the probability of the contractor making the trade-off decisions most beneficial to the Army. Finally, the structures of the incentive provisions should be kept as simple as practical; the object is to motivate the contractor, not to impress him with the erudition of Army personnel.

C-9. Structuring incentive provisions. a. The first step in structuring incentive provisions is to determine which performance characteristics to consider for inclusion. Only those characteristics should be considered whose levels are dependent mainly on the contractor's actions and whose levels determine or help determine the value of the contract product to the Army. If the contractor is unable to change the performance level of a characteristic because of conditions beyond his control, inclusion of the characteristic in the incentive provisions frustrates the contractor and reduces his motivation. If the performance level of a characteristic does not affect the value of the contract product to the Army, inclusion of the characteristic in the incentive provisions wastes the contractor's efforts and the Army's money.

b. The second step in structuring incentive provisions is to determine, for each performance characteristic considered, what level of performance is to be sought (i.e., the target level), what level of performance is the worst acceptable, and what level of performance is the best which can be reached under the current state of the art (i.e., the optimum level). The optimum level must be feasible. In those instances where the value of the contract product does not decrease after a certain performance level below the target level is reached, that certain level, for purposes of what follows, is the worst acceptable level. Similarly, in those instances where the value of the contract product does not increase after a certain performance level above the target level is reached, that certain level is the optimum level.

c. The third step is to determine the differences in value to the Army, for each performance characteristic considered, between optimum level and target level and between target level and worst acceptable level. For most performance characteristics, other than cost, these differences will be estimates rather than exact. For example, if reliability is one of the performance characteristics considered, one approach to estimating the differences is to consider how many more items would be needed if the reliability were at the worst acceptable level instead of at target level and how many fewer items would be needed if the reliability were at the optimum level instead of at the target level and to multiply these changes each by the target per-item-cost. Assuming that the impact of reliability upon procurement costs is several times that upon other costs, then the computed changes in value are accurate enough for use in structuring the incentive provisions.

d. The fourth step is to find the total difference in value between having all the characteristics at their optimum level and having all the characteristics at their target level and the total difference in value between having all the characteristics at their target level and having all the characteristics at their worst acceptable level. Each difference in value found in c between optimum level performance and target level performance is divided by the first difference to find the proportion of the maximum allowable incentive payment to be allocated to the corresponding characteristic. Similarly, each difference in value found in c between target level performance and worst acceptable level performance is divided by the second difference to find the proportion of the maximum incentive deduction to be allocated to the corresponding characteristic.

e. The fifth step is to relate achieved performance level for each characteristic to incentive payment or deduction. The relationship between performance level and incentive payment or deduction should be the same type as the relationship between performance level and value. Usually, the performance level-incentive relationship changes at the target level since the two sets of proportions found in d usually are not the same. Types of possible relationships include linear, exponential and step; a linear relationship may be applied to fraction defective when this is small; an exponential relationship may be applied to reliability when level is measured in mean time between failures; a step relationship may be used for such characteristics as schedules.

f. The sixth step is to determine what evidence is to be used to measure the level of performance for each characteristic. For reliability, this includes the sampling plans, estimates and test methods, and conditions to be used. The sampling plans selected should reflect a good balance between test cost and cost of a wrong estimate of performance level.

g. The final step is to convert the results of the preceding steps into the incentive provisions for the contract. These provisions must state in clear and unambiguous terms exactly how the incentive payment(s) and deduction(s) are to be determined.

h. As the structure of the incentive provisions evolves, judgment and experience may dictate that one or more of the characteristics be dropped or changed to achieve more effective incentive provisions; this should be done and the appropriate steps repeated using the new set of characteristics.

C-10. Example. a. Consider a program to develop a nonrepairable item having the following expected characteristics:

(1) Item failure rate remains constant with time.

(2) Mission duration will be 21 hours.

(3) Cost per production unit will be $36,000.

(4) At the target reliability level of 90%, an initial production run of 500 units is required.

(5) Each week's delay in completing the development of the item costs the Army $5000.

Because a cost-plus-incentive-fee contract was to be used to cover the development of the item, the performance characteristics selected for consideration were: contract cost, item reliability, and contract duration.

b. The three characteristics were studied to find their three key levels.

(1) The cost analysts arrived at a target contract cost of \$12,000,000; a minimum feasible (or optimum) contract cost of \$10,000,000; and a maximum allowable (or worst acceptable) contract cost of \$15,000,000.

(2) Engineering analysis indicated a maximum feasible (or optimum) mean time to failure of 2100 hours (or an optimum reliability of 99%) and a minimum allowable (worst acceptable) mean time to failure of 164 hours (or a worst acceptable reliability of 80%). The target reliability of 90% is equivalent to a mean time to failure of (21 hours)/(-ln 0.9) or 199.32 hours.

(3) Application of PERT to the work to be accomplished indicated that the contract would take at least 36 weeks from start to finish, was expected to have a duration of 52 weeks, and would be completed in no more than 68 weeks after its start. Therefore, the target contract duration was set at 52 weeks; the optimum contract duration was set at 36 weeks; and the worst acceptable contract duration was set at 68 weeks.

c. Further study was made of the impact of the three key levels of each characteristic.

(1) Little additional study was needed to show that the value to the Army of the optimum contract cost rather than the target contract cost would be \$12,000,000 - \$10,000,000, or \$2,000,000, and that the value to the Army of the target contract cost rather than the worst acceptable contract cost would be \$15,000,000 - \$12,000,000, or \$3,000,000.

(2) Further study of the reliability levels revealed that if the item reliability were equal to 90%, the target level, the initial production run of 500 units could be expected to complete (0.90)(500), or 450 successful missions. Also, it was found that if the item reliability were equal to 80%, the worst acceptable level, 450 successful missions would require on the average (450)/(0.80), or (rounding upward) 563 units and, if the item reliability were equal to 99%, the optimum level,

450 successful missions would require on the average (450)/(0.99), or (rounding upward) 455 units. At $36,000 per unit, the value to the Army of the optimum reliability level rather than the target reliability level was found to be (500-455)($36,000), or $1,620,000, and the value to the Army of the target reliability level rather than the worst acceptable reliability level was found to be (563-500) ($36,000), or $2,268,000.

(3) Based on $5,000 per week saving for speedy contract completion, the value to the Army of the optimum contract duration rather than the target contract duration was found to be (52-16)($5000), or $80,000, and the value to the Army of the target contract duration rather than the worst acceptable contract duration was found to be (68-52)($5000), or $80,000.

d. The total differences in value and the allocations which were found are shown in figure C-10.

	Optimum-Target		Target-Worst Acceptable	
Characteristic	Amount	Allocation	Amount	Allocation
Contract Cost	$2,000,000	54.05%	$3,000,000	56.10%
Reliability	1,620,000	43.78	2,268,000	42.41
Contract Duration	80,000	2.16	80,000	1.50
Total	$3,700,000	99.99%	$5,348,000	100.01%

Figure C-10

Table of Values and Allocations

At this stage, consideration was given to dropping the characteristic, contract duration, because of its small allocations; retention of the characteristic was decided upon because only three characteristics were under consideration and inclusion would serve to indicate the Army's active interest in the characteristic.

e. Based upon the judgment of the negotiator who was to negotiate the contract for the item, a maximum allowable incentive payment of $1,200,000 and a maximum incentive deduction of $600,000 were selected for use in setting up the incentive structure. When the final contract was written, the structure was adjusted easily by multiplying the incentive rates by the appropriate constant.

(1) The contract cost incentive structure was set at (0.5405)($1,200,000)/(2000), or $324.30, incentive paid for every $1000 that the contract cost fell below $12,000,000, and (0.5610) ($600,000)/(3000), or $112.20, incentive deducted for every $1000 that the contract cost exceeded $12,000,000. Any incentive payment earned for contract cost was to be added to the target fee and any other incentive payments; any incentive deduction for contract cost was to be deducted from the target fee and any incentive payments. (Essentially, this structure is that of two straight lines, since there are 2000 possible payment steps and 3000 deduction steps.)

(2) The reliability incentive structure required the most study. Since mean time to failure could be estimated, a table relating incentive to that variable was prepared. Also, a table relating incentive to estimated reliability was prepared. The second table showed the incentive-to-reliability relationship to be nearly linear. To achieve a simple structure, it was decided to pay (0.4378) ($1,200,000)/9, or (rounding off) $58,400, for each percent that the demonstrated reliability exceeded 90%; and to deduct (0.4241) ($600,000)/10, or (rounding off) $25,450, for each percent that the demonstrated reliability fell below 90%. (In effect, the incentive structure for reliability consisted of 19 steps: 9 payment steps and 10 deduction steps.)

(3) The contract duration incentive structure was set at (0.0216)($1,200,000)/16, or $1620, incentive paid for each week less than 52 weeks that the contract takes to complete; and (0.015)($600,00)/16, or $562.50, incentive deducted for each week more than 52 weeks that the contract takes to complete. (The incentive structure for contract duration also was a step one: 16 each for payments and deductions.

f. Having developed the skeleton of the incentive structure, the team for developing the incentive provisions turned to the question of how the level of performance was to be measured for each characteristic.

(1) The team member from the comptroller's office had no problem, since the Armed Services Procurement Regulation and precedence provided ample guidance.

(2) The team member from the reliability office realized that his efforts to achieve maximum efficiency would have a large payoff. Past experience with similar units indicated that the mean times to failure of the units in the field and the mean times to failure of units tested, using Reliability Laboratory Test H, were the same, and that the total test cost (including the cost of performing the test, the cost of the components degraded by the test, and the cost of disassembly) averaged $250 per unit tested for Reliability Laboratory Test H. Since this test permitted the test units to be returned to as-good-as-new condition, a replacement test was chosen. A time-terminated plan had to be used, since only one week (168 hours) could be allowed for running the test. Unable to arrive at the best sample size by theory, the team member selected a sample size of 20 on the basis of his best judgment and recommended that the estimate, exp[-(number of failures occurring)/160], be used to determine achieved reliability.

(3) The contract was to be considered completed when the contractor had met all contractual requirements and had delivered the required technical data package in its final form.

g. The incentive provisions submitted for legal review stated:

"The cost of the work covered by this contract, the reliability of the item developed under this contract, and the contract duration, i.e., award of this contract to delivery of the final draft of the required technical data package) shall be used to adjust the fee for this contract. For each $1000 that the cost of the work covered by this contract exceeds $12,000,000, $112.20 will be deducted from the target fee. For each week that the contract duration is less than 52 weeks, $1620 will

be added to the target fee; and for each week that the contract duration exceeds 52 weeks, \$562.50 will be deducted from the target fee. For each percent that the estimated item reliability, as determined by the procedure given in paragraph __.2, below, exceeds 90%, \$58,400 will be added to the target fee; and for each percent that the estimated item reliability, as determined by the procedure given in paragraph __.2, below, is less than 90%, \$25,450 will be deducted from the target fee. The total fee paid shall be equal to the target fee plus any additions and minus any deductions required by the above provisions.

"__.2. <u>Procedure for Determining Estimated Item Reliability</u>. Twenty units, prior to sealing, plus forty additional widget assemblies, will be furnished to the Army for conducting Reliability Laboratory Test H of KLG Test Manual 702-8 at a total cost of \$724,000. This cost will not be used in determining the total fee. The Army will test the twenty units for 168 hours, replacing each failure by disassembling the failing unit, inserting a new widget assembly, reassembling the unit, and continuing to test the unit until its total time under test is 168 hours. The number of failures observed, r, will be inserted in the expression $e^{-(r/160)}$; the value of this expression will be rounded off to the nearest percent and this percentage used as the estimated reliability."

APPENDIX D

RELIABILITY PREDICTION AND APPORTIONMENT PROCEDURES

Section I. INTRODUCTION

D-1. General. The contents of this appendix are intended to provide some analysis methodology which may be used to implement the discussions in chapter IV. Since both prediction and apportionment are dependent upon reliability models, several basic models are formulated and illustrated by example. In addition, some specific prediction and apportionment procedures are included.

Section II. MATHEMATICAL MODELS

D-2. General. The reliability mathematical model is a set of functions depicting the reliability characteristics of the system. The basic laws of probability will be utilized in this appendix to describe the reliability models associated with certain basic system functional configurations. These models may be combined to form a model for more complex configuration. Discussion is confined to models for which independence of all components and/or subsystems is assumed. However, the models may be modified if dependence between subsystems is known.

D-3. Series model. a. When a group of components or subsystems must function properly for the system to succeed, they are said to function in series. A system consisting of a series arrangement of n components is illustrated in block diagram form in figure D-1.

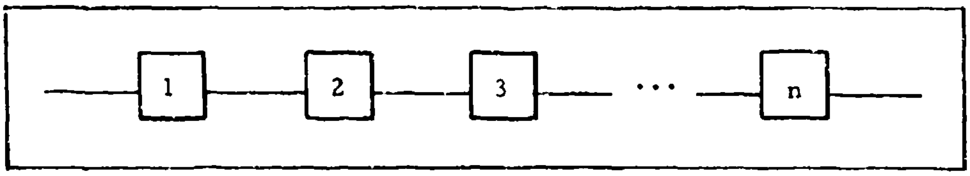


Figure D-1
Block Diagram of Series Components

b. In order to state the reliability model, the following symbols will be used:

R_s = system reliability or probability that the system will function properly.

R_i = reliability of the i^{th} component (or subsystem) or probability that it will function properly.

$Q_s = 1 - R_s$ = unreliability of the system.

$Q_i = 1 - R_i$ = unreliability of the i^{th} component (or subsytem).

Using the multiplication law of probability describing the intersection of n events,

$$R_s = R_1 R_2 \ldots R_n = \prod_{i=1}^{n} R_i$$

If all subsystems have equal reliability R,

$$R_s = R^n$$

c. As an example of this model, consider a tracked vehicle. The two tracks function as a series because the vehicle's locomotion is unsatisfactory if either track breaks. Historical information indicates that each track has a probability of 0.95 of surviving a specific mission. Find the reliability of the two tracks as a system.

(1) Figure D-2 is a block diagram representing this system and showing the subsystem reliabilities for the specific mission.

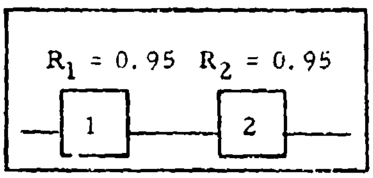


Figure D-2
Block Diagram - Series Example

(2) The resultant system reliability is

$$R_s = R_1 R_2 = (0.95)(0.95) = 0.9025$$

D-4. Parallel model. a. System or subsystem reliability can sometimes be increased by including redundant components so that success is achieved as long as at least one is operating satisfactorily. First we shall consider active redundancy where all the redundant components are simultaneously subjected to operation unless they have failed. Such components are said to be in parallel. A parallel system of n components (or subsystems) is illustrated in block diagram form in figure D-3.

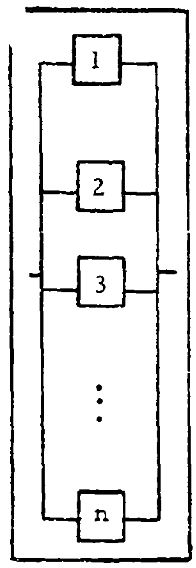


Figure D-3
Block Diagram of Parallel Components

b. Since system failure occurs only when all subsystems fail, the reliability model can best be approached by finding system unreliability as an intersection of component unreliabilities. Then the system reliability is the complement of the system unreliability.

$$Q_s = Q_1 Q_2 Q_3 \ldots Q_n = \prod_{i=1}^{n} Q_i = \prod_{i=1}^{n} (1-R_i) = 1-R_s$$

$$R_s = 1-Q_s = 1-\prod_{i=1}^{n} (1-R_i)$$

If the n components are identical, defining component reliability as R, the above model becomes

$$R_s = 1 - (1-R)^n$$

c. An example of this model might be a volley of 4 artillery shells fired at a specified target. Each shell has a probability of 0.3 of destroying the target. Find the probability that the target will be destroyed by the volley.

(1) Figure D-4 shows a block diagram representing the parallel (redundant) function of the four rounds.

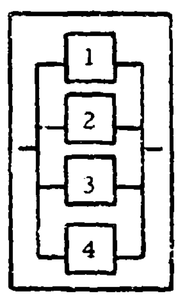


Figure D-4
Block Diagram - Parallel Example

(2) Since the 4 shells are identical, the reliability of the volley is

$$R_s = 1-(1-R)^n = 1-(1-0.3)^4 = 0.7599$$

D-5. Partial redundancy model. a. In the previous redundancy model, the system succeeded if at least one of the parallel elements is successful. There may be cases where at least k out of n identical elements must be successful. Such a configuration is sometimes called partial redundancy. The binomial distribution may be used to develop the reliability model if all elements are identical. The general block diagram is shown in figure D-5.

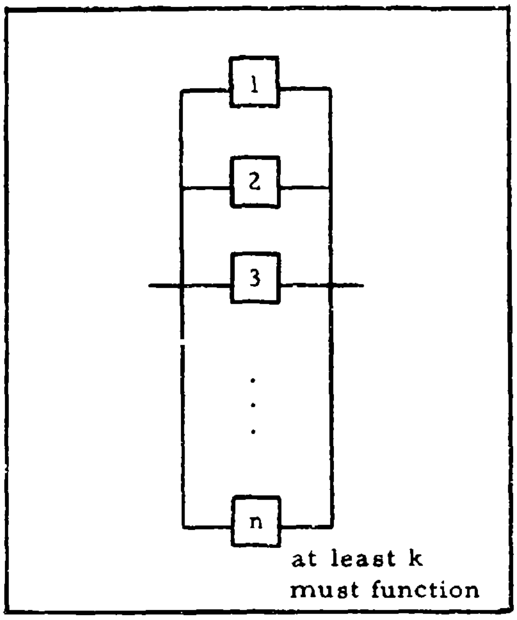


Figure D-5
Block Diagram - Partial Redundancy

b. If the reliability of each element is R, the model is

$$R_s = \sum_{x=k}^{n} \binom{n}{x} R^x(1-R)^{n-x} = \sum_{x=k}^{n} \frac{n!}{x!(n-x)!} R^x(1-R)^{n-x}$$

c. Suppose an eight cylinder engine can operate successfully if at least seven of the eight spark plugs are functioning properly. A single plug has a reliability of 0.95 for a certain mission. Find the reliability of the partial redundancy system of eight plugs for this mission.

(1) Figure D-6 shows this system as a block diagram.

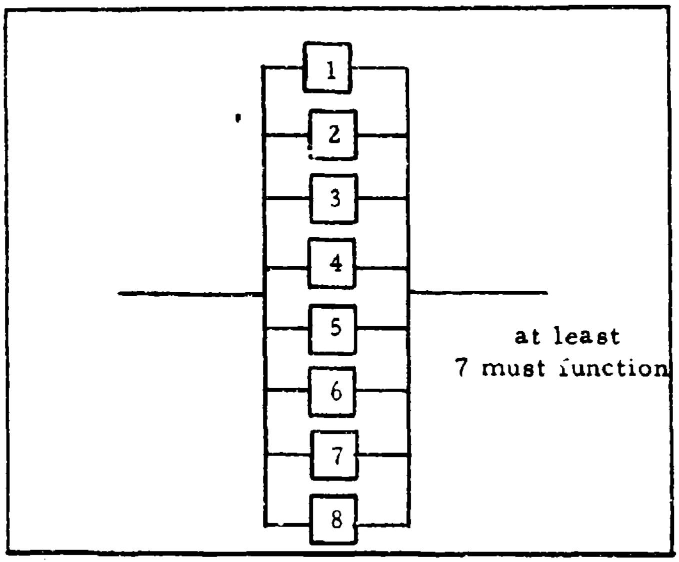


Figure D-6
Block Diagram - Partial Redundancy Example

(2) The system reliability, probability that at least 7 of 8 plugs will function properly, is

$$R_s = \sum_{x=7}^{8} \binom{8}{7} (.95)^x (.05)^{8-x} \doteq 0.943$$

D-6. Standby redundancy models.

a. Definition. The foregoing models have had to do with active redundancy, i.e., all elements are operating. In some instances, redundancy may take the form of elements standing by to take over when the primary element fails. Standby redundancy arrangements require failure-sensing and switching devices which contribute to system unreliability.

b. Exponential model. Consider n + 1 identical elements where only one is functioning when the system is initiated and the remaining n elements are standing by until preceding elements fail. Assuming a perfect sensory and switching procedure, system reliability is the probability of n or less failed elements within the prescribed mission time. Determination of this probability depends upon the distribution of failure

times. The model which follows depends upon exponential failure times.

$$R_s(x) = \sum_{k=0}^{n} \frac{(\lambda x)^k \exp(-\lambda x)}{k!} = \left[1 + \lambda x + \frac{(\lambda x)^2}{2!} + \ldots + \frac{(\lambda x)^n}{n!}\right] \exp(-\lambda x)$$

where x is a time variable

λ is the failure rate of the individual elements.

c. General model

(1) Consider a system composed of n + 1 elements where n are standing by and all elements are not identical and may even have different failure time density functions. The general model for system reliability, assuming independence of the elements and a perfect switching procedure, is as follows:

(2) System failure occurs only when the n + 1 elements fail. The system failure time density function, i.e., the probability density function for the time to failure (n+1) is:

$$f_s(x) = \int_{t_n=0}^{x} \int_{t_{n-1}=0}^{t_n} \ldots \int_{t_1=0}^{t_2} f_1(t_1)\, f_2(t_2-t_1) \ldots$$

$$f_{n+1}(x-t_n)\, dt_1\, dt_2 \ldots dt_n$$

where t_i is the time from mission beginning till the i^{th} element will fail, x is the variable indicating system failure time; i.e., $(n+1)^{th}$ element failure; and $f_i(x)$ is the probability density function of the i^{th} element failure times. Then system reliability is

$$R_s(x) = \int_x^{\infty} f_s(t)dt$$

(3) As a special case, consider a two-element standby system; i.e., one primary and one standby element; where each element failure time distributi n is exponential but with different failure

rates. The density functions are

$$f_1(x) = \lambda_1 \exp(-\lambda_1 x)$$

and

$$f_2(x) = \lambda_2 \exp(-\lambda_2 x)$$

$$f_s(x) = \int_{t_1=0}^{x} \lambda_1 \lambda_2 \{\exp(-\lambda_1 t_1)\} \{\exp[-\lambda_2(x-t_1)]\} dt_1$$

$$= \lambda_1 \lambda_2 \exp(-\lambda_2 x) \int_{t_1=0}^{x} \exp[t_1(\lambda_2 - \lambda_1)] dt_1$$

$$= \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} [\exp(-\lambda_1 x) - \exp(-\lambda_2 x)]$$

Then system reliability becomes

$$R_s(x) = \int_x^{\infty} \frac{\lambda_1 \lambda_2}{\lambda_2 - \lambda_1} [\exp(-\lambda_1 t) - \exp(-\lambda_2 t)] dt$$

$$= \frac{\lambda_2 \exp(-\lambda_1 x) - \lambda_1 \exp(-\lambda_2 x)}{\lambda_2 - \lambda_1}$$

(4) The density function for a three element standby system (assuming exponential distributions) becomes

$$f_s(x) = \int_{t_2=0}^{x} \int_{t_1=0}^{x_2} \lambda_1 \lambda_2 \lambda_3 \{\exp(-\lambda_1 t_1)\}$$

$$\{\exp[-\lambda_2(t_2-t_1)]\} \{\exp[-\lambda_3(x-t_2)]\} dt_1 \, dt_2$$

(5) The same method can be used to expand to any number of elements where the appropriate density function is used for each.

d. Imperfect switching model.

(1) Preceding inactive redundancy models have assumed a perfect switching procedure. In practice, however, the probability of switch failure has a significant affect upon system reliability. Some general rules for building switch reliability into the models will be considered. (For limitations, see (12) below.)

(2) Two modes of failure which are often associated with switching mechanisms are: the switch may fail to operate when it should, and the switch may operate prematurely. The model for a two-element standby redundancy is shown in block diagram form in figure D-7.

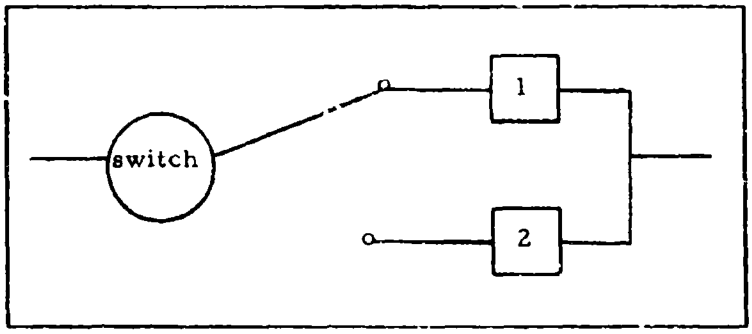


Figure D-7
Block Diagram - Two-Element Standby Redundancy

(3) The diagram indicates that whenever element 1 fails, the switch activates the standby element 2. The symbols to be used to develop a model are:

p_i = probability that the i^{th} element succeeds

q_i = $1-p_i$

q_w = probability that the switch will fail to operate when it should

$p_w \;= 1-q_w$

q'_w = probability that the switch will operate prematurely

$p'_w = 1-q'_w$

(4) The system has three possible element states which could lead to system failure:

(a) Element 1 succeeds, switch operates prematurely and element 2 fails.

(b) Element 1 fails, switch fails to operate.

(c) Element 1 fails, properly switches to 2 and 2 fails.

Then system unreliability becomes

$$Q_s = p_1 q'_w q_2 + q_1 q_w + q_1 p_w q_2$$

and system reliability is

$$R_s = 1-(p_1 q'_w q_2 + q_1 q_w + q_1 p_w q_2)$$

(5) To exemplify the model, consider a two-element standby redundant system where the identical elements each have a 0.9 probability of successfully completing its mission. The switching mechanism is such that there is a 0.05 probability of premature switching and 0.01 probability of failing to switch when a switch should occur. Find the system reliability.

(6) The resulting system reliability is shown below.

$p_1 \;= p_2 = 0.9$

$q_1 \;= q_2 = 0.1$

$q_w \;= 0.01$

$q'_w = 0.05$

$$R_s = 1 - [0.9(0.05)(0.1) + 0.1(0.01) + 0.1(0.99)(0.1)] =$$

$$1 - 0.0155 = 0.9845$$

(7) The model for a three-element standby redundant system is more complex and is shown below. The switching of elements into operating status can take place only in ascending numerical order shown in figure D-8.

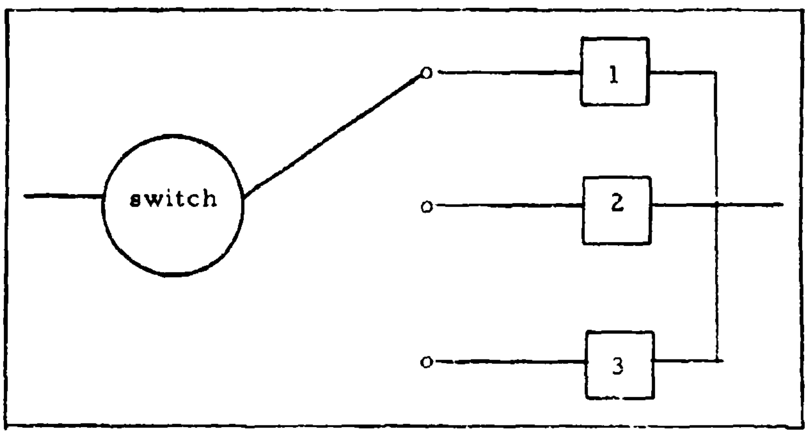


Figure D-8
Block Diagram - Three Element Standby Redundancy

(8) The symbols to be used are:

$p_i = 1-q_i$ = probability that the i^{th} element will succeed.

$p_{w1} = 1-q_{w1}$ = probability that the mechanism will switch to element 2 if element 1 fails.

$p_{w2} = 1-q_{w2}$ = probability that the mechanism will switch to element 3 if element 2 fails.

$q'_{w1} = 1-p'_{w1}$ = probability that, given element 1 is operating properly, the mechanism will wrongly switch to element 2.

$q'_{w2} = 1 - p'_{w2}$ = probability that, given element 2 is operating properly, the mechanism will wrongly switch to element 3.

(9) System failure can happen only in the following seven element states.

(a) Element 1 succeed, wrongly switch to 2, 2 succeed, wrongly switch to 3 and 3 fail

(b) Element 1 succeed, wrongly switch to 2, 2 fail, properly switch to 3 and 3 fail.

(c) Element 1 succeed, wrongly switch to 2, 2 fail and no switch to 3.

(d) Element 1 fail and no switch to 2.

(e) Element 1 fail, a proper switch to 2, 2 fail and no switch to 3.

(f) Element 1 fail, a proper switch to 2, 2 succeed, wrongly switch to 3 and 3 fail.

(g) Element 1 fail, a proper switch to 2, 2 fail, a proper switch to 3 and 3 fail.

(10) Then system unreliability is

$$Q_s = p_1 q'_{w1} p_2 q'_{w2} q_3 + p_1 q'_{w1} q_2 p_{w2} q_3 + p_1 q'_{w1} q_2 q_{w2} +$$

$$q_1 q_{w1} + q_1 p_{w1} q_2 q_{w2} + q_1 p_{w1} p_2 q'_{w2} q_3 + q_1 p_{w1} q_2 p_{w2} q_3$$

and

$$R_s = 1 - Q_s$$

(11) The method may be extended to standby redundant systems of more than three elements. Other failure modes may also be added, e.g., switch contact failure is sometimes encountered.

(12) The standby redundancy models, with imperfect switching mechanisms, shown above, have considered success or failure of an element as being independent of time, i.e., the remaining mission time when an element is activated does not affect the probability of its success. The models may be further refined by considering failure distributions of the elements as well as the incorrect switch time distribution. Development of such models would be similar to the method used for redundant models with perfect switching mechanisms. Such models, however, are not included herein.

D-7. Conditional probability and the reliability model.

a. General.

(1) There exist configurations of subsystems which are not series, parallel, or standby as previously defined. Conditional probability may be used to facilitate the modeling of such configurations. A useful theorem is stated:

$$P(A) = \sum_{i=1}^{m} P(A/B_i)P(B_i)$$

where the B_i are mutually exclusive events such that

$$\sum_{i=1}^{m} P(B_i) = 1$$

A special case of this theorem where $m = 2$ is sometimes useful for reliability modeling

$$P(A) = P(A/B_1)\,P(B_1) + P(A/B_2)\,P(B_2)$$

where B_1 and B_2 are mutually exclusive and exhaustive events.

(2) For example, consider a configuration as shown by figure D-9.

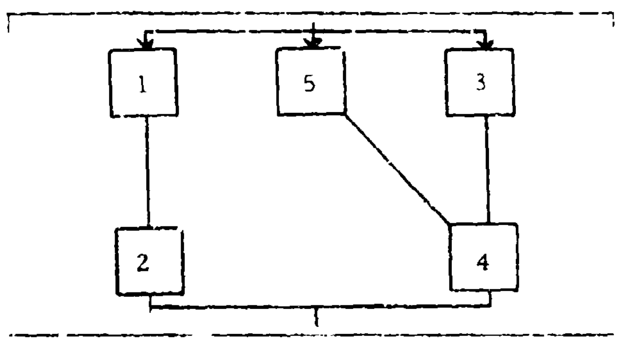


Figure D-9
Block Diagram - Conditional Probability Illustration

The conditions for a successful operation of the system are: proper functioning of subsystems 1 and 2, 3 and 4, or 5 and 4. The model for this configuration may be stated in terms of conditional probability as

$$P(S) = P(S/S_5)P(S_5) + P(S/\bar{S}_5)P(\bar{S}_5)$$

where R_s = system reliability

$P(S/S_5)$ = system reliability given subsystem 5 does not fail

$P(S/\bar{S}_5)$ = system reliability given subsystem 5 fails

$P(S_5)$ = reliability of subsystem 5

$P(\bar{S}_5)$ = unreliability of subsystem 5

b. Example.

(1) To illustrate the use of conditional probability, consider an anti-aircraft system with primary subsystems of detection (D), tracking (T), and firing (F). A backup detection subsystem (BD) is to be used only when D fails. The method for determining when D fails and switching the function to BD is referred to as subsystem M and has only one mode of failure, that of failing to switch to BD if D fails. Figure D-10 shows a block diagram.

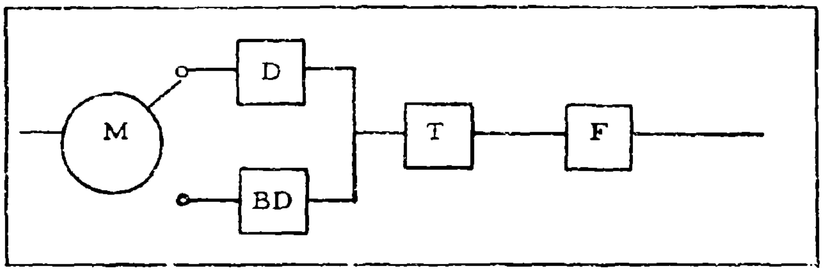


Figure D-10
Block Diagram - Conditional Probability Example

(2) First consider D to be operative. Since M cannot fail for this mode of operation and D is successful, the system reduces to the diagram in figure D-11, and the conditional system reliability becomes $P(S/S_D) = P(S_T)P(S_F)$.

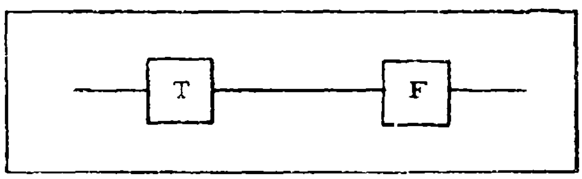


Figure D-11
Block Diagram - Primary Detection Subsystem Operative

(3) Next consider D to be inoperative. The system reduces to the diagram in figure D-12 and the conditional system reliability becomes

$$P(S/\overline{S}_D) = P(S_M)P(S_{BD})P(S_T)P(S_F)$$

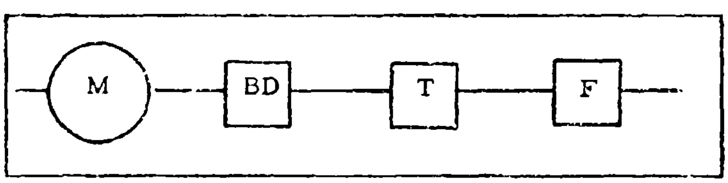


Figure D-12
Block Diagram - Primary Detection System Inoperative

(4) The unconditional system reliability then is

$$P(S) = P(S_T)P(S_F)P(S_D) + P(S_M)P(S_{BD})P(S_T)P(S_F)P(\bar{S}_D)$$

c. Conditional probability may be applied to a variety of component configurations. Complex combinations of series and parallel models are often more easily analyzed by applying the conditional probability theorem one or more times, considering a key item first as operative and then as inoperative.

Section III. RELIABILITY PREDICTION

D-8. General. Reliability prediction is the use of the reliability model to combine reliability information concerning components and subsystems to determine a prediction of system reliability. The key activities associated with reliability prediction are discussed in chapter 4.

D-9. Mixed model example. a. Most practical systems will not fit any single model of section II, but rather are represented by combinations and/or variations of these models. To show this, the following paragraphs contain an example of reliability prediction for a mixed system.

b. An automobile is to be used for a 500 mile trip and the reliability is to be predicted for this trip. Reliability information is available concerning automobile subsystems: tires, lighting group, spark plugs, brake system and all other subsystems collectively.

c. The tire subsystem (subsystem T) is comprised of 5 tires, four originals and a spare, where the spare is a standby to r lace the first, if any blowout. The trip length is such that wearout does not occur, but blowouts may occur because of chance random stresses encountered. Past information indicates miles between blowouts to be exponentially distributed with failure rate of 0.0001 blowouts per mile. Figure D-13 shows block diagrams for the four potential configurations which mav occur. T1, T2, T3 and T4 are the original tires and T5 is the spare.

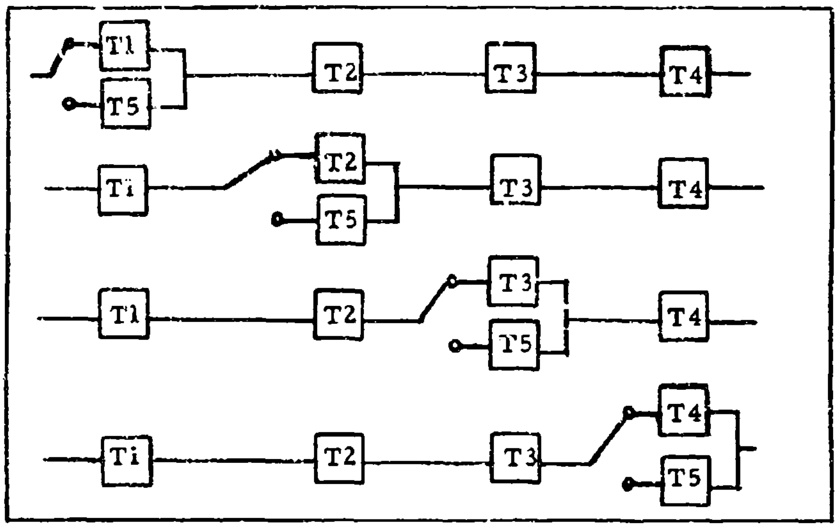


Figure D-13
Block Diagram - Tire Subsystem

(1) Since only one replacement tire is available, the reliability of subsystem T for a 500 mile trip is merely the probability of not more than one failure in 2000 miles of tire use, i.e.,

$$R_T = \sum_{k=0}^{1} \frac{(\lambda x)^k}{k!} \exp(-\lambda x)$$

where λ = 0.0001 blowouts per mile and

x = 2000 tire-miles mission.

(2) Then the numerical reliability of subsystem T is

$$R_T = \sum_{k=0}^{1} \frac{(0.2)^k}{k!} \exp(-0.2) = 0.9825$$

d. The subsystem of spark plugs (subsystem P) is comprised of eight plugs. Although undesirable, the automobile can continue the trip with only 7 operating spark plugs. Since a 500 mile trip will not cause the plugs to wear out, the failure distribution for each plug is exponential with failure rate of 0.00003 failures per mile during the trip. Figure D-14 is a block diagram for this partially redundant subsystem.

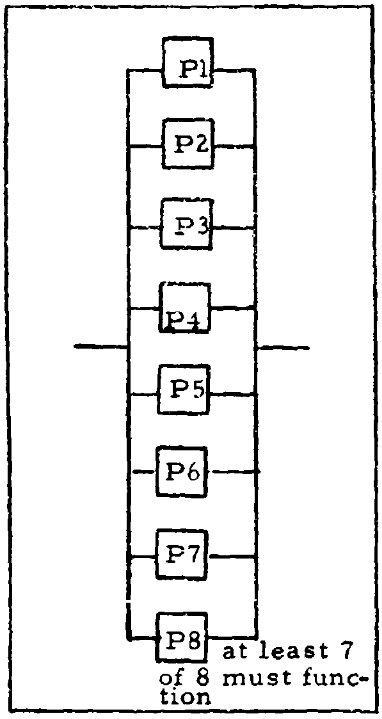


Figure D-14
Block Diagram - Spark Plug Subsystem

(1) The reliability of each individual spark plug for the 500 mile trip is

$$\exp(-\lambda x) = \exp(-0.015) = 0.9851$$

where λ = 0.00003 failures per mile and

x = 500 miles

(2) The numerical reliability of subsystem P is

$$R_P = \sum_{k=0}^{1} \binom{8}{k} (0.0149)^k (0.9851)^{8-k} = 0.9937$$

e. The braking subsystem (subsystem B) is comprised of two independent and identical brake units functioning in parallel. Past information indicates that such a unit is 0.99 reliable for a 500 mile trip. Figure D-15 is a block diagram for this redundant subsystem.

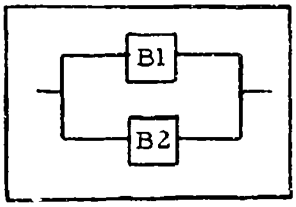


Figure D-15
Block Diagram - Braking Subsystem

Then the reliability of subsystem B is

$$R_B = 1-(1-0.99)^2 = 0.9999$$

f. The lighting subsystem (subsystem L) considers the headlights and rear lights. For safety during nondaylight driving, both headlights and at least one of the two rear lights must function properly. For the 500 mile trip, an average of three hours of nondaylight driving can be expected. Failures occur (for this typical trip) exponentially, with a failure rate of 0.005 failures per hour for a headlight and 0.004 failures per hour for a rear light. Figure D-16 shows the block diagram for subsystem L.

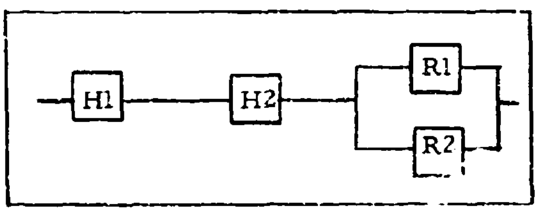


Figure D-16
Block Diagram - Lighting Subsystem

(1) The reliability of each headlight for a 3 hour mission is

$$R_{LH} = \exp[-3(0.005)] = \exp(-.015) = 0.9851$$

The reliability of each rear light for a 3 hour mission is

$$R_{LR} = \exp[-3(0.003)] = \exp(-0.009) = 0.9910$$

(2) Then the reliability of subsystem L is found as follows:

(a) Define subsystem A as the parallel configuration of Rl and R2. Then

$$R_A = 1-(1-R_{LR})^2 = 1-(1-0.9910)^2 = 0.999919$$

(b) Treating Hl, H2 and A as subsystems in series,

$$R_L = R_{LH}R_{LH}R_A = (0.9851)^2(0.999919) = 0.9703$$

g. All other functional subsystems of the vehicle are grouped as a single subsystem denoted by "other" (subsystem O). Data from past records indicate the reliability of this subsystem for the given trip to be

$$R_O = 0.9990$$

h. The entire system is represented by a functional series of subsystems T, P, B, L and O as shown by the block diagram in figure D-17.

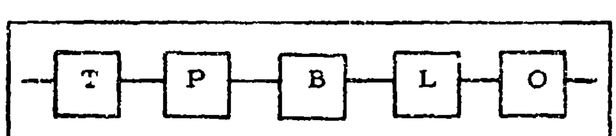


Figure D-17
Block Diagram - Vehicle System

Then the system reliability may be predicted as

$$R_S = R_T R_P R_B R_L R_O$$

$$= (0.9825)(0.9937)(0.9999)(0.9703)(0.9990) = 0.9463$$

Section IV. RELIABILITY APPORTIONMENT

D-10. General. Reliability apportionment utilizes the reliability model to assign combinations of compatible goals among subsystems such that subjection of the model to these subsystem goals will yield the system goal. Discussion of apportionment in this appendix will be confined to some specific techniques of apportionment, two of which approach apportionment with minimum expenditure of effort.

D-11. Equal apportionment technique. a. The equal apportionment technique assumes a series of n subsystems each of which is to be assigned the same reliability goal. A prime weakness of the method is that the subsystem goals are not assigned in accordance with the degree of difficulty associated with achievement of these goals. For this technique, the model is

$$R^* = \prod_{i=1}^{n} R^*_i$$

or

$$R^*_i = (R^*)^{1/n} \quad \text{for } i = 1, 2, \ldots, n$$

Where R^* is the required system reliability and

R^*_i is reliability requirement apportioned to subsystem i.

b. To exemplify this technique, consider a proposed communication system which consists of 3 subsystems (transmitter, receiver and coder) each of which must function if the system is to function. Each of these subsystems is to be developed independently. Assuming each to be equally expensive to develop, what reliability requirement should be assigned to each subsystem in order to meet a system requirement of 0.729.

c. The apportioned subsystem requirements are found as

$$R^*_T = R^*_R = R^*_C = (R^*)^{1/n} = (0.729)^{1/3} = 0.90$$

Then a reliability requirement of 0.90 should be assigned to each subsystem.

D-12. AGREE apportionment technique. a. This technique takes into consideration both the complexity and importance of each subsystem for electronic systems. It assumes a series of k subsystems each with exponential failure distributions. Then the minimum acceptable mean life of the i^{th} subsystem is defined as

$$\theta_i = \frac{N w_i t_i}{n_i \left[-\ln R^*(t)\right]}.$$

and the corresponding i^{th} subsystem reliability requirement becomes

$$R_i^*(t_i) = \exp\left(\frac{-t_i}{\theta_i}\right)$$

where

$i = 1, 2, 3, \ldots, k$

t = required mission time of the system

t_i = required mission time of the i^{th} subsystem

w_i = importance factor expressed as the probability that failure of the i^{th} subsystem will result in system failure.

n_i = number of modules in the i^{th} subsystem

$N = \sum_{i=1}^{k} n_i$ = total number of modules in the system

$R^*(t)$ = the required system reliability for system mission time

$R_i^*(t_i)$ = the reliability apportioned to the i^{th} subsystem for its mission time

θ_i = apportioned mean time to failure for the i^{th} subsystem

b. A concept of module is used in this technique for three purposes: (1) so that the relative complexity inherently required can be taken into account; (2) so that the minimum acceptable reliability figures will not be grossly inconsistent; and (3) so that reliability requirements will be dynamic and state of art changes can be incorporated as they occur. A module is designated as the basic electronic building block and is considered to be a group of electronic parts. This is a fictitious way of partitioning an electronic system for reliability purposes. For systems involving electron tubes, it has been found that for one tube there are approximately fifteen additional electronic parts -- this is considered to be a module. Thus, the number of modules for an equipment is defined as the number of electron tubes.

c. To illustrate the AGREE apportionment method, consider a fictitious system composed of four subsystems. The system has a mission time of 4 hours and required reliability of 0.9. Figure D-18 shows the number of modules, importance factor and mission time for each subsystem.

Subsystem (i)	No. Modules (n_i)	Importance Factor (w_i)	Mission Time (t_i)
1	20	0.7	4
2	30	0.5	4
3	200	0.8	4
4	50	0.2	4
	N = 300		

Figure D-18

d. The apportionment to each subsystem is found as follows:

$$\theta_i = \frac{Nw_i t_i}{n_i\left[-\ln R^*(t)\right]}$$

and

$$R^*_i(t_i) = \exp\left(\frac{-t_i}{\theta_i}\right)$$

where $R^*(4) = 0.90$

$$\theta_1 = \frac{300(0.7)(4)}{20(0.1054)} = \frac{840}{2.108} = 398 \text{ hours}$$

$$R^*_1(4) = \exp(-4/398) = \exp(-0.01) = 0.990$$

$$\theta_2 = \frac{300\ (0.5)(4)}{30(0.1054)} = \frac{600}{3.162} = 189 \text{ hours}$$

$$R^*_2(4) = \exp(-4/189) = \exp(-0.021) = 0.979$$

$$\theta_3 = \frac{300(0.8)(4)}{200(0.1054)} = \frac{960}{21.08} = 45 \text{ hours}$$

$$R^*_3(4) = \exp(-4/45) = \exp(-0.089) = 0.911$$

$$\theta_4 = \frac{300(0.2)(4)}{50(0.1054)} = \frac{240}{5.27} = 45 \text{ hours}$$

$$R^*_4(4) = \exp(-4/45) = \exp(-0.089) = 0.911$$

D-13. The ARINC apportionment technique. a. This method assumes series subsystems with constant failure rates such that any subsystem failure causes system failure and that subsystem mission time is equal to system mission time.

b. The apportionment technique requires expression of reliability requirements in terms of failure rate. The following steps apply:

(1) The objective is to choose λ^*_i such that

$$\sum_{i=1}^{n} \lambda^*_i \leq \lambda^*$$

where

λ^*_i is the failure rate allocated to subsystem i;

λ^* is the required system failure rate.

(2) Determine the subsystem failure rates (λ_i) from past observation or estimation.

(3) Assign a weighting factor (w_i) to each subsystem according to the failure rates determined in step 2.

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}$$

(4) Allocate subsystem failure rate requirements

$$\lambda^*_i = w_i \lambda^*$$

c. To illustrate this method, consider an antiaircraft system composed of three subsystems (detection equipment, tracking equipment and firing equipment) with predicted failure rates of $\lambda_1 = 0.003$, $\lambda_2 = 0.001$ and $\lambda_3 = 0.004$ failures per hour respectively. The system has a mission time of 20 hours and 0.90 reliability is required. Find the subsystem requirements.

d. The apportioned failure rates and reliability goals are found as follows:

(1) $R^*(20) = \exp[-\lambda^*(20)] = 0.90$

Then

$\lambda^* = 0.005$ failures per hour.

(2) $\lambda_1 = 0.003, \lambda_2 = 0.001, \lambda_3 = 0.004$

(3) $$w_1 = \frac{0.003}{0.003 + 0.001 + 0.004} = 0.375$$

$$w_2 = \frac{0.001}{0.003 + 0.001 + 0.004} = 0.125$$

$$w_3 = \frac{0.004}{0.003 + 0.001 + 0.004} = 0.5$$

(4) $\lambda_1^* = 0.375\ (0.005) = 0.001875$

$\lambda_2^* = 0.125(0.005) = 0.000625$

$\lambda_3^* = 0.5\ (0.005) = 0.0025$

(5) The corresponding apportioned subsystem reliability requirements are

$$R^*_1(20) = \exp\ [-20(.001875)] = 0.96$$

$$R^*_2(20) = \exp\ [-20(.000625)] = 0.99$$

$$R^*_3(20) = \exp\ [-20(.0025)] = 0.95$$

D-14. Minimization of effort algorithm. a. This apportionment technique considers minimization of total effort expended to meet the system reliability requirements. A system is considered where n subsystems are arranged in series.

b. Let $R_1, R_2, \ldots, R_n$ denote subsystem reliabilities and the system reliability $\bar{R}$ would be given by

$$\bar{R} = \prod_{i=1}^{n} R_i$$

c. Let R^* be the required reliability of the system, where $R^* > R$. It is then required to increase at least one of the values of the R_i to the point that the required reliability R^* will be met. To accomplish such an increase takes a certain effort, which is to be alloted in some way among the subsystems. The amount of effort would be some function of number of tests, amount of engineering manpower applied to the task, etc..

d. The algorithm assumes that each subsystem has associated with it the same effort function $G(R_i, R^*_i)$ which measures the amount of effort needed to increase the reliability of the i^{th} subsystem from R_i to R^*_i.

e. The assumptions on $G(x, y)$ where $y > x$ are:

(1) $G(x, y) \geqslant 0$

(2) $G(x, y)$ is nondecreasing in y for fixed x and non-increasing in x for fixed y. That is

$$G(x, y) \leqslant G(x, y + \Delta y) \text{ and}$$

$$G(x, y) \geqslant G(x + \Delta x, y)$$

(3) $G(x, y) + G(y, z) = G(x, z)$ where $x < y < z$.

(4) $G(0, x)$ has a derivative $h(x)$ such that $xh(x)$ is strictly increasing in $(0 < x < 1)$.

f. The problem then is to determine R_i^* such that

$$\sum_{i=1}^{n} G(R_i, R_i^*)$$

is minimized subject to the condition

$$\prod_{i=1}^{n} R_i^* = R^*$$

g. With the preceding assumptions, it was shown in the aforementioned reference that the unique solution is

$$R^*_i = \begin{cases} R^*_o & \text{if } i \le K_o \\ R_i & \text{if } i > K_o \end{cases}$$

where the subsystem reliabilities $R_1, R_2, \ldots, R_n$ are ordered in nondecreasing fashion (assuming such an ordering is implicit in the notation).

$$R_1 \le R_2 \le \ldots \le R_n$$

and the number K_o is determined as

K_o = maximum value of j such that

$$R_j < \left[\frac{R^*}{\left(\prod_{i=j+1}^{n+1} R_i\right)}\right]^{1/j} = r_j$$

where $R_{n+1} = 1$ by definition.

h. The number R^*_o is determined as

$$R^*_o = \left[\frac{R^*}{\left(\prod_{j=K_o+1}^{n+1} R_j\right)}\right]^{1/K_o}$$

i. It is evident that the system reliability will then be R^*, since new reliability $(R^*_o)^{K_o} R_{K_o+1} \ldots R_n = (R^*_o)^{K_o} \left(\prod_{j=K_o+1}^{n+1} R_j\right) = R^*$ when the relationship for R^*_o is substituted.

j. As an example, consider a system that consists of three subsystems (A, B and C), all of which must function without failure

in order to achieve system success. The system reliability requirement has been set at 0.70. We have predicted subsystem reliabilities as $R_A = 0.90$, $R_B = 0.80$, and $R_C = 0.85$. How should we apportion reliability to the subsystems in order that total effort be minimized and that the system reliability requirement be satisfied? Assume identical effort functions for the three subsystems.

k. The resulting minimum effort apportionment goals are found as follows:

(1) Arrange subsystem reliability values in ascending order:

$$R_1 = R_B = 0.80,\ R_2 = R_C = 0.85 \text{ and } R_3 = R_A = 0.90$$

(2) Determine K_o, the maximum value of j, such that

$$R_j < \left[\frac{R^*}{\left(\prod_{i=j+1}^{n+1} R_j\right)}\right]^{1/j} = r_j$$

(3) When j = 1,

$$R_1 = 0.80 < r_1 = \left[\frac{0.70}{R_2R_3(1.00)}\right]^1 = \left[\frac{0.70}{(0.85)(0.90)(1.00)}\right] = \left[\frac{0.70}{0.765}\right] = 0.915$$

(4) When j = 2,

$$R_2 = 0.85 < r_2 = \left[\frac{0.70}{(0.90)(1.00)}\right]^{1/2} = \left(\frac{7}{9}\right)^{1/2} = \frac{\sqrt{7}}{3} = 0.882$$

(5) When i = 3,

$$R_3 = 0.90 > r_3 = \left[\frac{0.70}{1.00}\right]^{1/3} = (0.70)^{1/3} = .888$$

(6) Since $R_1 < r_1$, $R_2 < r_2$, but $R_3 > r_3$, then $K_o = 2$ because 2 is the largest subscript j such that $R_j < r_j$. Thus,

$$R^*_o = \left[\frac{0.70}{0.90}\right]^{1/2} = 0.882$$

which means that the effort is to be allotted so that subsystem B increases in reliability from 0.80 to 0.882, and subsystem C increases in reliability from 0.85 to 0.882; whereas subsystem A is left alone with a reliability of 0.90. The resulting reliability of the entire system is, as required, $0.70 = (.882)^2 (.90)$. This means that effort should be expended on subsystem C and B to raise their respective reliabilities to .882 with no developmental effort spent on subsystem A. This policy would minimize the total expended effort required to meet system reliability requirements. The minimization, however, is dependent upon the effort function meeting the initial assumptions.

D-15. Dynamic programming approach to reliability apportionment.

a. General. The preceding minimization of effort algorithm requires that all subsystems be subject to the same effort function. If all subsystems are not equally difficult to develop, dynamic programming provides an approach to reliability apportionment with minimum effort expenditure when the subsystems are subject to different but identifiable effort functions.

b. Introduction to dynamic programming. To serve as a basis for formulation of such problems, a brief summary of the essential elements of the dynamic programming procedure follows.

(1) The dynamic programming technique is applicable to multi-stage (or sequential) decision problems. The technique converts such a problem to a series of single-stage optimization problems.

(2) In addition to defining the stages of such a process, four attributes of the problem must be identified if the technique is to be applied.

(a) S_k is the set of all possible states of stage k. Its elements are designated as s_k, i.e., $s_k \in S_k$.

(b) D_k is the set of all possible decision alternatives available at stage k. Its elements are designated as $d_k \in D_k$.

(c) $T_k(s_k, d_k)$ is a function transforming s_k to s_{k-1}

depending on the existing state, s_k, of stage k and the decision alternative, d_k, selected at stage k.

(d) $R_k(s_k, d_k)$ is a function defining the return realized at stage k resulting from state s_k and alternative d_k.

(3) An n-stage process is displayed by figure D-19.

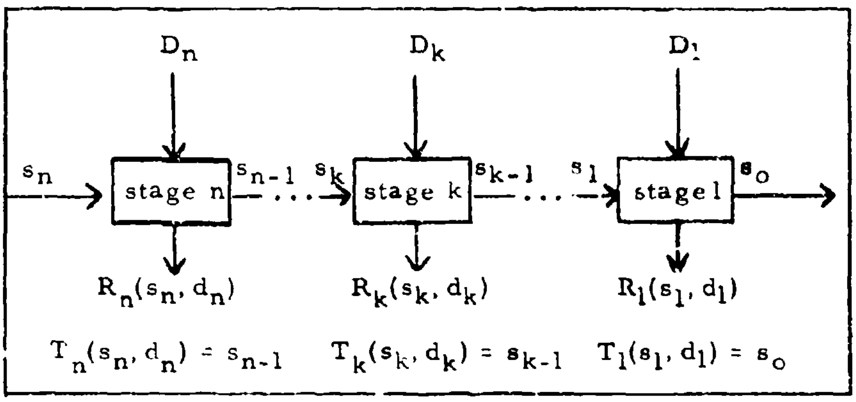


Figure D-19
n-Stage Dynamic Programming Representation

(4) The multi-stage decision problem may then be converted to a series of single-stage decision problems as reflected by a set of recursion equations.

$$f_k(s_k) = \min_{d_k \in D_k} \left[Q_k(s_k, d_k) \right], \quad k = 1,\ 2,\ \ldots,\ n$$

$$Q_k(s_k, d_k) = R_k(s_k, d_k), \qquad k = 1$$

$$= R_k(s_k, d_k) \bullet f_{k-1}(s_{k-1}), \quad k = 2,\ 3,\ \ldots,\ n$$

where $\bullet$ may be interpreted as either an addition or multiplication operator. However, it is used as a multiplication operator on condition that the operands are non-negative.

(5) Then

$$f_n(s_n) = \min_{d_n D_n} \left[R_n(s_n, d_n) \bullet f_{n-1}(s_{n-1}) \right]$$

is the total return which results from the optimal set of decision alternatives.

$$d^* = (d_1^*, d^*_2, \ldots, d^*_n).$$

(6) The above formulation may be applied to a maximization objective by the substitution of max for min..

c. Subsystems operating in series.

(1) The dynamic programming formulation contained herein pertains to apportionment of system reliability requirement among series subsystems in such a manner to minimize the total expenditure of development effort. Some basic assumptions which are fundamental to the formulation are:

(a) At any particular stage of the development program (at time of apportionment), the system can be partitioned into n subsystems and that the present reliability level can be estimated for each subsystem. Failure of any subsystem will cause system failure. In addition, it is assumed that the subsystem goal cannot be less than its estimated present level.

(b) The n subsystems function independently so that expected system reliability resulting from the subsystem goals can be expressed as the product of these subsystem goals:

$$y = \prod_{i=1}^{n} y_i$$

where y is the system reliability goal and y_i is the goal for the i^{th} subsystem.

(c) An effort function can be identified for each subsystem, defining the number of units of development effort required

to raise its reliability level from the present value to any potential reliability goal. The effort may represent a single important resource or a combination of resources if these can be expressed by a common unit. The effort function may be either continuous or discrete. A continuous mathematical function allows the reliability goal to assume any value between the estimated present level and one. A discrete function limits potential subsystem goals to particular values.

(2) Consider a proposed system comprised of n subsystems, each of which are to be developed independently. These subsystems are to function independently and in series. What reliability goal should be assigned to each subsystem in order that the system goal be satisfied at a minimum expenditure of development effort? Symbols to be used in problem formulation are defined as:

$\bar{y}$ = system reliability goal, $0 \leq \bar{y} \leq 1$

x_i = reliability level of subsystem i at the present state of development, $0 \leq x_i \leq 1$

y_i = reliability goal apportioned to subsystem i, $x_i \leq y_i \leq 1$

$G_i(x_i, y_i)$ = units of development effort required to raise the reliability level of subsystem i from x_i to y_i

n = number of subsystems

y_i^* = reliability goal apportioned to subsystem i such that total development effort is minimized.

(3) The problem may be formulated as:

$$\text{minimize } \sum_{i=1}^{n} G_i(x_i, y_i)$$

$$\text{subject to } \prod_{i=1}^{n} y_i = y$$

$$x_i \leq y_i \leq 1, \quad i = 1, 2, \ldots, n$$

(4) The problem may be converted to a dynamic programming problem as follows:

(a) Identify each of the n subsystems as a stage such that an apportionment goal must be determined at each stage. A specific numbering sequence for the stages (subsystems) is not necessary, but each subsystem must maintain its assigned identity throughout the entire procedure.

(b) Define the set, S_k, of all possible states, s_k, at stage k such that:

$$1 = s_n \geq s_{n-1} \geq \cdots \geq s_1 \geq s_o = y$$

(c) Define the set, D_k, of all possible decision alternatives, $d_k = y_k$, at stage k such that:

$$x_k \leq y_k \leq 1, \qquad k = 1, 2, \ldots, n.$$

(d) Define the transformation function for stage k:

$$T_k(s_k, d_k): \; s_k y_k = s_{k-1}, \qquad k = 1, 2, \ldots, n.$$

(e) Define the return realized at stage k as the function:

$$R_k(s_k, d_k) = G_k(x_k, y_k), \qquad k = 1, 2, \ldots, n.$$

(5) The problem is displayed by figure D-20. The resulting recursion equations are:

$$f_k(s_k) = \min_{y_k} \left[Q_k(s_k, y_k) \right] \qquad k = 1, 2, \ldots, n.$$

$$Q_k(s_k, y_k) = G_k(s_k, y_k), \qquad k = 1$$

$$= G_k(s_k, y_k) + f_{k-1}(s_{k-1}), \quad k = 2, 3, \ldots, n.$$

and the optimal set of apportioned goals will be defined as

$$d^* = (y^*_1, y^*_2, \ldots, y^*_n).$$

This problem can be solved by means of a digital computer.

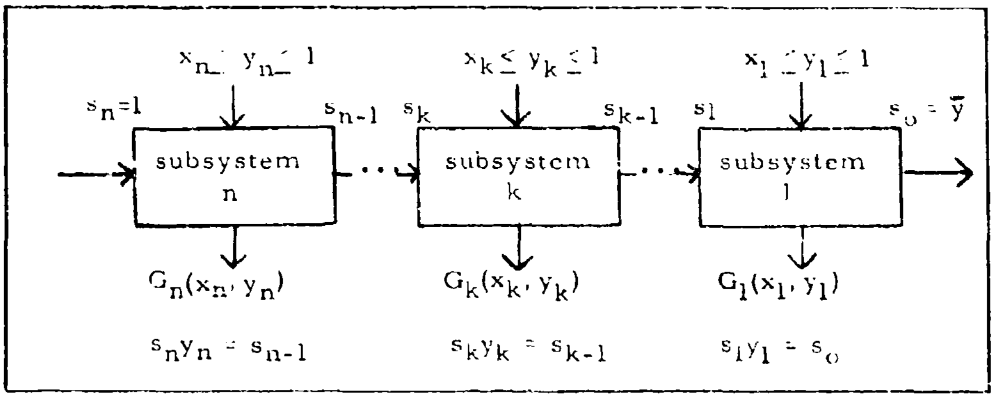


Figure D-20
Dynamic Programming Apportionment Formulation

(b) To exemplify the use of the technique, consider a proposed weapon system which is to be developed as three independent subsystems. The system can be functionally successful if, and only if, each of the three subsystems function properly. In order that the system fulfill its intended role, it should be 0.90 reliable. Based on engineering analysis and historical information of similar type equipment, estimates of the state of the art reliability levels of the subsystems are 0.95, 0.95 and 0.97. What reliability goal should be assigned to each subsystem in order to minimize the total expenditure of development funds? The estimated effort (funds) functions for the three subsystems are contained in figure D-21 where $G_i(x_i, y_i)$ is expressed in \$1000 units. Potential apportioned goals are

limited to those contained in these tabled functions.

y_1	$G_1(0.95, y_1)$	y_2	$G_2(0.95, y_2)$	y_3	$G_3(0.97, y_3)$
0.95	0	0.95	0	0.97	0
0.96	1.0	0.96	20.0	0.98	25.0
0.97	3.9	0.97	46.0	0.99	55.6
0.98	16.5	0.98	81.2	0.995	99.7
0.99	34.0	0.99	126.8		
0.995	65.0	0.995	179.8		

Figure D-21
Table of Effort Functions

(7) First, $(0.95)(0.95)(0.97) = 0.875425 < 0.90$ indicates ... further development is necessary to meet the system goal.

$n = 3$

$\bar{y} = 0.90$

$x_1 = 0.95$

$x_2 = 0.95$

$x_3 = 0.97$

(8) The general formulation follows:

$$\min \; G_1(0.95, y_1) + G_2(0.95, y_2) + G_3(0.97, y_3)$$

subject to $\prod_{i=1}^{3} y_i \geq 0.90$

$y_1 = 0.95,\ 0.96,\ 0.97,\ 0.98,\ 0.99$ or 0.995.

$y_2 = 0.95,\ 0.96,\ 0.97,\ 0.98,\ 0.99$ or 0.995.

$y_3 = 0.97,\ 0.98,\ 0.99$ or 0.995.

(9) Since discrete effort functions allow only specific values to be considered as potential subsystem goals, the system goal might not be met as an equality; hence, the inequality constraint.

(10) The dynamic programming format and elements are shown in figure D-22.

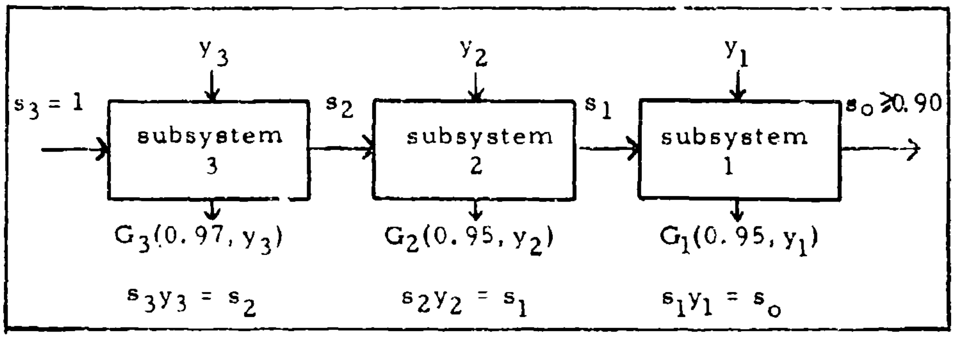


Figure D-22
Dynamic Programming Formulation Example

(11) The recursion equations are:

$$f_1(s_1) = \min_{y_1} \left[G_1(0.95, y_1) \right]$$

$$f_2(s_2) = \min_{y_2} \left[G_2(0.95, y_2) + f_1(s_1) \right]$$

$$f_3(s_3) = \min_{y_3} \left[G_3(0.97, y_3) + f_2(s_2) \right]$$

(12) Figures D-23a, b, and c contain the calculated state transformations for stages 3, 2, and 1, respectively, utilizing the following relations.

$$s_3 = 1$$

$$s_2 = s_3y_3$$

$$s_1 = s_2y_2$$

$$s_o = s_1y_1$$

s_2	y_3			
	0.97	0.98	0.99	0.995
$s_3 = 1$	0.97	0.98	0.99	0.995

Figure D-23a
State Transformations for Stage 3

s_1		y_2					
		0.95	0.96	0.97	0.98	0.99	0.995
s_2	0.97	0.9215	0.9312	0.9409	0.9506	0.9603	0.9652
	0.98	0.9310	0.9408	0.9506	0.9604	0.9702	0.9751
	0.99	0.9405	0.9504	0.9603	0.9702	0.9801	0.9851
	0.995	0.9453	0.9552	0.9652	0.9751	0.9851	0.9900

Figure D-23b
State Transformations for Stage 2

s_0	y_1 = 0.95	0.96	0.97	0.98	0.99	0.995
0.9215	0.8754	0.8846	0.8939	0.9031	0.9123	0.9169
0.9310	0.8845	0.8938	0.9031	0.9124	0.9217	0.9263
0.9312	0.8846	0.8940	0.9033	0.9126	0.9219	0.9265
0.9405	0.8935	0.9029	0.9123	0.9217	0.9311	0.9358
0.9408	0.8938	0.9032	0.9126	0.9220	0.9314	0.9361
0.9409	0.8939	0.9033	0.9127	0.9221	0.9315	0.9362
0.9453	0.8979	0.9074	0.9169	0.9263	0.9358	0.9405
0.9504	0.9029	0.9124	0.9219	0.9314	0.9409	0.9456
0.9506	0.9031	0.9126	0.9221	0.9316	0.9411	0.9458
s_1 0.9552	0.9074	0.9170	0.9265	0.9361	0.9456	0.9504
0.9603	0.9123	0.9219	0.9315	0.9411	0.9507	0.9555
0.9604	0.9124	0.9220	0.9316	0.9412	0.9508	0.9556
0.9652	0.9169	0.9265	0.9362	0.9458	0.9555	0.9603
0.9702	0.9217	0.9314	0.9411	0.9508	0.9605	0.9653
0.9751	0.9263	0.9361	0.9458	0.9556	0.9653	0.9702
0.9801	0.9311	0.9409	0.9507	0.9605	0.9703	0.9752
0.9851	0.9358	0.9456	0.9555	0.9653	0.9752	0.9801
0.9900	0.9405	0.9504	0.9603	0.9702	0.9801	0.9851

Figure D-23c

State Transformations for Stage 1

(13) Figure D-24a shows the calculated $Q_1(s_1, y_1)$ for stage 1.

$$Q_1(s_1, y_1) = G_1(0.95, y_1)$$

The circled values represent

$$\min_{y_1}\left[Q_1(s_1, y_1)\right] = f_1(s_1)$$

The blanks represent s_0 values which do not satisfy the problem constraint that

$$s_0 \geq 0.90.$$

	y_1					
$Q_1(s_1, y_1)$	0.95	0.96	0.97	0.98	0.99	0.995
0.9215	-----	-----	-----	$\boxed{16.5}$	34.0	65.0
0.9310	-----	-----	$\boxed{3.9}$	16.5	34.0	65.0
0.9312	-----	-----	$\boxed{3.9}$	16.5	34.0	65.0
0.9405	-----	$\boxed{1.0}$	3.9	16.5	34.0	65.0
0.9408	-----	$\boxed{1.0}$	3.9	16.5	34.0	65.0
0.9409	-----	$\boxed{1.0}$	3.9	16.5	34.0	65.0
0.9453	-----	$\boxed{1.0}$	3.9	16.5	34.0	65.0
s_1 0.9504	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9506	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9552	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9602	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9604	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9652	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9702	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9751	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9801	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9851	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0
0.9900	$\boxed{0}$	1.0	3.9	16.5	34.0	65.0

Figure D-24a

Returns for Stage 1

(14) Figure D-24b shows the calculated $Q_2(s_2, y_2)$ for stage 2.

$$Q_2(s_2, y_2) = G_2(0.95, y_2) + f_1(s_1)$$

The circled values are

$$f_2(s_2) = \min_{y_2} \left[Q_2(s_2, y_2) \right]$$

$Q_2(s_2, y_2)$	y_2: 0.95	0.96	0.97	0.98	0.99	0.995
s_2 = 0.97	0+16.5 =$\boxed{16.5}$	20.0+3.9 =23.9	46.0+1.0 =47.0	81.2+0 =81.2	126.8+8 =126.8	179.8+0 =179.8
0.98	0+3.9 =$\boxed{3.9}$	20.0+1.0 =21.0	46.0+0 =46.0	81.2+0 =81.2	126.8+0 =126.8	179.8+0 =179.8
0.99	0+1.0 =$\boxed{1.0}$	20.0+0 =20.0	46.0+0 =46.0	81.2+0 =81.2	126.8+0 =126.8	179.8+0 =179.8
0.995	0+1.0 =$\boxed{1.0}$	20.0+0 =20.0	46.0+0 =46.0	81.2+0 =81.2	126.8+0 =126.8	179.8+0 =179.8

Figure D-24b

Cumulative Returns for Stage 2

(15) Figure D-24c shows the calculated $Q_3(s_3, y_3)$ for stage 3.

$$Q_3(s_3, y_3) = G_3(0.97, y_3) + f_2(s_2)$$

The circled value is

$$f_3(s_3) = \min_{y_3} \left[Q_3(s_3, y_3) \right]$$

$Q_3(s_3, y_3)$	y_3: 0.97	0.98	0.99	0.995
$s_3=1$	0.+16.5 =$\boxed{16.5}$	25.0+3.9 =28.9	55.6+1.0 =56.6	99.7+1.0 =100.7

Figure D-24c

Cumulative Returns for Stage 3

(16) Then the optimal decision at stage 3 is

$$y_3^* = 0.97$$

as indicated by the circled value in figure D-24c and

$$s_2 = s_3 y_3^* = 0.97$$

The optimal decision at stage 2, given $s_2 = 0.97$, is

$$y_2^* = 0.95$$

as indicated in figure D-24b, and

$$s_1 = s_2 y_2^* = 0.97(0.95) = 0.9215$$

Similarly, the optimal decision at stage 1, given that $s_1 = 0.9215$, is

$$y_1^* = 0.98$$

as indicated by figure D-24c, and the resulting

$$s_0 = s_1 y_1^* = 0.9215(0.98) = 0.903$$

which meets the system reliability goal.

(17) Summarizing, the optimal reliability subsystem goals are

$$y_3^* = 0.97$$

$$y_2^* = 0.95$$

$$y_1^* = 0.98$$

and the total required expenditure of development funds to achieve these goals is

$$\$1000 f_3(s_3) = \$1000(16.5) = \$16,500$$

as indicated in figure D-24c.

d. Other uses of dynamic programming relative to reliability. The dynamic programming optimization technique has application potential in other areas of reliability analysis. For example, useful models have been developed for determining an optimal number of redundant components (subsystems) subject to restraints such as weight, cost, volume, opposing failure modes, etc. Also, a dynamic programming model has been developed for providing a systems approach to test planning, i.e., planning for an optimal number of tests.

APPENDIX E

RELIABILITY DESIGN

Section I. INTRODUCTION

E-1. General. This appendix is intended to supplement the reliability design discussion of chapter 5. It consists of an example illustrating the role of prediction and apportionment activities during the development phase. The elements of apportionment and prediction techniques are described in chapter 4 and illustrated in appendix D.

Section II. EXAMPLE OF RELIABILITY PREDICTION AND APPORTIONMENT DURING THE DEVELOPMENT PHASE

E-2. Statement of Example. a. A weapon system to be used by the U. S. Army field units was selected. Since this exercise is for exemplary purposes only, duplicate activities will be eliminated where possible. In other words, each and every subsystem will not be subjected to the entire analysis but only exempiary subsystems. All classified areas of the problem were evaded and data is hypothetical.

b. The system to be used is the 115 MM XM70E4 towed Rocket Launcher. The objectives are to:

(1) Predict from the design and available data the reliability of the proposed 115 MM XM70E4 automatic rapid fire field artillery weapon system.

(2) Determine whether the reliability predicted will satisfy the requirement.

(3) Propose an alternate set of requirements if the predicted reliability does not satisfy the requirement.

c. The product is defined as follows.

(1) Description of mission environment and use factors.

(a) The launcher rocket 115 MM XM70E4 is a light weight, towed, close support field artillery weapon system capable of automatically firing a six round burst of 115 MM ammunition, highly mobile, helicopter transportable, quickly emplaced in position to fire.

(b) Transportable by any helicopter capable of a 3400 lb. load, can be towed by any present in service vehicle at 40 mph highway travel and 10 mph cross country, also capable of being air dropped.

(c) Programmed mode of fire from single shot to bursts of 2 to 6 rounds of 115 MM ammunition.

(d) Environment.

1. Thirty percent of the operation will be under prevailing environmental conditions with a temperature range between 0 to 80 degrees Farenheit.

2. Sixty percent of the operation will be under prevailing environmental conditions with a temperature range between 80 and 105 degrees Farenheit.

3. Ten percent of the operation will be under prevailing environmental conditions with a temperature range between -25 to 0 degrees Farenheit.

(e) Use conditions. Twenty-five miles of cross country towing per 1000 rounds fired under standard field conditions; 25 miles of helicopter sling transport per 1000 rounds of fire.

(2) Performance parameters and allowable limits, operating modes and functions, mission profiles and duty cycles.

(a) The 115 MM XM70E4 can be elevated from -5° to 70°. Hand-wheeled powered track traverse 20° each side of center line or 40° total. Rapid traverse of 360° by lifting trail.

(b) Fire 115MM ammunition in single shot fashion 12 rounds per minute or automatically in bursts of 2 to 6 rounds or semi-automatically in single shot fashion where the number of rounds and mode of fire is preselected. The gun has a six round magazine.

(c) Emplaced and trained on target in less than 3 minutes.

(d) Range: Minimum range at maximum elevation 1000 m; maximum range 15,000 m. The weapon must be capable of changing range at the rate of 100m/sec. while simultaneously changing the deflection lay of the gun at the rate of 5 degrees of arc/sec.

(e) Accuracy: Horizontal range error at 80% maximum range is to be 90% of the rounds within 15m of the center of the set range; deflection error at 80% maximum range is 90% of the rounds within 5m of center of set aiming point.

(3) Reliability mission profiles.

(a) Mission A. Fire six, six round bursts; fire in the automatic mode. Minimum acceptable reliability is 0.90.

(b) Mission B. Fire 20 bursts - each burst less than six rounds; fire in the automatic mode two to five round bursts. Minimum acceptable reliability is 0.95.

(c) Mission C. Same as mission B. Firing rate is single shot in the semiautomatic mode. Mission is 120 rounds or 20 full load semiautomatic single round bursts. Minimum acceptable reliability is 0.95.

(4) Function and physical boundaries, physical constraints, operating environment, physical parameters and configuration.

(a) Weights:

Complete weapon with cover	3400 lbs
Recoiling parts	1528 lbs
Tipping parts	1827 lbs
Carriage assembly	1553 lbs

(b) Tires:

Carriage wheel	7.00 x 16, 6 ply
Pressure	45 PSI

(c) Recoiling Parts:

Breech Block	Swing Pivot
Firing Mechanism	Percussion Pin
Recoil System	Hydro Pneumatic

(d) Performance:

Towing Speed — 40 mph on highway
10 mph cross country

Angle of departure (long tow position)-27°

Ground clearance - 10 in.

Traverse - 360°

(e) Ammunition:
115 MM Booster Rocket or
Conventional Projectile

(f) Firing Capability
Single round
Semiautomatic
Full automatic - 6 rounds in 2 seconds

(g) Crew: 5

(h) Basic Lists Required
Parts List
Listing of Parts Lists
List of Drawings
List of Specifications
Equipment List

(i) Mobility
Towed
Air Drop
Helicopter Transportable

(5) Product failure is defined as a stoppage of function with no time for repair allowed. The system will have failed its mission if:

(a) Mission A fails to successfully complete an automatic six round burst (stoppage).

(b) Mission B fails to complete a burst of at least two rounds and no more than five rounds in automatic firing mode (stoppage).

(c) Mission C fails to fire a selected single shot in the semi-automatic or manually loaded single shot.

d. Establish reliability model.

(1) The reliability model. The block diagram for the 115mm XM70E4 weapon system is a series of subsystems as shown in figure E-1.

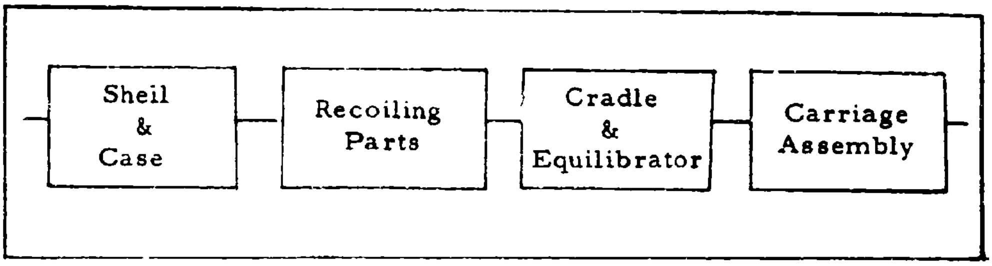


Figure E-1

Weapon Reliability Block Diagram

(2) Alternate modes of operation. This weapon system can function in either the single shot or automatic mode (see mission profiles).

(3) Mission time requirements for each block. Artillery weapon system mission time may be defined as mission duration in terms of number of rounds fired. Since each block must function during the firing of each round, the mission times is the same for all blocks and is defined as:

(a) Mission A - 36 rounds

(b) Mission B - maximum of 120 rounds

(c) Mission C - 120 rounds

(4) Define reliability of the product. Reliability of the weapon system may be defined as the probability of performing as indicated by the mission profile statements.

E-3. Analysis of reliability requirements.

a. Mission A. (1) Let p be the probability of successfully firing a single round. Then the reliability for a six-round burst is p^6 and the reliability for 6 six-round bursts is $(p^6)^6 = p^{36}$.

(2) Assuming the number of rounds between failure (stoppage) to be exponentially distributed, the reliability requirement may be translated to the probability of successfully firing a single round (p), the mean rounds between failure(MRBF or θ)or to failure rate (λ).

(a) The Mission A system reliability requirement is R_s(36 rounds) = 0.90.

(b) The required probability of successfully firing a single round may be found as follows:

$$p^{36} = 0.90$$

$$p = 0.90^{1/36} = 0.9971$$

(c) Then, a comparable required MRBF may be found as follows:

$$\exp\left(-\frac{36}{\theta}\right) = 0.90$$

or

$$\theta = \frac{-36}{\ln(0.90)} = 342 \text{ rounds between failure}$$

(d) The required failure rate may be expressed as

$$\lambda = \frac{1}{342} = 0.00292 \text{ failures per round}$$

b. Mission B. (1) Since this mission allows for the automatic firing of 2 to 5 rounds in a burst for 20 bursts or any combination of 2 to 5 rounds (e. g., 20 bursts of 2 rounds, 3 rounds, 4 rounds, 5 rounds or a sequence of varied bursts, e. g., 2, 3, 5, 4, 2, 2, 2, 4, 3, 2, 2, 3, 3, 3, 5, 5, 4, 4, 2 and 2) or a total of $20^4 = 160,000$ possible mission combinations. It is an impractical task to determine each possible combination. For the purpose of demonstration, ten bursts of 2 rounds each and ten bursts of 4 rounds each will be considered as mission duration requirement. (Note total rounds are even multiples of six) In practice we may consider those combinations representing the most severe requirement.

(2) The system reliability requirement may be stated in any of the following ways.

(a) The mission B system reliability requirement is $R_s(60) = 0.90$.

(b) The comparable p requirement may be found as follows:

$$R_s(60) = p^{60} = 0.95$$

$$p = (0.95)^{1/60} = 0.999147$$

(c) The required MRBF is

$$\theta = \frac{-60}{\ln(0.95)} = 1170 \text{ rounds between failure}$$

(d) The required failure rate is

$$\lambda = \frac{1}{1170} = 0.000855 \text{ failures per round}$$

c. Mission C. (1) This mission requires the firing of 20 one shot full loads semi-automatically (a total of 120 rounds).

(2) The system reliability requirement may be stated in any of the following ways.

(a) The mission C system reliability requirement is $R_s(120) = 0.95$.

(b) The comparable p requirement is

$$p = (0.95)^{1/120} = 0.9995734$$

(c) The required MRBF is

$$\theta = \frac{-120}{\ln(0.95)} = 2340 \text{ rounds between failures}$$

(d) The required failure rate is

$$\lambda = \frac{1}{2340} = 0.000428 \text{ failures per round}$$

(3) The mission C requirement of MRBF = 2340 rounds does not necessarily preclude the MRBF = 1170 rounds for missions A and B because A and B involve fully automatic firing and mission C involves semiautomatic firing.

d. Assumptions and simplifications of analysis. In order to determine MRBF of the various modes it is assumed that when firing the weapon a failure is just as likely to occur on the first round as it is on any other round, regardless of the mode of fire. In the mission profile analysis it was further assumed that all primary systems and other operating requirements were met. Human element is assumed to have reliability of 1.

E-4. Functional complexity and active elements.

a. The four primary subsystems are shown in block diagram form by figure E-2.

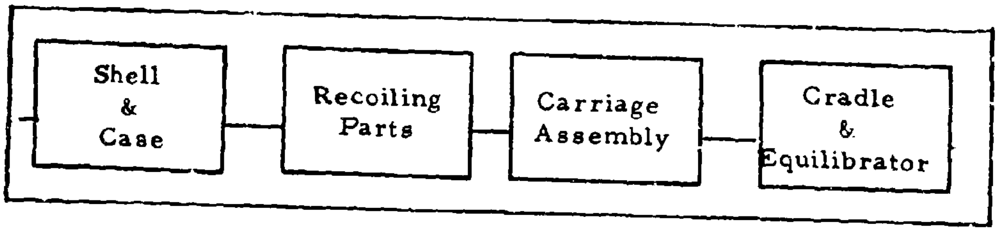


Figure E-2

Block Diagram of Primary Subsystems

b. The recoiling parts subsystem is selected to demonstrate further analysis approaches. Figure E-3 shows a block diagram for this primary subsystem.

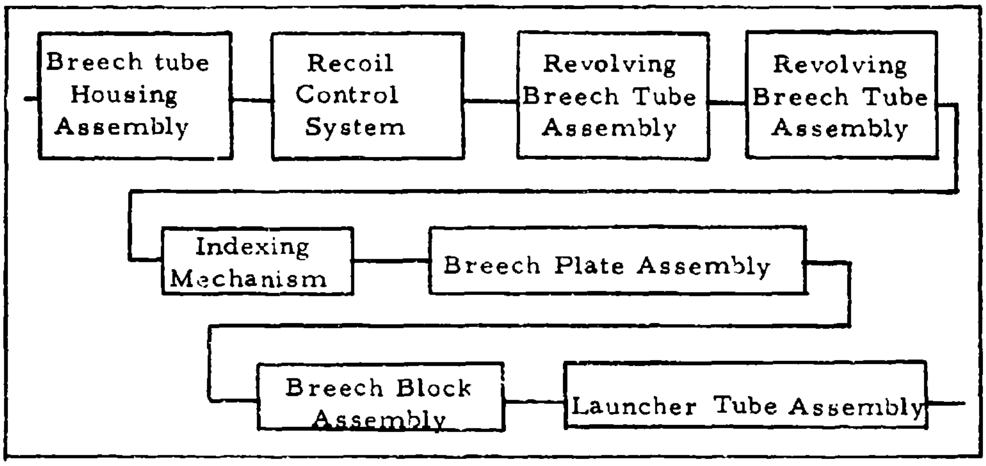


Figure E-3

Block Diagram of Recoiling Parts

c. A functional complexity description for each recoiling part follows:

(1) Breech tube housing assembly. (a) The breech tube housing assembly consists of the breech housing pawls and pawl shafts, indexing make-up energy torsion bar, V-runners, auto-cocking shaft, breech tube indicate signal shaft, and slow counter-recoil signal shaft.

(b) The part population is:

Breech housing pawls	8
Pawl shafts	4
Indexing make-up energy torsion bar	1
V-runners	4
Auto-cocking shaft	1
Breech tube indicator signal shaft	1
Shaft and slow counter recoil signal shaft	1

(2) Recoil control system. (a) The recoil system is a hydro-pneumatic arrangement. Two single acting recoil cylinders are connected to a hydraulic accumulator. Interposed between the recoil cylinders and the accumulator is a control system which will be referred to as the hydraulic recoil control.

(b) The part population is:

Single acting recoil cylinders	2
Hydraulic accumulator	1
Manifold	1
Recuperator	1
Recuperator piston	1
Recoil pistons	1
Buffer pistons	1
Buffer cylinders	1
Cocker pistons	1
Hydraulic recoil control block	1
Recoil throttle valve	1
Recoil flow control release	1
Recoil signal orifice	1
Check valve	4
Counter recoil flow control valve	1
Counter recoil orifice spool	1
Orifice	1
Anti-cavitation valve	1
Spring actuated relief valve	1
Indexing cock and release actuator	1
Recoil timing valve	1
Pressure actuated relief valve	1
Recoil pressure regulator	1

(3) Breech tube assemblies. (a) Two assemblies, consisting of three breech tubes each, are located in the breech tube housing between the front and rear bulkheads. Each assembly is composed of three smoothbore tubes fastened to a spider weldment.

(b) The part population is:

Breech tubes	3
Spider	1

(4) Indexing mechanism. (a) The indexing mechanism is an energy-conserving type operated by torsion bar springs. With this mechanism, indexing motion is divorced from recoil motion.

(b) The part population is:

Cams	2
Cam rollers	2
Torsion bars	2
Gears	6
Worm gear	1

(5) Breech plate assembly. (a) The breech plate assembly covers the rear of the breech tube housing and contains the manual indexing linkage, firing mechanism and extractor linkage.

(b) The part population is:

Firing mechanism	1
Extractor linkage	1
Torsion bar wind-up linkage	1
Pawl release linkage	1

(6) Breech block assembly. The breech block is a swinging type featuring center position loading. This allows firing single shots from the same chamber without indexing.

(7) Launcher tube. The single 115mm launcher tube is mounted in the indexing mechanism housing. The tube which weighs 147 pounds is progressively rifled. Gunner's quadrant pads are provided on the top of the tube for checking tube elevation.

E-5. <u>Prediction of failure rates for primary subsystems.</u> a. Recoil-parts subsystem. In order to establish failure rates, available test

data, supplemented by data from FARADA, were used.

(1) Breech tube housing assembly. Past firing data on early experimental rapid fire weapons of this type have shown that approximately four failures per 6,000 rounds occurred to the breech tube housing assembly. Therefore, we will assign a failure rate of 4 failures per 6,000 rounds or 1500 MRBF.

(2) Recoil control system. The recoil control system is basically a hydraulic system consisting of hydraulic pistons and a hydraulic valve network in a single manifold. Figure E-4 shows a tabulation of the number of different types of valves in the system, and the estimated failure rates.

Valve Type	Quantity in System	Estimated Failure Rate*	Total Failure Rate*
Check Valves	4	$50/10^6$ hrs.	$200/10^6$ hrs.
Control Valves	3	$100/10^6$ hrs.	$300/10^6$ hrs.
Relief Valves	2	$100/10^6$ hrs.	$200/10^6$ hrs.
Estimated Total System Failure Rate			$= 700/10^6$ hrs.

Figure E-4

Valve Failure Rate Data

*To convert failures per 10^6 hours to failures per round, we assume an average of approximately 10 rounds per hour. Estimated failure rate for this system then is 7 failures/100,000 rounds.

(3) Indexing mechanism. The indexing mechanism is essentially a gear train, therefore gear failure rates may be used to estimate the MRBF. Figure E-5 shows failure rates obtained from FARADA on similar parts.

Part	Quantity	Failure Rate*	Total Failure Rate*
Cams	2	$1/10^4$ hrs.	$2/10^4$ hrs.
Rollers	2	$0.5/10^4$ hrs.	$1/10^4$ hrs.
Gears (spur)	6	$2/10^4$ hrs.	$12/10^4$ hrs.
Worm Gear Assembly	1	$10/10^4$ hrs.	$10/10^4$ hrs.
Subsystem Total Failure Rate		=	$25/10^4$ hrs.

Figure E-5

Indexing Mechanism Parts Failure Rate Data

*Using 10 rounds per hour as an average firing rate, the failure rate in terms of rounds fired is 25 failures/100,000 rounds.

b. Other subsystems. (1) No historical failure data are available for the following items:

(a) Breech tube assemblies

(b) Breech plate assemblies

(c) Breech block assemblies

(d) Launcher tube

(2) Test firing of 6000 rounds on an early feasibility model resulted in no failures of any of the above items. Under the assumption that the exponential distribution (within the useful life of the gun) applies as a model, a lower 50% confidence limit may be found for the MRBF.

$$\theta > \frac{2 \times t}{\chi^2_{2,2r+2}} = \frac{(2)(6000)}{\chi^2_{.50,2}} = \frac{12000}{1.386}$$

θ > 8658 rounds between failure

Then an upper 50% confidence limit for the failure rate is

$$\frac{1}{8658} = .00011554 \text{ failures per round.}$$

This failure rate is divided equally among the twelve subsystems indicated by * in figure E-6. This figure shows the predicted failure rates including those for the primary subsystems, cradle and equilibrator, carriage assembly, and shell and case.

(3) Historical data indicates the failure rate due to shell and case to be about 0.00001 failures per round.

Subsystems	Failures per 10^5 rounds	MRBF
Recoiling Parts		
1) Breech tube housing assembly	66.	1,516
2) Recoil control system	7.	14,286
3) Breech tube assemblies*	0.96283	108,605
4) Indexing mechanism	25.	4,000
5) Breech plate assembly*	0.96283	108,605
6) Breech block assembly*	0.96283	108,605
7) Launcher tube*	0.96283	108,605
Total	101.85132	982
Cradle and Equilibrator		
1) Cradle*	0.96283	108,605
2) Equilibrators*	0.96283	108,605
Total	1.92566	51,814
Carriage Assembly		
1) Box frame*	0.96283	108,605
2) Upper firing base*	0.96283	108,605
3) Lower firing base*	0.96283	108,605
4) Suspension*	0.96283	108,605
5) Traversing mechanism*	0.96283	108,605
6) Elevating mechanism*	0.96283	108,605
Total	5.77698	17,272
Shell and Case	1.	100,000

* See preceding page

Figure E-6

Predicted Failure Rates

E-6. Predicted system failure rate. The predicted system failure rate may be calculated as the sum of the primary system failure rates. $101.85 + 1.93 + 5.78 + 1.00 = 110.56$ failures per 10^5 rounds.

E-7. Comparison of predicted failure rate with requirements.

a. Mission A. Failure rate is required not to exceed 292 failures per 10^5 rounds. Then the present state of development is sufficient to satisfy Mission A reliability requirements.

b. Mission B. Failure rate is required not to exceed 85.5 failures per 10^5 rounds. Then the system requires further development to satisfy Mission B reliability requirements.

c. Mission C. Since Mission C pertains to semiautomatic firing which has performance features different than those for A and B, the prediction procedure would be similar to that for A and B, and will be omitted for this example. Due to the nature of Mission C there would be certain vital differences in the block diagram of time system, for example only (1) breech tube assembly need be operational, the indexing mechanism and recoil control assembly would not need to work.

E-8. Apportionment of reliability goals to primary subsystems.

a. In order for the system to meet its Mission B requirements it is necessary to apportion the reliability requirements. Certain components of the system will require improved reliability so that the overall system reliability will be increased sufficiently to meet the Mission B requirement. A systematic method of reapportioning reliability is needed. Some factors to be considered are cost and technical complexity.

b. The minimization of effort algorithm (appendix D) may be utilized to apportion the goals for Mission B.

(1) Determine the present predicted reliability level for each primary subsystem and rank from lowest to the highest in value.

Recoiling Parts	Cradle and Equilibrator	Carriage Assembly	Shell and Case
$R_{rp} = e^{-60/982}$	$R_{ce} = e^{-60/51,814}$	$R_{ca} = e^{-60/17272}$	$R_{sc} = 1$
$R_{rp} = e^{-0.0611}$	$R_{ce} = e^{-0.00117}$	$R_{ca} = e^{-0.00347}$	
$R_{rp} = 0.94073$	$R_{ce} = 0.99883$	$R_{ca} = 0.99653$	

Let $R_{rp} = R_1$

Let $R_{ca} = R_2$

Let $R_{ce} = R_3$

$\therefore \quad R_1 < R_2 < R_3$

Ignore R_{sc} since it is one.

Compute
$$r_j = \left[\frac{R}{\prod_{i=j+1}^{n+1} R_i} \right]^{1/j}$$

R* is the mission reliability

$$r_1 = \left[\frac{R*}{R_2 R_3} \right]^{1/1} \frac{0.95}{(0.99653)(0.99883)} = 0.954425$$

$\therefore \quad (R_1 = 0.94073) < (r_1 = 0.954425)$

$$r_2 = \left[\frac{R*}{R_3} \right]^{1/2} = \left[\frac{0.95}{0.99883} \right]^{1/2} = 0.97525$$

$(R_2 = 0.99653) > (r_2 = 0.97525)$

$\therefore$ R_1 is the only system that must be improved, i.e., R_{rp} must be increased from 0.94073 to 0.954425.

(2) It is now necessary to determine which of the components of the primary system should become more reliable. Repeating the above procedure for the recoiling parts subsystem for a required level of $R^*_{rp} = 0.954425$.

Breech tube housing assembly	$R = e^{-60/1516} = 0.96120 = R_1$
Recoil control system	$R = e^{-60/14,286} = 0.99581 = R_3$
Breech tube assembly	$R = e^{60/108,605} = 0.99945 = R_4$

Indexing mechanism	$R = e^{-60/4000} = 0.98511 = R_2$
Breech plate assembly	$R = e^{-60/108,605} = 0.99945 = R_5$
Breech block assembly	$R = e^{-60/108,605} = 0.99945 = R_6$
Launcher tube assembly	$R = e^{-60/108,605} = 0.99945 = R_7$

$R_1 \leq R_2 \leq R_3 \leq R_4 \leq R_5 \leq R_6 \leq R_7$

$$r_j = \left[\frac{R^*}{\prod_{i=j+1}^{n+1} R_i}\right]^{1/j}$$

$$r_1 = \left[\frac{0.954425}{R_2R_3R_4R_5R_6R_7}\right]^{1/1} = \left[\frac{0.954425}{(0.98511)(0.99581)(0.99945)^4}\right] = 0.97507$$

$$(R_1 = 0.96120) < (r_1 = 0.97507)$$

$$r_2 = \left[\frac{0.9544255}{(0.99581)(0.99945)}\right]^{1/2} = 0.96055$$

$$(R_2 = 0.98511) > (r_2 = 0.96055)$$

The breech tube housing assembly is the only subsystem that must be improved. Therefore in order for the recoiling parts to acquire a reliability 0.954425, the breech tube housing assembly reliability must be increased from 0.96120 to 0.97507.

(3) Then for Mission B

$$R_s = R_{rp}R_{ce}R_{ca}R_{sc}$$

$$R_s = (0.954425)(0.99883)(0.99653)(1)$$

$$R_s = 0.95$$

The breech tube housing assembly must have a reliability of 0.97507 or a maximum failure rate of

$$\lambda = \frac{-\ln(0.97507)}{60} = 41 \text{ failures per } 10^5 \text{ rounds}$$

E-9. <u>An alternate method of apportionment.</u> a. The procedure of Minimum Effort illustrated above assumes that the effort (time, money, resources, etc.) to be expended in improving any of the subsystems is equivalent. Obviously this is not always a good assumption. A more general approach to the problem is the technique of dynamic programming described in appendix D.

b. This technique, though general in nature, requires that a cost and improvement relationship be specified. As a result it is more definitive and requires knowledge about the economy of improving specific subsystems such as cost of improvement per incremental jump.

c. Since there are no such data available for the preceding system, a hypothetical cost function will be used as an example. The problem may be formulated as follows.

$$\text{Minimize} \quad \sum_{i=1}^{3} G_i(x_i, y_i)$$

$$\text{Subject to } y \geq \bar{y}$$

$$x_i \leq y_i \leq 1$$

x_i is the existing reliability level of subsystem i

y_i is the level to which subsystem i should be improved.

$G_i(x_i, y_i)$ = amount of effort required to raise the reliability level of subsystem from x_i to y_i.

$\overline{y}$ is the system reliability goal or requirement for Mission B.

$$R(\text{Mission B}) = 0.95$$

$$R_{rp} = 0.94073 = x_1$$

$$R_{ce} = 0.99883 = x_2$$

$$R_{ca} = 0.99653 = x_3$$

$R_{sc} \approx 1$, i.e., it will be ignored since it is not likely that reliability can be improved. For this example let the cost be as described in figure E-7 with cost expressed in thousand dollar units required for additional development. (Note: Due to lack of cost data, the following figures are purely hypothetical).

y_1	$G(0.94, y_1)$	y_2	$G_2(0.99883, y_2)$	y_3	$G_3(0.9965, y_3)$
0.94	0	0.99883	0.0	0.9965	0
0.945	40.0	0.9990	200.0	0.9970	100
0.95	120.0	0.9992	600.0	0.9975	200
0.955	480.0	0.9994	2400.0	0.9980	600
0.96	2400.0	0.9996	12000.0	0.9985	2400
0.965	14,400.0	0.9998	72000.0	0.9990	12,000
0.97	100,800.0	0.9999	504000.0	0.9995	72,000
0.975	907,200.0	0.99999	4032000.0	0.9999	504,000

Figure E-7

Cost Function for Increasing Reliability

The computer yields the following solution

Subsystem 1 Goal = 0.95500

Subsystem 2 Goal = 0.99883

Subsystem 3 Goal = 0.99650

Requiring 480.00 units of effort (cost)

$R_s = R_1R_2R_3 = 0.95054$

This indicates that the recoiling parts subsystem reliability should be increased to 0.955.

E-10. Stress-strength design. In the design of mechanical hardware, the interrelationship of stress-strength distributions is often investigated to insure that the probability of over-stress failures is at an acceptable level. This method of statistical design evaluation utilizes the estimated or known stress levels to be encountered in conjunction with material properties, in order to obtain a design which normally minimizes weight and cost due to overdesign. Design engineers who use strength of materials techniques will find this method easy to master and useful when reliability is of prime concern.

E-11. Summary. a. The entirety of this appendix has been to exemplify early prediction and apportionment. The numerical accuracy used has been unrealistic in the face of grossly estimated data and simplifying assumptions.

b. It should also be noted that there are several procedures available for prediction and apportionment, all of which must be modified to satisfy a given project. The methodology described herein was to demonstrate the procedure and was chosen primarily for ease of discussion.

APPENDIX F

DEMONSTRATION AND TESTING

Section I. INTRODUCTION

F-1. General. a. The materials contained in this appendix are intended to supplement the discussions in chapter 6. The content includes some typical statistical analysis techniques associated with reliability demonstration and testing along with some brief examples. This is not an exhaustive coverage of available techniques, but a summary of some typical and basic techniques. A more thorough coverage may be found in the various published reliability literature.

b. These techniques pertain to inferences about certain reliability related parameters from sample test results. Coverage includes both estimation of and hypothesis tests about such parameters. The distributions and parameters covered are those discussed in appendix A.

Section II. PARAMETER ESTIMATION

F-2. General. a. The principles of statistical estimation may be applied to the results of sample tests to estimate reliability parameters. The principle of parameter estimation may be illustrated by considering a random variable X with a density function f(x) which is described by a parameter θ. Suppose we make n independent observations on X, i.e., $x_1, x_2, \ldots, x_n$. Analysis of these values will allow us to estimate θ by:

(1) A point estimate designated as $\hat{\theta}$ and/or

(2) A confidence statement that

$$A < \theta < B$$

where A and B are dependent upon the specified degree of confidence required and may be found by performing an appropriate analysis on the sample observations.

F-3. Nonparametric analysis. a. Nonparametric procedures for estimation of reliability are useful when the underlying distribution of failure times cannot be identified. The method considered herein will utilize the binomial distribution approach to estimation of equipment reliability and may be used to obtain estimates of reliability from a nonreplacement, time-terminated test if the mission time is equal to test termination time. The reliability for a mission time of T, i.e. R(T), may be defined as that portion of the population which would not fail during a mission whose length is equal to the test duration time T.

b. Consider a nonreplacement life test which will be terminated at time T where n sample items are placed on test. Let r be the number of failed items and d be the number of items which did not fail.

(1) A nonparametric point estimate of R(T) is

$$\hat{R}(T) = \frac{d}{n}$$

(2) Table H-7 of appendix H provides 90%, 95%, or 99% lower one-sided confidence limits for R(T) if n $\leq$ 30. The table headings contain directions for its use. Table H-8 provides two-sided confidence limits.

(3) A lower 100(1- α)% confidence limit for R(T) may also be found by

$$R(T) \geq \frac{1}{1 + \left(\frac{r+1}{n-r}\right) F_{\alpha, 2r+2, 2n-2r}}$$

where α is the likelihood that the statement is incorrect (and is the complement of the confidence level); n, T and r are as defined above; and $F_{\alpha, 2r+2, 2n-2r}$ may be found in table H-5 of appendix H.

This method is not limited to 90%, 95%, and 99% confidence levels encountered above.

c. To illustrate the estimation technique and the concept of confidence statements, consider a nonreplacement test which is to be terminated after 40 hours. Twenty type xxx batteries were placed on test and after 40 hours, 12 items had not yet failed. Assuming the underlying failure distribution to be unknown, estimate battery reliability for a 40-hour mission by means of a point estimate and a 90% confidence interval.

(1) $n = 20$, $r = 8$ failures, and $d = 12$ survivors.

(2) $\hat{R}(40) = \frac{12}{20} = 0.60$ which, based on these sample test results, represents a point estimate of R(40) for such batteries.

(3) Using table H-7, a 90% lower confidence limit for $R(40) \geq 0.433$. That is, we are 90% confident that the battery reliability for a 40-hour mission time is greater than 0.433.

(4) Using the preceding formula for $\alpha = 0.10$ and $F_{.10, 18, 24} = 1.75$ (as found by interpolation in table H-5):

$$R(40) \geq \frac{1}{1 + \left(\frac{8 + 1}{20 - 8}\right) 1.75} = \frac{1}{1 + 1.31} = 0.433$$

This agrees with the tabular determination.

d. The same procedures may be applied to nontime dependent or one shot items for purposes of estimating the portion of items (p) which successfully fulfill a mission. To exemplify this, consider a sample of twenty rounds of ammunition which have been proof fired and two did not function properly. Estimate p by means of a point estimate and by a 95% lower confidence limit.

(1) Define the following:

$n = 20$
$r = 2$ failures
$d = 18$ successes
$\alpha = .05$
$F_{.05, 4, 36} = 2.642$ from Table H-5.

(2) A point estimate of p is $\hat{p} = \frac{d}{n} = \frac{18}{20} = 0.90$.

(3) Using table H-7, a 95% lower confidence limit for p is

$$p \geq 0.717$$

In other words, there is 95% confidence that at least 0.717 of all rounds will perform successfully.

(4) By formula, a 95% lower confidence limit for p may be found as

$$p \geq \frac{1}{1 + \left(\frac{3}{8}\right)(2.64)} = \frac{1}{1.44} = 0.694$$

F-4. <u>Determination of underlying distribution.</u> a. More precise estimates of reliability parameters can be obtained if the underlying distribution of failure times can be determined. Certain goodness of fit tests can be applied to test failure times for determining whether a hypothesized distribution is a reasonable model. Both graphical procedures and statistical methods are included herein for this purpose.

b. Exponential graphical procedure.

(1) Semi-log graph paper may be used to determine the validity of the assumption that the underlying distribution is exponential. The vertical scale of this graph paper is logarithmic and the horizontal is arithmetic. Given a sample of size n resulting in failure times $x_1, x_2, \ldots, x_n$, the graphical procedure follows:

(a) For each failure time, compute $\frac{n+1}{n+1-i}$ where $i = 1, 2, 3, \ldots, n$ and stands for the failure number.

(b) Plot the points $\left(x_i, \frac{n+1}{n+1-i}\right)$ where x_i is found on the horizontal arithmetic scale and $\frac{n+1}{n+1-i}$ on the vertical logarithmic scale.

(c) Draw a straight line, through the origin, which best fits the trend indicated by these points.

(d) If this line provides a good indication of the trend, the underlying distribution may be considered exponential. The decision is qualitative and is dependent upon the decision maker.

(2) To exemplify the use of this graphical procedure, consider a life test of 10 sample widgets which resulted in failure times of 0.8, 0.9, 1.9, 2.4, 2.8, 4.1, 4.4, 6.2, 10.2 and 12.4 hours. Using semi-log paper, determine graphically whether the underlying distribution of failure time may be considered exponential.

(a) Figure F-1 contains the calculated coordinates to be plotted (n = 10).

(b) Figure F-2 is a semi-log graph plotted from these coordinates.

i	x_i	$\frac{n+1}{n+1-i}$
1	0.8	1.10
2	0.9	1.22
3	1.9	1.38
4	2.4	1.57
5	2.8	1.83
6	4.1	2.20
7	4.4	2.75
8	6.2	3.67
9	10.2	5.50
10	12.4	11.00

Figure F-1
Exponential Coordinates

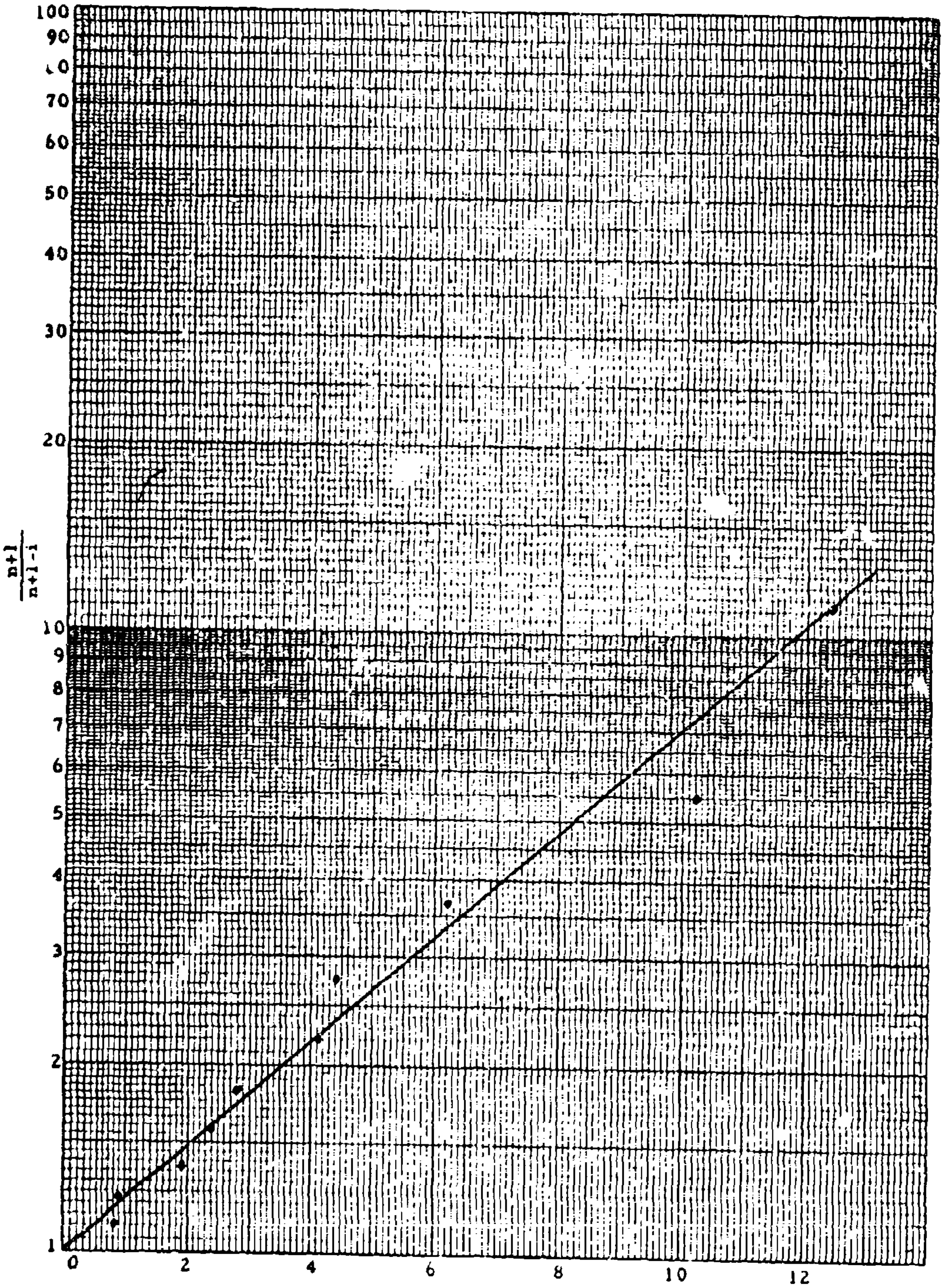


Figure F-2
Plot of Data from Figure F-1

(c) The resulting decision concerning the underlying distribution is qualitative and depends upon how well a straight line fits the plotted points. In this example, a straight line trend is rather pronounced and an underlying exponential distribution is reasonable.

c. Weibull graphical procedure.

(1) Various types of Weibull probability paper are available for determining the validity of the assumption that the underlying distribution of failure times is Weibull. The Weibull paper used below utilizes four scales. For purposes of discussion, the scales shall be identified A, B, C and D as shown in figure F-3. Given a sample of n items with failure times $x_1, x_2, \cdots, x_n$, the graphical procedure follows:

(a) For each failure time compute $\frac{100\,i}{n+1}$ where $i = 1, 2, \ldots, n$ stands for the failure number.

(b) Plot the coordinates $\left(x_i, \frac{100i}{n+1}\right)$ on scales A and B, respectively,

(c) Draw a straight line which best fits the trend indicated by these points.

(d) The decision concerning the validity of the Weibull assumption depends upon how well the line fits the plotted points. A good fit indicates that a Weibull distribution is a reasonable failure model.

(e) The parameters, β and η, of the Weibull distribution may also be estimated by graphical procedures.

1 Draw a line through the point (1, 0), referring to scales C and D respectively, parallel to the original trend line.

2 Find the value from the D scale corresponding to the point of intersection of the parallel line with the 0.0 axis of the C scale. This value is an estimate of β.

3 Find the value from the A scale corresponding to the point of intersection of the trend line with the 0.0 axis of the D scale. This value is an estimate of the parameter η.

(f) If the trend of plotted points is something other than a straight line, the assumption of a two-parameter Weibull distribution is rejected.

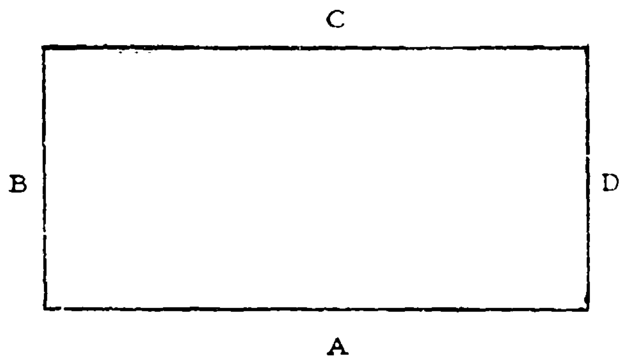


Figure F-3
Weibull Paper Scales

(2) To exemplify the use of this graphical procedure, consider a life test of 9 vacuum tubes resulting in failure times of 2.0, 3.7, 5.3, 6.2, 8.5, 9.4, 11.1, 16.0, and 18.8 hours. Using Weibull probability paper, determine graphically the validity of the assumption of an underlying Weibull distribution. If the assumption is valid, estimate the parameters β and η.

(a) Figure F-4 shows the coordinates to be plotted and the plot of $\left(x_i, \frac{100i}{n+1} \right)$ is shown by figure F-5.

(b) Since the trend line fits the plotted points quite well, the Weibull model of failure times is reasonable. As indicated by the graph, estimates of β and η are 1.35 and 10.5 respectively.

i	x_i	$\frac{100i}{n+1}$
1	2.0	10
2	3.7	20
3	5.3	30
4	6.2	40
5	8.5	50
6	9.4	60
7	11.1	70
8	16.0	80
9	18.8	90

Figure F-4
Weibull Coordinates

d. Normal graphical procedure.

(1) Normal probability paper may be used to determine the validity of the assumption that the underlying distribution is normal. This graph paper consists of a horizontal arithmetic scale and a vertical probability scale. Given a sample of n items with failure times $x_1 \leq x_2 \leq \ldots \leq x_n$, the graphical procedures follow:

(a) For each failure time, compute $\frac{100i}{n+1}$ where i = 1, 2, ..., n stands for the failure number.

(b) Plot the points $\left(x_i, \frac{100i}{n+1}\right)$ where x_i is found on the arithmetic scale and $\frac{100\,i}{n+1}$ on the probability scale.

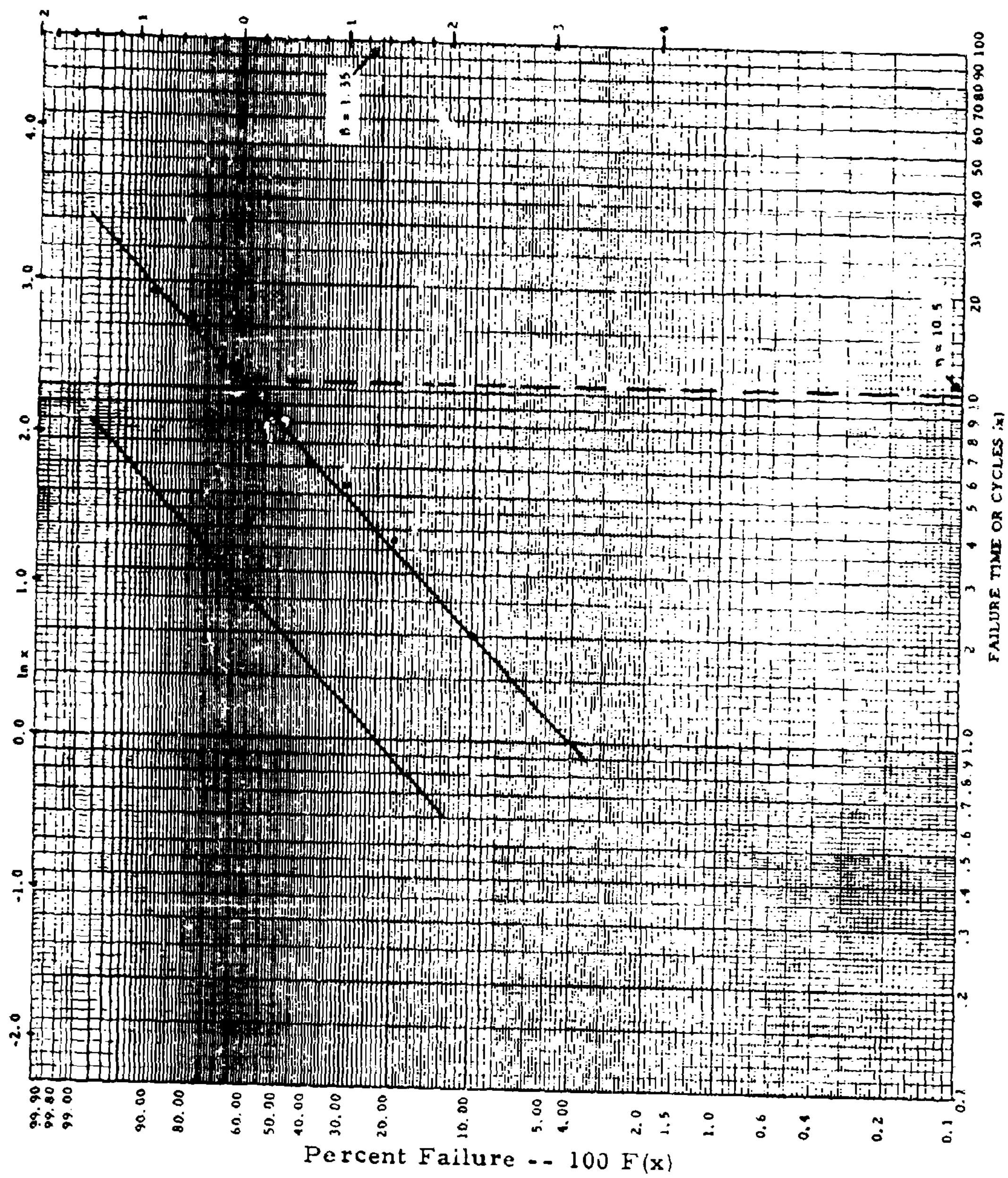


Figure F-5
Weibull Plot

(c) If the trend of the points can be represented by a straight line, the assumption of a normal underlying distribution is reasonable.

(2) Nine rounds have been fired in a certain type gun. The resulting chamber pressure values were 1,100; 1,500; 2,300; 2,500; 2,600; 3,200; 3,300; 3,400 and 3,600 psi. Using normal probability paper, determine graphically the validity of the assumption of an underlying normal distribution pressure values.

(a) Figure F-6 indicates the coordinates to be plotted. the plot of $\left(x_i, \dfrac{100\,i}{n+1}\right)$ on normal probability paper is as shown by figure F-7.

i	x_i	$\frac{100i}{n+1}$
1	1,100	10
2	1,500	20
3	2,300	30
4	2,500	40
5	2,600	50
6	3,200	60
7	3,300	70
8	3,400	80
9	3,600	90

Figure F-6
Normal Coordinates

(b) Since a straight line fits the trend of plotted points, the assumption of normality is reasonable.

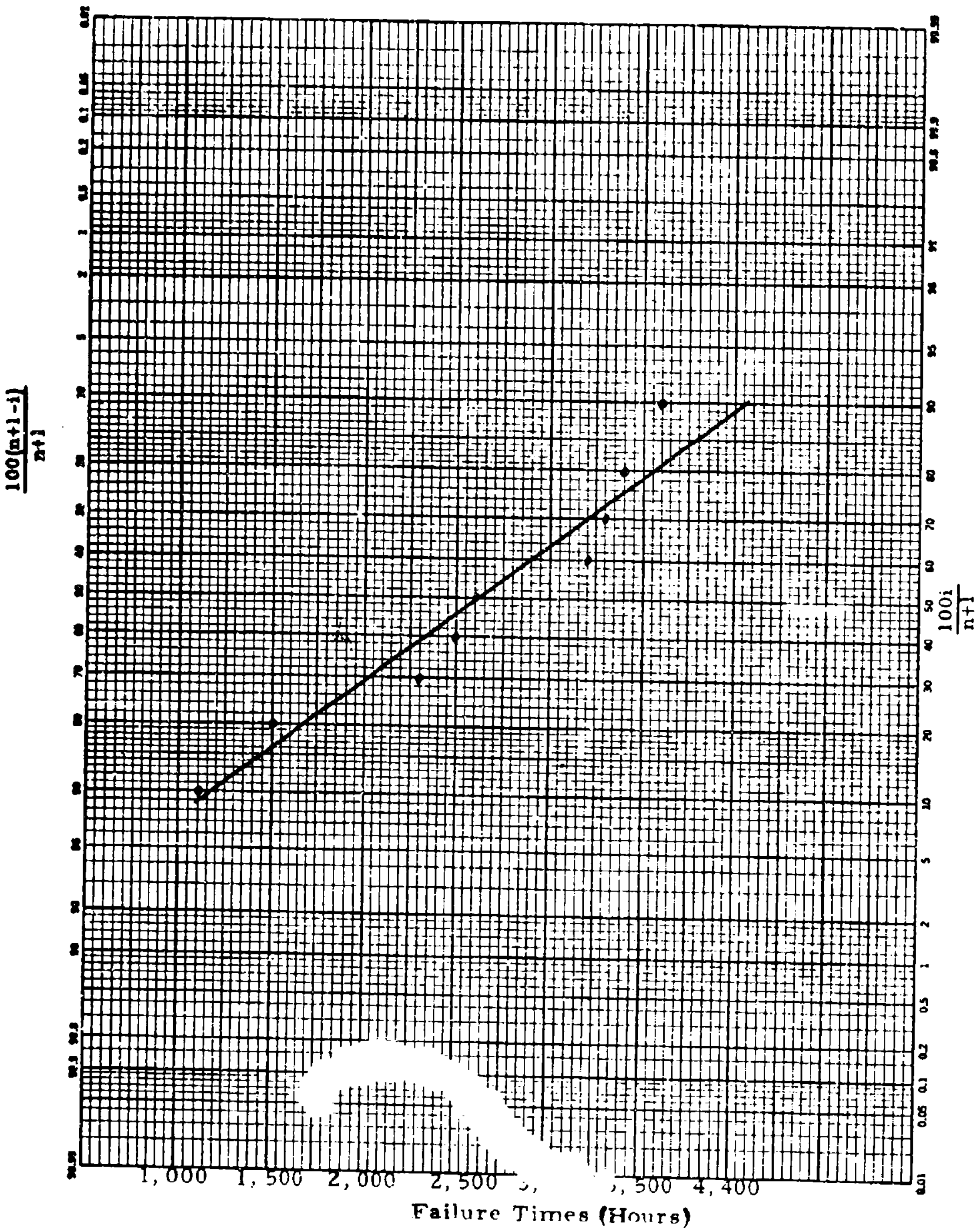


Figure F-7

Normal Plot

e. Chi-squared goodness of fit test.

(1) The χ^2 goodness of fit test may be used to test the validity of any assumed distribution, either discrete or continuous. The test may be summarized as follows for a continuous distribution.

(a) Determine the underlying distribution to be tested.

(b) Determine a level of significance, α, which is defined as the risk of rejecting the underlying distribution if it is, in fact, the real distribution.

(c) Divide the continuous scale into intervals. For reliability analysis, this scale is usually time.

(d) Determine the number of sample observations falling within each interval.

(e) Using the assumed underlying distribution, determine the expected number of observations in each interval. Combining of intervals may be required because the expected number of observations in an interval must be at least 2.5. This determination may require an estimation of the distribution parameters from the sample data.

(f) Compute

$$\chi^2 = \sum_{i=1}^{k} \frac{\left(O_i - E_i\right)^2}{E_i}$$

where

O_i = number of sample observations in the i^{th} interval.

E_i = expected number of observations in the i^{th} interval.

k = number of intervals

(g) If $\chi^2 = \sum_i^k \frac{\left(O_i - E_i\right)^2}{E_i} > \chi^2_{\alpha,k-w-1}$

Where w is the number of parameters estimated and $\chi^2_{\alpha,k-w-1}$ may be found in table H-3 of appendix H, reject the distribution under test. Otherwise, we do not have sufficient evidence to reject the assumed underlying distribution.

(2) To illustrate the use of the χ^2 goodness of fit test, consider the data in figure F-8 indicating the failure times obtained from testing a sample of 100 fuel systems. Using a significance level of .05, test whether the assumption of an exponential distribution is reasonable. The sample mean was found to be 8.9 hours.

Interval (Hours	Frequency
0 - 5 05	48
5.05 - 10.05	22
10.05 - 15.05	11
15.05 - 20.05	7
20.05 - 25.05	3
25.05 - 30.05	5
30.05 - 35.05	2
35.05 - 40.05	0
40.05 - 45.05	1
45.05 - 50.05	0
50.05 - 55.05	1
	100

Figure F-8
Fuel System Failure Times

(a) Figure F-9 is used as a means of computing

$$\sum_{i=1}^{k} \frac{\left(O_i - E_i\right)^2}{E_i}$$

Interval (hrs) $(L_i - U_i)$	Observed Frequency (O_i)	Expected Frequency (E_i)	$O_i - E_i$	$(O_i - E_i)^2$	$\frac{(O_i - E_i)^2}{E_i}$
0 - 5.05	48	43	5	25	.58
5.05 - 10.05	22	24	-2	4	.38
10.05 - 15.05	11	14	-3	9	.64
15.05 - 20.05	7	8	-1	1	.12
20.05 - 25.05	3	5	-2	4	.80
25.05 - 30.05	5	3	2	4	1.33
30.05 - 35.05	2				
35.05 - 40.05	0				
40.05 - 45.05	1 } 4	3	1	1	.33
45.05 - 50.05	0				
50.05 - ∞	1				4.18

Figure F-9
Computation

(b) The expected frequency (E_i) is found by multiplying the sample size by the probability of falling within the i^{th} interval if the assumed (exponential) distribution is true.

$$E_i = n\left[\exp\left(\frac{-L_i}{\hat{\theta}}\right) - \exp\left(\frac{-U_i}{\hat{\theta}}\right)\right] = 100\left[\exp\left(\frac{-L_i}{8.9}\right) - \exp\left(\frac{-U_i}{8.9}\right)\right]$$

where U_i and L_i are the upper and lower limits of the i^{th} interval, $U_i = L_{i+1}$, and $\hat{\theta} = 8.9$ hours.

(c) Some of the original intervals were combined to satisfy the requirement that no E_i value be less than 2.5.

$$\chi^2 = \sum_{i=1}^{7} \frac{(O_i - E_i)^2}{E_i} = 4.18$$

$$\chi^2_{\alpha, k-w-1} = \chi^2_{.05, 7-1-1} = \chi^2_{.05, 5} = 11.070 \text{ (table H-3, appendix H).}$$

Since $\sum_{i-1}^{7} \frac{(O_i - E_i)^2}{E_i} = 4.18 < \chi^2_{.05, 5} = 11.070$, we do not have sufficient evidence to reject the exponential distribution as a model for these failure times.

f. Kolmogorov-Smirnov goodness of fit test.

(1) The preceding χ^2 goodness of fit test is limited to a reasonably large sample size. The Kolmogorov-Smirnov test is useful for a small sample, as well as for large samples, but is limited to tests concerning continuous distributions. This test requires that the parameters, as well as the underlying distribution type, be included as a part of the assumption or hypothesis being tested. The Kolmogorov-Smirnov goodness of fit test procedure is as follows:

(a) Determine the underlying distribution and associated parameter values to be tested.

(b) Determine the level of significance.

(c) Using the assumed distribution and parameters, compute $F(x_i) = P(X \leq x_i)$ where x_i is the i^{th} sample observation.

(d) From the sample data, compute $\hat{F}(x_i)$ = the portion of the sample observations which are less than or equal to x_i.

(e) Determine the maximum value of

$$\left| F(x_i) - \hat{F}(x_i) \right| = d.$$

(f) If $d > d_\alpha$, reject the assumed distribution and associated parameters. Rejection would be the result of evidence that either the underlying assumed distribution or the assumed parameters or both are not realistic. Values of d_α may be found in table H-6 appendix H.

(2) To exemplify the Kolmogorov-Smirnov test, consider five sample components which have been life tested with the following failure times: 1, 5, 6, 8 and 10 hours. Test the assumption that the underlying distribution of failure times is normal. Use an α risk of 0.20.

(a) Since no assumed parameters are given, we shall use the sample mean $(\bar{x})$ and sample standard deviation(s).

x_i	x_i^2
1	1
5	25
6	36
8	64
10	100
30	226

$$\bar{x} = \frac{30}{5} = 6$$

$$s^2 = \frac{\Sigma (x - \bar{x})^2}{n - 1} = \frac{n \Sigma x^2 - (\Sigma x)^2}{n(n-1)} = \frac{5(226) - (30)^2}{5(4)} = 11.5$$

$$s = \sqrt{11.5} = 3.4$$

(b) The calculation of d is shown in figure F-10.

x_i	$\frac{x_i - \bar{x}}{s}$	$F(x_i)$	$\hat{F}(x_i)$	$\lvert F(x_i) - \hat{F}(x_i) \rvert$
1	-1.47	.07	1/5 = .20	.13 max
5	-0.29	.39	2/5 = .40	.01
6	0	.50	3/5 = .60	.10
8	.59	.72	4/5 = .80	.08
10	1.18	.88	5/5 = 1.00	.12

Figure F-10
Calculation of d Statitistic

$$F(x_i) = P(X \leq x_i) = P\left(Z \leq \frac{x_i - \mu}{\sigma} \right) \quad \text{which becomes}$$

$$F(x_i) = 1 - P\left(Z > \frac{x_i - \bar{x}}{s} \right)$$ when the sample values are used as parameters.

(c) Then d = maximum absolute difference = .13.

$$d_\alpha = d_{.20} = 0.446 \text{ (table H-6)}$$

Since $d = .13 < d_{.20} = .446$, we have no reason to reject the assumption of normality with $\mu = 6$ and $\sigma = 3.4$

F-5. Exponential distribution of failure times.

a. General.

(1) The exponential distribution is often assumed as the failure time model. In the case of complex equipment, this assumption is generally reasonable.

The exponential probability density function has been defined as

$$f(x) = \lambda \exp(-\lambda x) = \frac{1}{\theta} \exp\left(-\frac{x}{\theta}\right) \text{if } x \geq 0$$

$$f(x) = 0 \text{ if } x < 0$$

where λ = failure rate

and $\theta = \frac{1}{\lambda}$ = mean failure time.

(2) Then the parameter to be estimated from sample life test data is either θ or λ. A point estimate of the exponential parameter leads to a point estimate of the reliability function. Each can also be estimated by means of confidence statements. The symbols $\hat{\theta}$, $\hat{\lambda}$ and $\hat{R}(x)$ are used herein as point estimates of the parameters θ, λ and $R(x)$ respectively. The value for $\hat{\lambda}$ will not be shown but may be determined as $\hat{\lambda} = \frac{1}{\hat{\theta}}$.

(3) Sample data may be generated from different types of life tests. Life tests may be conducted either with or without replacement of failed components and may be terminated either upon the occurrence of a preassigned number of failures or at a preassigned time.

b. Life tests terminated upon the occurrence of a preassigned number of failures.

(1) Assuming an exponential distribution of failure times, consider a life test which is to be terminated upon the occurrence of the r^{th} failure -- i.e., n items are placed on test until the r^{th} failure occurs. The failure times are expressed as $x_1 \leq x_2 \leq \ldots \leq x_r$. The method for estimating θ or λ depends upon whether the test procedure involves replacement or nonreplacement of field items.

(2) Point estimates of θ and $R(x)$ are

$$\hat{\theta} = \frac{\sum_{i=1}^{r} x_i + x_r(n-r)}{r}$$ for the nonreplacement test, failure terminated

$$\hat{\theta} = \frac{n\, x_r}{r} \quad \text{for the replacement test, failure terminated}$$

$$\hat{R}(x) = \exp\left(-\frac{x}{\hat{\theta}}\right) \quad \text{for either case of failure terminated test}$$

(a) Ten engines are initially to be placed on test without replacement, until the third failure occurs. At the conclusion of the test, the resulting failure times were found to be 6, 8, 20 hours. Assuming an exponential distribution of failure times, find a point estimate of the mean failure time θ and of R(5).

$$r = 3$$

$$n = 10$$

$$x_r = 20$$

$$\sum_{i=1}^{r} x_i = 34$$

$$\hat{\theta} = \frac{34 + (7)20}{3} = \frac{174}{3} = 58 \text{ hours}$$

$$\hat{R}(5) = \exp\left(\frac{-5}{58}\right) = \exp(-0.09) = 0.91$$

(b) Twenty vehicles were originally placed on test until the fifth failure. Failed vehicles were replaced at the time of test Conduct of the test resulted in the fifth vehicle failure occurring 40 hours after the beginning of the test. Assuming an exponential distribution of failure times, estimate the population mean failure time θ and R(100).

$$n = 20$$

$$x_r = 40$$

$$r = 5$$

$$\hat{\theta} = \frac{20(40)}{5} = \frac{800}{5} = 160 \text{ hours}$$

$$\hat{R}(100) = \exp\left(-\frac{100}{160}\right) = \exp(-0.625) = 0.53$$

(3) It is sometimes desirable to find a lower 100(1 - α)% confidence limit on reliability parameters. For the failure terminated test, such limits may be found for θ and R(x) as follows:

$$\theta \geq \frac{2r\hat{\theta}}{\chi^2_{\alpha,2r}}$$

$$R(x) \geq \exp\left(\frac{-x\,\chi^2_{\alpha,2r}}{2r\hat{\theta}}\right)$$

where $\chi^2_{\alpha,\,2r}$ may be found in table H-3, appendix H. In other words, we are 100(1-α)% confident that θ and consequently R(x) are greater than or equal to the respective computed lower limits.

(a) For example, consider a non-replacement life test which is to be terminated at the time of the 3rd failure. Ten widgets are initially placed on test. The resulting failures were 6, 8 and 20 hours. Assuming an exponential distribution of failure times, find a lower 90% confidence limit for the population reliability for a 1-hour mission time.

$n = 10$

$r = 3$

$\alpha = 0.10$

$\chi^2_{\alpha,2r} = \chi^2_{.10,6} = 10.645$, table H-3.

Mission time = 1 hour

$$\hat{\theta} = \frac{6 + 8 + 20 + 7(20)}{3} = \frac{174}{3} = 58 \text{ hours}$$

$$R(1) \geq \exp\left[\frac{-(1)(10.645)}{2(3)(58)}\right]$$

$$R(1) \geq \exp(-.03) = 0.97$$

Based upon the test data, we are 90% confident that the widget population reliability is no less than 0.97 for a mission time of 1 hour.

(b) To exemplify the replacement test, consider a replacement test which is to be terminated at the time of the fifth failure. Twenty fuel systems were initially placed on test. The fifth failure occured at 40 hours from the beginning of the test. Assuming an exponential distribution of failure times, find a 90% lower confidence limit for the population reliability for a 10-hour mission time.

$$n = 20$$

$$r = 5$$

$$x_r = 40$$

$$\alpha = 0.10$$

$$\chi^2_{\alpha,2r} = \chi^2_{.10,10} = 15.987, \text{ table H-3.}$$

Mission time = 10 hours

$$\hat{\theta} = \frac{20(40)}{5} = 160$$

$$R(10) \geq \exp\left[-\frac{(10)(15.987)}{2(5)(160)}\right] = \exp(-0.10) = 0.90$$

We are 90% confident that at least 0.90 of the population of fuel systems would survive a mission of 10 hours.

c. Life tests terminated at preassigned time T.

(1) Another common life test concerns placing of n items on test, continuing the test until preassigned termination time T and recording the number of failures, r, which have occurred. Such tests may be conducted using either the replacement or nonreplacement procedure.

(2) Point estimates for θ and R(x) are

$$\hat{\theta} = \frac{T}{\ln n - \ln (n-r)}$$ for the nonreplacement test, time terminated

$$\hat{\theta} = \frac{nT}{r}$$ for the replacement test, time terminated

$$\hat{R}(x) = \exp\left(-\frac{x}{\hat{\theta}}\right)$$ for either type of time terminated test

(3) Lower $100(1-\alpha)\%$ confidence limit for θ and R(x) are

$$\theta \geq \frac{2x_t}{\chi^2_{\alpha, 2r+2}}$$

$$R(x) \geq \exp\left(\frac{-x\,\chi^2_{\alpha, 2r+2}}{2x_t}\right)$$

where $x_t = \sum_{i=1}^{r} x_i + (n-r)T$ for nonreplacement tests

$x_t = nT$ for replacement tests

It will be noted that a lower confidence limit can be found even if no failures have occurred. This is possible since the degrees of freedom are $2r + 2$.

(4) For example, consider a nonreplacement life test which is to be terminated after 10 hours. Twenty vacuum tubes are initially placed on test. The observed failure times are 3, 5 and 9 hours. Find point estimates of θ and R(1/2) in addition to a 95% lower confidence limit for reliability for a 1/2 hour mission time.

$$r = 3$$

$$x_t = 3 + 5 + 9 + 17(10) = 187 \text{ hours}$$

$$\alpha = 0.05$$

$$\chi^2_{\alpha,\, 2r+2} = \chi^2_{0.05,8} = 15.507$$

$$\hat{\theta} = \frac{T}{\ln n - \ln(n-r)} = \frac{10}{\ln 20 - \ln 17} = \frac{10}{2.99573 - 2.83321} = 61.5 \text{ hrs.}$$

$$\hat{R}(1/2) = \exp \frac{-1/2}{61.5} = \exp(-0.008) = 0.992$$

$$R(1/2) \geq \exp\left[\frac{-(1/2)(15.507)}{2(187)}\right]$$

$$R(1/2) \geq \exp(-0.02) = 0.98$$

F-6. Weibull distribution of failure times.

a. General.

(1) When the Weibull density function is used as a failure model, there are two parameters, β and η, plus the reliability function to be estimated. The density function has been defined as

$$f(x) = \frac{\beta}{\eta}\left(\frac{x}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{x}{\eta}\right)^{\beta}\right] \text{ for } x \geq 0$$

$$f(x) = 0 \text{ for } x < 0$$

(2) The methods contained herein for estimating the Weibull parameters are concerned only with nonreplacement tests. Methods for determining confidence limits for the Weibull parameters are not included.

(3) Two methods for obtaining point estimates of β, η and $R(x)$ are included; namely, the method of matching moments and the maximum likelihood method.

b. Method of matching moments.

(1) If we conduct a life test in which n randomly selected items are placed on test until all fail, the method of matching moments can be used to find point estimates of the Weibull parameters. In this method we set sample mean and sample variance equal to the mean and variance of the density function and solve for the desired parameters. The actual operations are:

let $b = 1/\hat{\beta}$

then

$$\bar{x} = \hat{\eta}\Gamma(b+1)$$

and

$$s^2 = \eta^2 \{\Gamma(2b+1) - [\Gamma(b+1)]^2\}$$

Solving these yields

$$\hat{\eta} = \bar{x}[\Gamma(b+1)]^{-1}$$

and

$$\frac{2\Gamma(2b)}{b[\Gamma(b)]^2} - 1 = \frac{s^2}{\bar{x}^2}$$

These equations cannot be solved for b (or $\hat{\beta}$). Figure F-11 is used to find values of b directly from $s^2/\bar{x}^2$. The relationship $\hat{\beta} = b^{-1}$ is then used to find $\hat{\beta}$. The first equation is then used to find $\hat{\eta}$.

(2) As an example, a sample of 3 items was placed on test until all failed. The failure times were 1, 3 and 8 hours. Assuming the Weibull distribution, estimate the Weibull parameters by the method of matching moments.

	x	x^2
	1	1
	3	9
	8	64
Σx =	12	74 = Σx^2

$$\bar{x} = \frac{\Sigma x}{n} = \frac{12}{3} = 4$$

$$s^2 = \frac{n\Sigma x^2 - (\Sigma x)^2}{n(n-1)} = \frac{3(74) - (12)^2}{3(2)}$$

$$= \frac{222 - 144}{6} = 13$$

$$s^2/\bar{x}^2 = 13/4^2 = 0.8125$$

Solving for b, using figure F-11, we find b = 0.9. Then

$$\hat{\beta} = (0.9)^{-1} = 1.11$$

and

$$\hat{\eta} = \frac{4}{\Gamma(1.9)} = 4.16$$

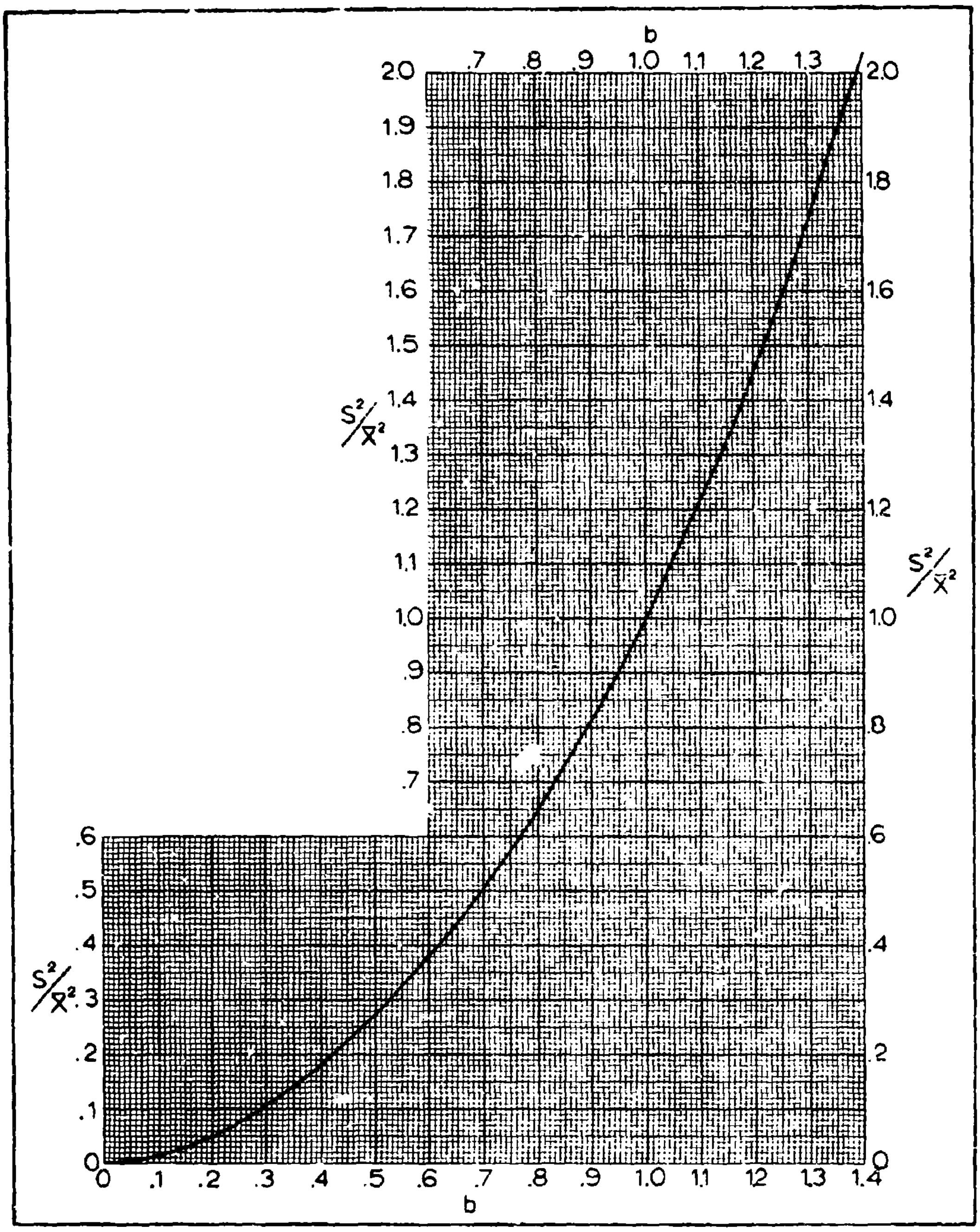


Figure F-11. Solution Curve for Weibull Matching Moments Method

c. Maximum likelihood method. Maximum likelihood estimates of the Weibull parameters may be found by solving iteratively the following equations for $\hat{\beta}$ and $\hat{\eta}$.

$$\hat{\eta} = \exp\left[\frac{1}{\hat{\beta}}\ln\left(\frac{\sum_{i=1}^{n} x_i^{\hat{\beta}}}{n}\right)\right]$$

$$\hat{\beta} = \frac{n}{\sum_{i=1}^{n}\left(\frac{x_i}{\hat{\eta}}\right)^{\hat{\beta}}\ln x_i - \sum_{i=1}^{n}\ln x_i}$$

F-7. Normal distribution of failure times.

a. General.

(1) The normal probability density function is sometimes useful as a failure model in reliability analysis, especially when failures are a result of wearout. The normal density function is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right], \quad -\infty < x < \infty$$

Estimation of the parameters μ and σ^2 will be considered only for a life test where n items are placed on test until all fail with resulting failure times $x_1, x_2, \ldots, x_n$.

b. Then point estimates of μ, σ^2 and R(x) are respectively

$$\bar{x} = \frac{\Sigma x}{n}$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{n\Sigma x^2 - (\Sigma x)^2}{n(n-1)}$$

$$\hat{R}(x) = P\left(Z > \frac{x-\bar{x}}{s}\right)$$

c. The 100(1-α)% confidence intervals for the two parameters are

$$\bar{x} - t_{\alpha/2,\,n-1}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + t_{\alpha/2,\,n-1}\left(\frac{s}{\sqrt{n}}\right)$$

and

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

where

n = sample size

n-1 = degrees of freedom

$t_{\alpha/2,\,n-1}$ may be found in table H-4, appendix H.

$\chi^2_{\alpha/2,\,n-1}$ may be found in table H-3, appendix H.

d. To illustrate the estimation of μ and σ, consider 4 randomly selected vehicles which have been tested till failure. The failure times were 110, 114, 116 and 120 hours. Assuming failure times to be normally distributed, estimate both μ and σ by a point estimate and by a 95% confidence interval. Also find a point estimate for the reliability for a mission time of 120 hours.

(1)

x	$x-\bar{x}$	$(x-\bar{x})^2$
110	-5	25
114	-1	1
116	1	1
120	5	25
460		52

$$\bar{x} = \frac{\Sigma x}{n} = \frac{460}{4} = 115$$

$$s^2 = \frac{\Sigma(x-\bar{x})^2}{n-1} = \frac{52}{3} = 17.33$$

Then an estimate of μ is $\bar{x} = 115$ and an estimate of σ is

$s = \sqrt{17.33} = 4.15$

(2) For 95% confidence, $\alpha = .05$.

$$t_{\alpha/2,n-1} = t_{.025,3} = 3.182$$

$$\chi^2_{\alpha/2,n-1} = \chi^2_{.025,3} = 9.348$$

$$\chi^2_{1-\alpha/2,n-1} = \chi^2_{.975,3} = .216$$

Then

$$\bar{x} - t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right) < \mu < \bar{x} + t_{\alpha/2,n-1}\left(\frac{s}{\sqrt{n}}\right)$$

$$115 - 3.182\left(\frac{4.15}{\sqrt{4}}\right) < \mu < 115 + 3.182\left(\frac{4.15}{\sqrt{4}}\right)$$

$$108.40 < \mu < 121.60$$

Therefore, we are 95% confident that the population mean life is between 108.40 and 121.60 hours. This statement is dependent upon the failure times being normally distributed.

$$\frac{(n-1)s^2}{\chi^2_{\alpha/2,n-1}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2,n-1}}$$

$$\frac{3(17.33)}{9.348} < \sigma^2 < \frac{3(17.33)}{.216}$$

$$5.56 < \sigma^2 < 240.74$$

$$2.36 < \sigma < 15.51$$

Dependent upon the failure times being normally distributed, we are 95% confident that σ is between 2.36 and 15.51 hours.

(3) The population reliability for a 120 hour mission time can be estimated as

$$\hat{R}(120) = P\left(Z > \frac{120-115}{4.15}\right) = P(Z > 1.20) = 0.12$$

as found from table H-2 of appendix H.

Section III. TESTS OF HYPOTHESES

F-8. General. a. The general procedures for conducting a test of hypothesis are:

(1) Set up the plan format including the analysis and decision rules.

(a) Determine a hypothesis (H_o) to be tested and an alternative hypothesis (H_1) which will be assumed true if H_o is rejected. Both hypotheses will pertain to a particular parameter of interest. For illustrative purposes, suppose this parameter is designated as θ. Then a typical set of hypotheses would be

$$H_o: \theta = \theta_o$$

$$H_1: \theta < \theta_o$$

where θ_0 is defined as a specified acceptable (nominal) value of θ. $H_1: \theta < \theta_0$ indicates that H_0 should be rejected if the real θ value becomes significantly less than θ_0

(b) Determine a level of significance, α, which is defined as the probability of rejecting H_0 if, in fact, H_0 is true. This value provides a measure of the sampling risk of wrongly rejecting H_0.

(c) Determine the test statistic, to be found from the sample data, which will be used as the basis of the decision to accept or reject H_0.

(d) Determine the acceptance region; that is, those values of the test statistic which will result in acceptance of H_0.

(2) Select the n sample items randomly and conduct the life test.

(3) Using the sample life test results, determine the test statistic value.

(4) Compare the test statistic value to the acceptance region to determine acceptance or rejection of H_0.

b. Ideally, acceptance or rejection of H_0 would be directly dependent upon the actual value of the parameter θ, but this value would not be known short of 100% testing. Consequently, decisions must be based on sample data and as such are subject to two types of sampling error. The first type is that of rejecting H_0 if H_0 is true and the second type is that of accepting H_0 if H_1 is true. [see F-9b(3)]

c. The sensitivity or protection offered by a test of hypothesis may be evaluated by investigating the probability of accepting H_0 (P_a). P_a, expressed as a function of the parameter under test, is defined as the operating characteristic (OC) curve. Even though the real parameter value is unknown, P_a may be investigated over a range of possible values of the parameter. Each different test plan has its own OC curve from which the sampling risks may be evaluated.

d. The parameter under test may pertain to either a population of existing material or a process for generating a population of material. The latter would be useful for evaluating the state of development of a product.

e. The discussions which follow will be confined to a few typical tests of hypotheses concerning parameters associated with exponential density functions.

F-7. Exponential failure times. a. Reliability is related to only one parameter (θ) when failure times are exponentially distributed. Discussion will be confined to tests of hypothesis such that H_o will be rejected if θ is too small. Two basic types of life tests will be considered -- the failure terminated test and the time terminated test. Both replacement and nonreplacement procedures are included.

b. Life tests terminated upon the occurrence of the r^{th} failure.

(1) The test of hypothesis format concerning failure terminated tests is as follows:

$H_o: \theta = \theta_o$

$H_1: \theta < \theta_o$

Level of significance: α

Test statistic: $\hat{\theta}$

Acceptance region : $\hat{\theta} > \dfrac{\theta_o \chi^2_{1-\alpha,\, 2r}}{2r}$

where

$$\hat{\theta} = \frac{nx_r}{r} \quad \text{for replacement tests}$$

$$\hat{\theta} = \frac{\sum_{i=1}^{r} x_i + (n-r)x_r}{r} \quad \text{for nonreplacement tests}$$

θ_o = acceptable mean life

r = termination number

n = sample size

$n \geq r$ for nonreplacement tests

The OC curve is expressed by

$$P_a = P\left[\chi^2_{2r} \geq \left(\frac{\theta_o}{\theta} \right) \chi^2_{1-\alpha, 2r} \right]$$

(2) To exemplify this test of hypothesis, a new vehicle is under development and five prototype models are available for testing to evaluate the process by which these models were generated. If the process is such that it has the capability of generating items which have a mean time to failure of θ = 100 hours or more, no further development effort is required. A nonreplacement test is proposed such that the five models will be placed on test until the third failure occurs. The test of hypothesis can tolerate an α = .05 risk of rejecting the present process if it is actually capable of generating product with θ = 100 hours. Set up the test format; draw the OC curve for the plan, and answer the question, if failure times are 20, 40 and 50, should the item be subjected to further development ?

(a) Define n, r, θ_o and α as:

$n = 5$, $r = 3$, $\theta_o = 100$, $\alpha = .05$

(b) H_o: θ = 100 hours

H_1: $\theta <$ 100 hours

Level of significance: $\alpha = .05$

Test statistic: $$\hat{\theta} = \frac{\sum_{i=1}^{r} x_i + (n-r)x_r}{r} = \frac{\sum_{i=1}^{3} x_i + 2x_3}{3}$$

$$\text{Acceptance region: } \hat{\theta} > \frac{\theta_0 \chi^2_{1-\alpha, 2r}}{2r} = \frac{100\, \chi^2_{.95, 6}}{6}$$

$$= \frac{100(1.64)}{6} = 27.3 \text{ hours}$$

(c) In order to evaluate the plan, the OC curve may be drawn from

$$P_a = P\left[\chi^2_{2r} \geq \left(\frac{\theta_0}{\theta}\right) \chi^2_{1-\alpha, 2r}\right] = P\left[\chi^2_6 \geq \frac{100(1.64)}{\theta}\right]$$

Figure F-12 shows P_a for various values of θ. These values may be used to sketch the OC curve.

θ	P_a
200	0.991
100	0.95
50	0.77
25	0.37
10	0.012

Figure F-12
OC Curve Computations

1 **Computations for the tabular entries follow.**

If θ = 200 hours, $P_a = P\left(\chi^2_6 \geq \frac{164}{200}\right) = P\left(\chi^2_6 \geq 0.82\right) = 0.991$ as interpolated from Table H-3.

If θ = 100 hours, $P_a = P\left(\chi^2_6 \geq 1.64\right) = 0.95$ as found in Table I-3.

If θ = 50 hours, $P_a = P\left(\chi^2_6 \geq \frac{164}{50}\right) = P\left(\chi^2_6 \geq 3.28\right) = 0.77$ as interpolated from Table H-3.

If θ = 25 hours, $P_a = P\left(\chi^2_6 \geq \frac{164}{25}\right) = P\left(\chi^2_6 \geq 6.56\right) = 0.37$

If θ = 10 hours, $P_a = P\left(\chi^2_6 \geq \frac{164}{10}\right) = P\left(\chi^2 \geq 16.4\right) = 0.012$

2 Figure F-13 represents a sketch of the OC curve.

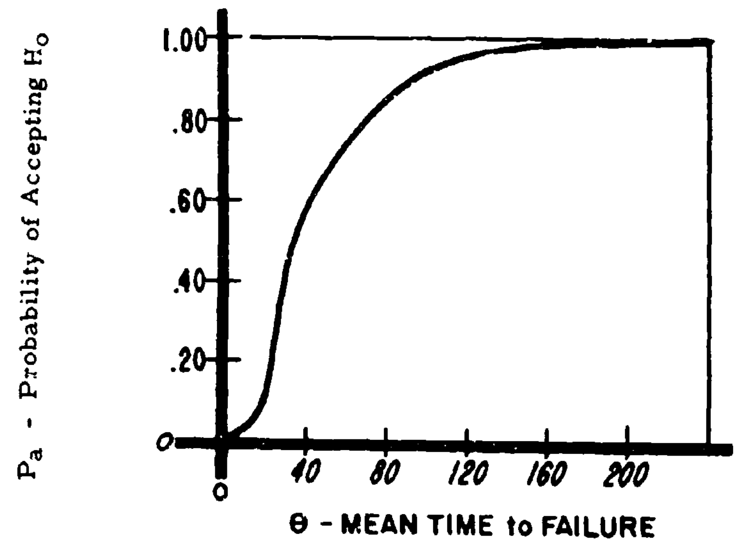


Figure F-13
OC Curve

investigation of this curve over the range of potential values of θ provides an evaluation of the proposed plan in terms of the protection offered. This evaluation may be performed prior to the decision to use the plan.

(d) $x_1 = 20$, $x_2 = 40$, $x_3 = 50$

$$\hat{\theta} = \frac{110 + 2(50)}{3} = \frac{210}{3} = 70$$

Since $\theta \geq 27.3$, accept the hypothesis that $\theta = 100$ hours and conclude that on the basis of these test results no further development is required.

(3) Inspection of the P_a function indicates that the number of failures (r), and not the sample size, determines the degree of protection offered. The preceding example specified that the test will be stopped upon occurrence of the third failure and that the test includes a risk of $\alpha = .05$ of rejecting H_0 if H_0 is actually true. If more rigid protection requirements were specified, the test termination number must be increased, i.e., testing must proceed until more failures have occurred.

(4) A test may be determined which will meet the following requirements for protection (i. e., required r value is determined).

(a) If the true mean failure time (θ) is equal to a specified nominal value (θ_0), the test should allow an α probability of rejecting the lot (process).

(b) If the true mean failure time (θ) is equal to a specified minimum value (θ_1), the test should allow a β probability of accepting the lot (process), where $\theta_0 > \theta_1$.

In other words, the test must be such that its OC curve will pass through the points (θ_0 , $1-\alpha$) and (θ_1 , β). The required value of r may be found from the following relationship.

$$\frac{\chi^2_{\beta, 2r}}{\chi^2_{1-\alpha, 2r}} = \frac{\theta_0}{\theta_1}$$

The relationship applies to either replacement or nonreplacement tests.

(5) To illustrate the above requirement, consider a lot of components which is being considered for use in a weapons system. Assume the failure times of these components to be exponentially distributed. Formulate a nonreplacement failure terminated test which would accept the lot with probability 0.95 if its true mean life is 500 hours and which would accept the lot with probability 0.10 if its true mean life is 200 hours.

(a) Define:

$\theta_o = 500, \quad \theta_1 = 200, \quad \alpha = 0.05, \quad \beta = 0.10$

(b) Test format:

$H_o : \theta = 500$

$H_1 : \theta < 500$

Level of significance : $\alpha = .05$

Test statistic : $\hat{\theta} = \dfrac{\sum_{i=1}^{r} x_i + (n-r)x_r}{r}$

Acceptance region : $\hat{\theta} \geq \dfrac{\theta \, \chi^2_{1-\alpha, 2r}}{2r} = \dfrac{500 \, \chi^2_{.95, 2r}}{2r}$

(c) The required value of r must be such that the following relationship is satisfied.

$$\frac{\chi^2_{.10, 2r}}{\chi^2_{.95, 2r}} = \frac{\chi^2_{\beta, 2r}}{\chi^2_{1-\alpha, 2r}} = \frac{\theta_o}{\theta_1} = \frac{500}{200} = 2.5$$

Solution of this equation may be obtained by trying different values of r until the ratio of χ^2 values is equal to 2.5. Figure F-14 shows an iterative method for determining r.

Trial Value of r	$\chi^2_{.10,2r}$	$\chi^2_{.95,2r}$	$\frac{\chi^2_{.10,2r}}{\chi^2_{.95,2r}}$
1	4.61	.103	44.8
5	15.99	3.94	4.1
7	21.06	6.57	3.2
10	28.41	10.85	2.6
11	30.81	12.34	2.49
12	33.20	13.85	2.39
13	35.56	15.38	2.31

Figure F-14

Determination of Termination Failure Number

(d) Thus, in order to meet the stated requirements, the test should be terminated upon occurrence of the 11th failure and consequently, the sample size can be no less than 11. Acceptance of H_0 for this test yields 90% confidence that $\theta \geq 200$ hours.

c. Life tests terminated at a preassigned time T.

(1) The test of hypothesis format concerning time terminated tests is as follows:

$H_0 \; : \; \theta = \theta_0$

$H_1 \; : \; \theta < \theta_0$

Level of significance : α

Test statistic: r = number of failures occurring during test

Acceptance region : $r < r_o$, where r_o is a rejection number which must satisfy the relationship

$$\sum_{k=0}^{r_o-1} \frac{\left(\frac{nT}{\theta_o}\right)^k \exp\left(\frac{-nT}{\theta_o}\right)}{k!} = 1-\alpha \quad \text{for a replacement test}$$

or

$$\sum_{k=0}^{r_o-1} \binom{n}{k} \left[1-\exp\left(\frac{-T}{\theta_o}\right)\right]^k \left[\exp\left(\frac{-T}{\theta_o}\right)\right]^{n-k} = 1-\alpha$$

for a nonreplacement test.

(2) The OC curve for the time terminated test depends upon whether the test is replacement or nonreplacement.

(a) The Poisson distribution is used to express probability of acceptance for replacement tests.

$$P_a = P(r < r_o) = \sum_{k=0}^{r_o-1} \frac{\left(\frac{nT}{\theta}\right)^k \exp\left(\frac{-nT}{\theta}\right)}{k!}$$

(b) The nonreplacement test uses the binomial distribution to express P_a.

$$P_a = P(r < r_o) = \sum_{k=0}^{r_o-1} \binom{n}{k} \left[1-\exp\left(\frac{-T}{\theta}\right)\right]^k \left[\exp\left(\frac{-T}{\theta}\right)\right]^{n-k}$$

(3) Four prototype developmental helicopter models are available for testing to determine if the present developmental process is capable of generating items with a mean time between failures (MTBF) of at least 100 hours. The decision to accept or reject the developmental process is needed within 10 hours and a 5% risk of rejecting, if the true MTBF is 100 hours, can be tolerated. Set up a nonreplacement test format; sketch the OC curve for this test; and answer the question: Should the development procedure be accepted if 3 failures are encountered during conduct of the test?

(a) Define:

$T = 10, \quad \theta_o = 100, \quad \alpha = .05, \; n = 4$

(b) Test format:

$H_o : \theta = 100$

$H_1 : \theta < 100$

$\alpha = .05$

Test statistic: r = number of failures found during test

Acceptance region : $r < r_o$ where r_o must satisfy the following relationship:

$$\sum_{k=0}^{r_o-1} \binom{4}{k} \left[1 - \exp\left(\frac{-10}{100}\right)\right]^k \left[\exp\left(\frac{-10}{100}\right)\right]^{4-k} = \sum_{k=0}^{r_o-1} \binom{4}{k} (.1)^k (.9)^{4-k} = .95$$

This relationship is approximately satisfied by $r_o = 2$. Exact satisfaction may be impossible because of the discrete characteristics of r_o.

(c) Some coordinates for sketching the OC curve are calculated below (binomial tables may be used in lieu of these calculations).

$$P_a = \sum_{k=0}^{1} \binom{4}{k} \left[1 - \exp\left(\frac{-10}{\theta}\right)\right]^k \left[\exp\left(\frac{-10}{\theta}\right)\right]^{4-k}$$

For θ = 100 hours, $\exp\left(\frac{-10}{100}\right) = \exp(-0.1) = 0.9$

$$P_a = (0.9)^4 + 4(0.1)(0.9)^3 = 0.66 + 0.29 = 0.95$$

For θ = 200 hours, $\exp\left(\frac{-10}{200}\right) = \exp(0.05) = 0.95$

$$P_a = (0.95)^4 + 4(0.05)(0.95)^3 = 0.81 + 0.17 = 0.98$$

For θ = 50 hours, $\exp\left(\frac{-10}{50}\right) = \exp(-0.2) = 0.82$

$$P_a = (0.82)^4 + 4(0.18)(0.82)^3 = 0.45 + 0.40 = 0.85$$

For θ = 25 hours, $\exp\left(\frac{-10}{25}\right) = \exp(-0.4) = 0.67$

$$P_a = (0.67)^4 + 4(0.33)(0.67)^3 = 0.20 + 0.40 = 0.60$$

For θ = 10 hours, $\exp\left(\frac{-10}{10}\right) = \exp(-1) = 0.37$

$$P_a = (0.37)^4 + 4(0.63)(0.37)^3 = 0.02 + 0.13 = 0.15$$

For $\theta = 5$ hours, $\exp\left(\frac{-10}{5}\right) = \exp(-2) = 0.14$

$$P_a = (0.14)^4 + 4(0.86)(0.14)^3 = 0.0004 + 0.009 \approx 0.0094$$

From these points, the OC curve is sketched as Figure F-15.

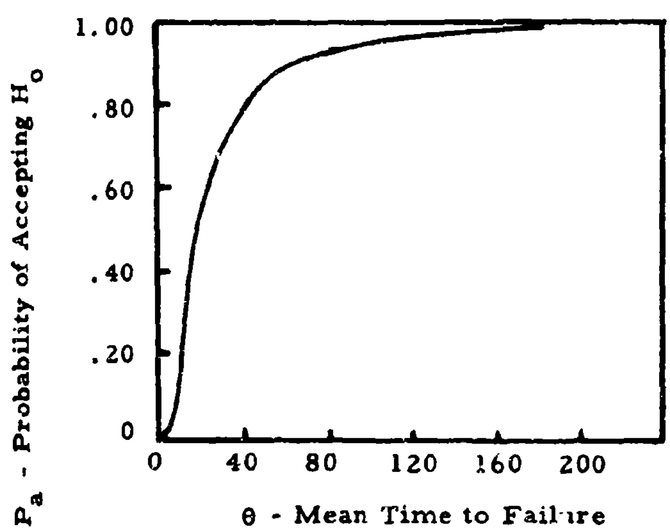


Figure F-15

OC Curve

(d) If 3 failures were encountered during the 10 hour test, H_0 would be rejected indicating that there is reason to believe the mean failure time to be less than 100 hours and development activities should continue. Since $r_0 = 2$ is a rejection number, the test could have been terminated when the 2nd failure occurred.

(4) Inspection of the P_a functions for a time terminated test shows that sample size contributes to the degree of protection offered for a specified termination time T. The required sample size and rejection number may be found if the following test requirements are specified.

(a) Test termination time (T).

(b) A lot (process) with $\theta = \theta_0$ would have a $(1-\alpha)$ probability of being accepted.

(c) A lot (process) with $\theta = \theta_1$ would have a β probability of being accepted.

The required sample size and rejection number must satisfy the following relationships. For a replacement test:

$$\sum_{k=0}^{r_o-1} \frac{\left(\frac{nT}{\theta_o}\right)^k \exp\left(\frac{-nT}{\theta_o}\right)}{k!} = 1-\alpha$$

and

$$\sum_{k=0}^{r_o-1} \frac{\left(\frac{nT}{\theta_1}\right)^k \exp\left(\frac{-nT}{\theta_1}\right)}{k!} = \beta$$

For a nonreplacement test

$$\sum_{k=0}^{r_o-1} \binom{n}{k} \left[1-\exp\left(\frac{-T}{\theta_o}\right)\right]^k \left[\exp\left(\frac{-T}{\theta_o}\right)\right]^{n-k} = 1-\alpha$$

and

$$\sum_{k=0}^{r_o-1} \binom{n}{k} \left[1-\exp\left(\frac{-T}{\theta_1}\right)\right]^k \left[\exp\left(\frac{-T}{\theta_1}\right)\right]^{n-k} = \beta$$

These determinations may be made by trying different values of r_o and n until the equations are satisfied. Figure H-12 in appendix H may be used for the replacement test determinations. Binomial tables would facilitate the nonreplacement test determinations.

d. Sequential tests.

(1) Often times a decision concerning acceptance or rejection of H_o can be reached more quickly by applying a sequential decision procedure. A sequential test involves a set of rules to be applied each time a failure occurs (theoretically applied continuously over time.) The decision will be one of these: Accept H_o and stop the test, reject H_o and stop the test, or continue testing. The test would continue until a decision to accept H_o or reject H_o is reached.

(2) The tests discussed herein are determined by applying Wald's probability ratio test to an exponential distribution of failure times. The test procedure will be designed to test the following set of hypotheses:

$$H_o : \theta = \theta_o$$

$$H_1 : \theta < \theta_o$$

(3) In order to design a test, the following requirements must be specified.

(a) Determine an acceptable mean life θ_o.

(b) Determine the magnitude of an α risk which may be tolerated, where α is the probability of rejecting H_o if the population mean life $\theta = \theta_o$.

(c) Determine an unacceptable or limiting mean life θ_1 where $\theta_1 < \theta_o$.

(d) Determine the magnitude of a β risk which may be tolerated, where β is the probability of accepting H_o if $\theta = \theta_1$.

(4) The protection offered by the test may be evaluated by examining its operating characteristic (OC) curve. Another factor which is of importance in selecting a plan is the expected duration of the test, i.e., how long must we wait for the decision?

(5) The sequential decision criteria are:

If $h_1 + k s < V(t) < h_o + k s$, continue the test.

If $V(t) \geq h_o + k s$, stop the test and accept H_o.

If $V(t) \leq h_1 + k s$, stop the test and reject H_o.

where

$$h_o = \frac{\ln\left(\frac{1-\alpha}{\beta}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_o}} \qquad h_1 = \frac{\ln\left(\frac{\alpha}{1-\beta}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_o}}$$

$$s = \frac{\ln\left(\frac{\theta_o}{\theta_1}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_o}}$$

$V(t) = nt$ for a replacement test

$V(t) = \sum_{i=1}^{k} x_i + (n-k)t$ for a non-replacement test

t is the test duration time

n is the original number placed on test

x_i is the time of the i^{th} failure measured from test beginning

(6) This sequential procedure may be applied at any point in time but must be applied at least upon the occurrence of each failure. Graphically, these decision rules appear as shown by figure F-16.

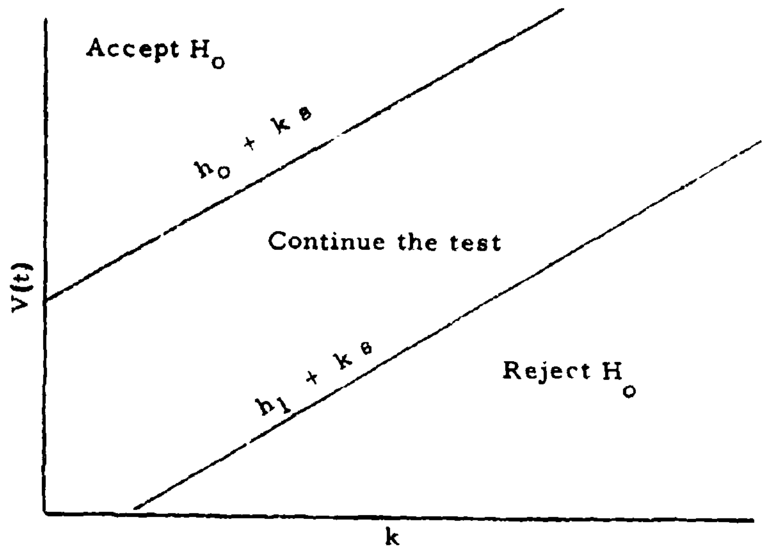


Figure F-16
Graphical Decision Rules of the Sequential Test

(7) The OC curve coordinates may be found by letting the parameter h run through all real values in the following parametric equations.

$$\theta = \frac{\left(\frac{\theta_o}{\theta_1}\right)^h - 1}{h\left(\frac{1}{\theta_1} - \frac{1}{\theta_o}\right)}$$

$$P_a = \frac{\left(\frac{1-\beta}{\alpha}\right)^h - 1}{\left(\frac{1-\beta}{\alpha}\right)^h - \left(\frac{\beta}{1-\alpha}\right)^h}$$

Figure F-17 shows five points which are easily found and enable us to sketch the OC curve.

θ	P_a
0	0
θ_1	β
s	$\dfrac{\ln\left(\frac{1-\beta}{\alpha}\right)}{\ln\left(\frac{1-\beta}{\alpha}\right) - \ln\left(\frac{\beta}{1-\alpha}\right)}$
θ	$1 - \alpha$
∞	1

Figure F-17
Coordinates for Sketching Sequential Test OC Curve

(8) The expected number of failures required to reach a decision, $E_\theta(r)$, is dependent upon the lot mean life θ and may be found as follows:

$$E_\theta(r) = \frac{-h_1 - (h_o - h_1) P_a}{s - \theta} \qquad \theta \neq s$$

$$E_\theta(r) = \frac{-h_o h_1}{s^2} \qquad \theta = s$$

where the random variable r is the number of failures required to reach a decision. The approximate values of $E_\theta(r)$ if

$\theta = 0, \theta_1, s, \theta_o$, or ∞ reduce to

$$E_o(r) = \frac{-h_1}{s}$$

$$E_{\theta_1}(r) \sim \frac{\beta \ln\left(\frac{\beta}{1-\alpha}\right) + (1-\beta)\ln\left(\frac{1-\beta}{\alpha}\right)}{\ln\left(\frac{\theta_o}{\theta_1}\right) - \left(1 - \frac{\theta_1}{\theta_o}\right)}$$

$$E_s(r) \sim \frac{-\ln\left(\frac{1-\beta}{\alpha}\right)\ln\left(\frac{\beta}{1-\alpha}\right)}{\ln^2\left(\frac{\theta_o}{\theta_1}\right)}$$

$$E_{\theta_o}(r) \sim \frac{(1-\alpha)\ln\left(\frac{\beta}{1-\alpha}\right) + \alpha\ln\left(\frac{1-\beta}{\alpha}\right)}{\ln\left(\frac{\theta_o}{\theta_1}\right) - \left(\frac{\theta_o}{\theta_1} - 1\right)}$$

$$E_\infty(r) = 0$$

(9) The expected waiting time to reach a decision, $E_\theta(t)$, is a function of θ and is found by :

$$E_\theta(t) = \frac{\theta}{n} E_\theta(r) \quad \text{for a replacement test}$$

and approximately

$$E_\theta(t) \sim \theta \ln\left[\frac{n}{n - E_\theta(r)}\right] \quad \text{for a nonreplacement test}$$

For either case, $\theta = \infty$ reduces to $E_\infty(t) = \frac{h_o}{n}$

(10) Assuming the exponential distribution, develop a sequential replacement test which would accept a submitted lot of material, with probability 0.95, if its mean life is 1500 hours and which would reject a submitted lot, with probability 0.90, if its mean life is 300 hours. Twenty items are to be placed on test. Calculate the decision rules; sketch the OC curve; sketch, as a function of θ, a curve showing the expected number of failures to be encountered before a decision of acceptance or rejection is reached; sketch a curve showing the expected test duration time as a function of θ; and if failures were encountered at the following times, should the lot be accepted? (Use only as many as needed to reach a decision.)

$$x_1 = 12, \; x_2 = 100, \; x_3 = 140, \; x_4 = 206$$

(a) Define and calculate:

$$\theta_o = 1500, \; \alpha = 0.05$$

$$\theta_1 = 300, \; \beta = 0.10$$

$$\ln\left(\frac{1-\alpha}{\beta}\right) = -\ln\left(\frac{\beta}{1-\alpha}\right) = \ln\left(\frac{0.95}{0.10}\right) = 2.252$$

$$\ln\left(\frac{\alpha}{1-\beta}\right) = -\ln\left(\frac{1-\beta}{\alpha}\right) = \ln\left(\frac{0.05}{0.90}\right) = -2.891$$

$$\frac{1}{\theta_1} - \frac{1}{\theta_o} = \frac{1}{300} - \frac{1}{1500} = \frac{4}{1500} = \frac{1}{375}$$

$$\ln\left(\frac{\theta_o}{\theta_1}\right) = \ln\left(\frac{1500}{300}\right) = 1.609$$

$$h_o = \frac{\ln\left(\frac{1-\alpha}{\beta}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_o}} = \frac{2.252}{1/375} = 844.5$$

$$h_1 = \frac{\ln\left(\frac{\alpha}{1-\beta}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_o}} = \frac{-2.891}{1/375} = -1084.1$$

$$s = \frac{\ln\left(\frac{\theta_o}{\theta_1}\right)}{\frac{1}{\theta_1} - \frac{1}{\theta_o}} = \frac{1.609}{1/375} = 603.4$$

If $-1084.1 + 603.4k < nt < 844.5 + 603.4k$, continue the test.

If $nt \geq 844.5 + 603.4k$, stop the test and accept the lot.

If $nt \leq -1084.1 + 603.4k$, stop the test and reject the lot.

(b) The OC curve may be sketched from points shown in figure F-18 and the sketch as shown in figure F-19.

θ	P_a
0	0
$\theta_1 = 300$	$\beta = 0.10$
$s = 603.4$	$\dfrac{\ln\left(\frac{1-\beta}{\alpha}\right)}{\ln\left(\frac{1-\beta}{\alpha}\right) - \ln\left(\frac{\beta}{1-\alpha}\right)} = \dfrac{2.891}{2.891 + 2.252} \doteq .56$
$\theta_o = 1500$	$1-\alpha = 0.95$
∞	1

Figure F-18
Coordinates for Sketching Sequential OC Curve

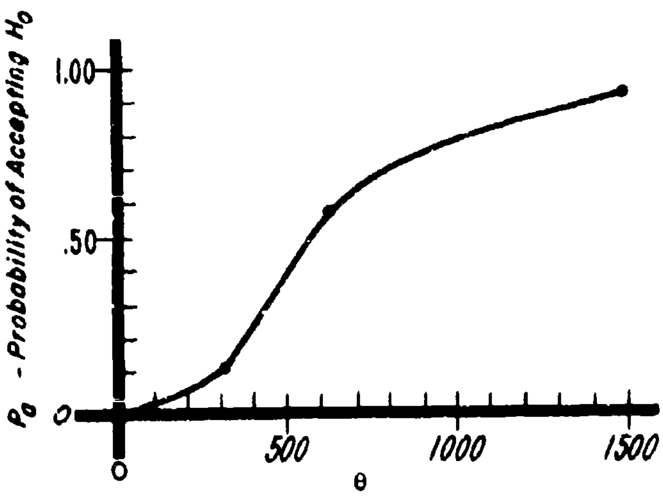


Figure F-19
Sketch of Sequential OC Curve

θ	$E_\theta(r)$
0	$\dfrac{-h_1}{s} = \dfrac{1084.1}{603.4} = 1.80$
$\theta_1 = 300$	$\dfrac{\beta \ln\left(\frac{\beta}{1-\alpha}\right) + (1-\beta)\ln\left(\frac{1-\beta}{\alpha}\right)}{\ln\left(\frac{\theta_o}{\theta_1}\right) - \left(1 - \frac{\theta_1}{\theta_o}\right)} = \dfrac{.10(-2.252) + .90(2.891)}{1.609 - (.8)} = 2.94$
$2s = 603.4$	$\dfrac{-\ln\left(\frac{1-\beta}{\alpha}\right)\ln\left(\frac{\beta}{1-\alpha}\right)}{\ln^2\left(\frac{\theta_o}{\theta_1}\right)} = \dfrac{-2.891(-2.252)}{(1.609)^2} = 2.51$
$\theta_o = 1500$	$\dfrac{(1-\alpha)\ln\left(\frac{\beta}{1-\alpha}\right) + \alpha \ln\left(\frac{1-\beta}{\alpha}\right)}{\ln\left(\frac{\theta_o}{\theta_1}\right) - \left(\frac{\theta_o}{\theta_1} - 1\right)} = \dfrac{(.95)(-2.252) + (.05)(2.891)}{1.609 - (4)} = .83$
∞	0

Figure F-20
Coordinates for Sketching Expected Number of Failure Curve

(c) The expected number of failures curve may be sketched from the points in figure F-20 and the resulting sketch is shown by figure F-21.

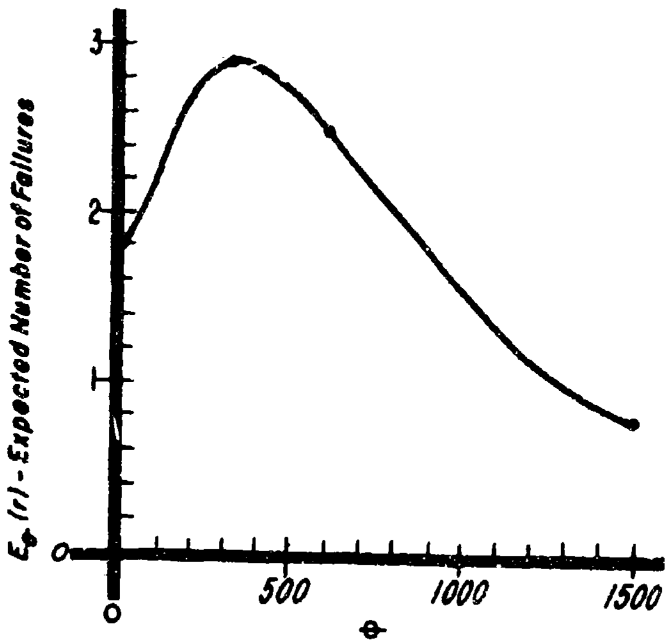


Figure F-21
Sketch of Expected Number of Failures Curve

(d) The expected test duration time curve may be sketched from the points in figure F-22 and the resulting sketch is shown in figure F-23.

θ	$E_\theta(t) = \frac{\theta}{n} E_\theta(r)$
0	0
$\theta_1 = 300$	$\frac{300}{20}(2.94) = 44.1$
$s = 603.4$	$\frac{603.4}{20}(2.51) = 75.7$
$\theta_o = 1500$	$\frac{1500}{20}(.83) = 62.3$
∞	$\frac{h_o}{n} = \frac{844.5}{20} = 42.2$

Figure F-22
Coordinates for Sketching the Expected Test Duration Time Curve

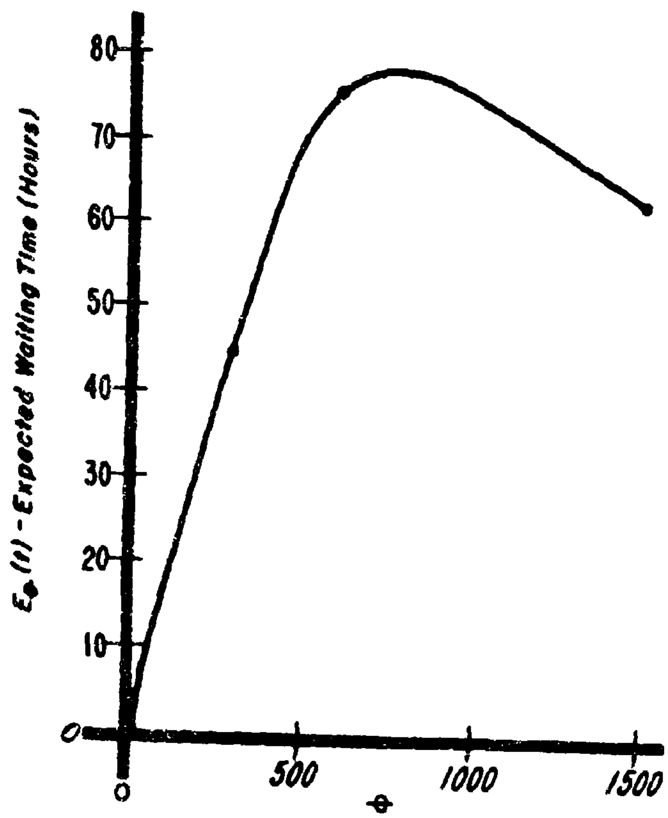


Figure F-23
Sketch of Expected Time Duration Curve

(e) Figure F-24 shows graphically the sequential decision criteria for this test. The cumulative nt line crosses into the acceptance region before the second failure occurs. Consequently, accept H_o and the test should be stopped at the time the nt value crosses the acceptance line.

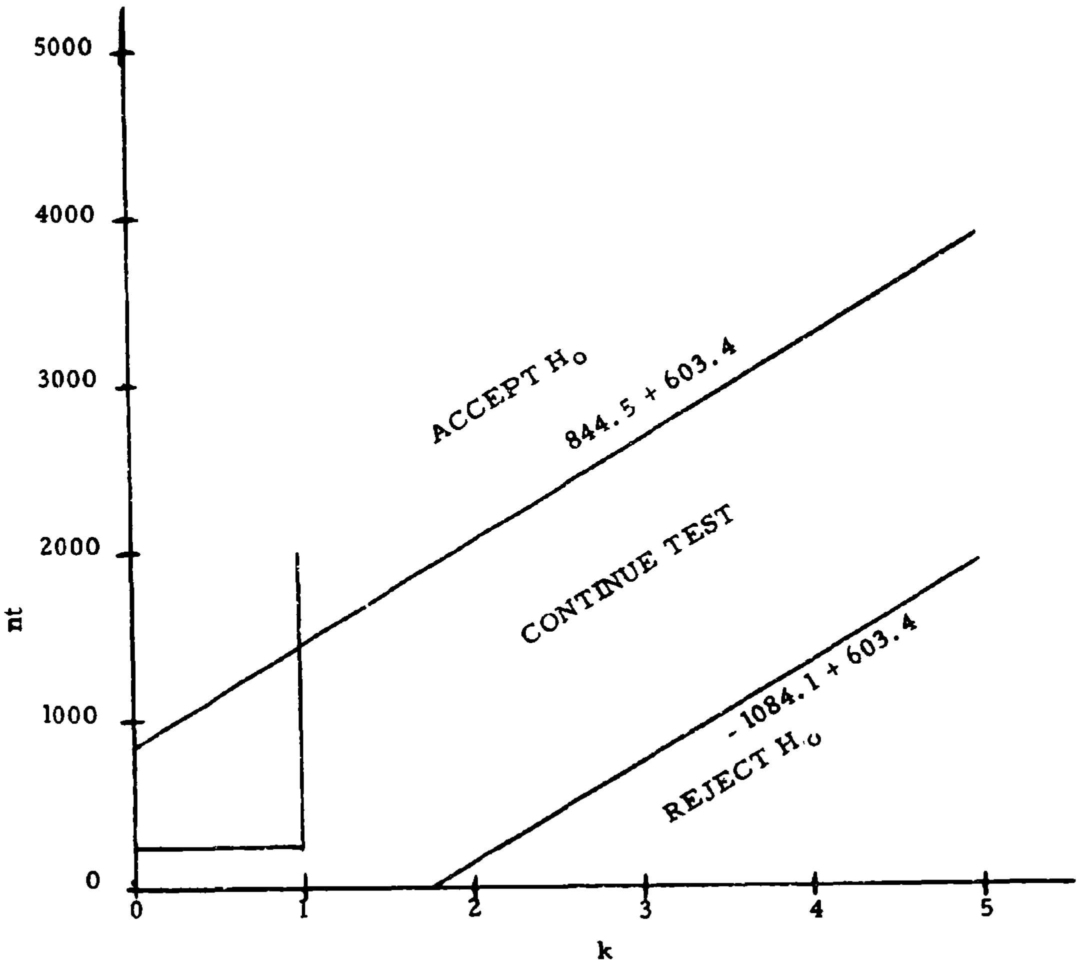


Figure F-24
Sequential Decision Criteria

(11) The primary advantage of sequential testing is that, on the average, less testing is required. Some important disadvantages of sequential testing are as follows:

(a) The exact amount of testing is not specified in advance with the result that planning and budgeting problems may develop.

(b) The greatest amount of testing is required if the value of θ lies in the zone of indifference between θ_1 and θ_o.

(c) The required record keeping becomes more complex.

F-10. Weibull failure times. The Weibull distribution depends upon two parameters β and η. Tests of hypotheses concerning these parameters are rather complex and will not be discussed herein. However, there are government sampling tables pertaining to the Weibull distribution.

F-11. Normal failure times. Tests of hypotheses may be applied to the parameters μ and σ, of the normal distribution. However, these tests are found in numerous elementary statistics textbooks and will not be discussed herein.

Section IV. ACCEPTANCE LIFE TESTING

F-12. General. a. The preceding tests were concerned with testing of hypothesis about reliability and reliability related parameters. This type of test, when used as a basis for accepting material, is sometimes referred to as an acceptance life test. There are a number of government documents which contain tables of test plans which were calculated using tests of hypothesis similar to those already discussed. These will be discussed under the title of acceptance life tests.

b. Tables are available for different failure time models. Those to be discussed herein will include the exponential distribution, Weibull distribution and normal distribution, as well as nonparametric tables Since these documents are self-explanatory and include examples, the contents will only be summarized.

F-13. Exponential failure times.

a. DoD Quality Control and Reliability Handbook H108, Sampling Procedures and Tables for Life and Reliability Testing (Based on the Exponential Distribution) contains tables of life testing sampling plans pertaining to exponential failure times. The plans are divided into three general categories: failure terminated tests, time terminated tests and sequential tests. The mean failure time θ is used as the reliability parameter under test. Minimal and nominal acceptable values of θ, along with allowable risks, are used as the basis for specifying a satisfactory sampling plan.

b. The sequential test procedures are outlined and the parameters of the test are tabled. The formulas of paragraph F-9d of this appendix may be used for expanding these tables.

c. Operating characteristic curves are drawn for most tabled plans. The methods in paragraph F-9 of this appendix may be used for constructing OC curves which are not included.

F-14. Weibull failure times. A series of technical reports contain tabled sampling plans when failure times are distributed in accordance with a Weibull distribution. Each is dependent upon a known β parameter

a. DOD Quality Control and Reliability Technical Report TR-3,[1] Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution, (Mean Life Criterion) provides tabled plans when mean failure time is the reliability parameter to be tested.

b. DOD Quality Control and Reliability Technical Report TR-4,[1] Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Hazard Rate Criterion) provides tabled plans when hazard rate or instantaneous failure rate is the parameter under test.

c. DOD Quality Control and Reliability Technical Report TR-6,[1] Sampling Procedures and Tables for Life and Reliability Testing Based on the Weibull Distribution (Reliable Life Criterion) provides plans when reliable life is the parameter under test. Reliable life is defined as that time beyond which a specified portion of a lot can be expected to survive.

d. DOD Quality and Reliability Assurance Technical Report TR-7,[1] Factors and Procedures for Applying MIL-STD-105D Sampling Plans to Life and Reliability Testing, provides instructions for selecting life test sampling plans from MIL-STD-105D.

[1] These reports may be secured from the Superintendent of Documents, U.S. Government Printing Office, Washington, D.C. 20402.

F-15. Normal failure times.

a. If failure times are known to be normally distributed, a life test sampling plan may be found from MIL-STD-414, Sampling Procedures and Tables for Inspection by Variables for Percent Defective. As indicated by the title, MIL-STD-414 expresses conformance as percent defective, i.e., the percent of the items falling outside specification limits. In the case of life testing, conformance may be expressed as the percent of items failing prior to a specified time.

b. A life test sampling plan from MIL-STD-414 would require that all sample items be tested until failure. The resulting failure times may then be symbolized as $x_1, x_2, \ldots, x_n$. A sampling plan consists of two parameters -- sample size (n) and maximum allowable percent defective (M) -- and may be obtained if an AQL, inspection level, degree of inspection, and lot size are specified. The first step is to find a code letter in table A-2 by using the lot size and inspection level. If normal or tightened inspection are to be used, enter table B-3 with AQL and code letter to find n and M. For reduced inspection, table B-4 is to be used.

c. The procedures of analyzing the sample data and deciding upon lot acceptance is summarized in the following steps.

(1) Compute the sample mean $\bar{x} = \frac{\Sigma x}{n}$

(2) Compute the sample standard deviation

$$s = \sqrt{\frac{n \Sigma x^2 - (\Sigma x)^2}{n(n-1)}}$$

(3) Calculate the quality index

$$Q_L = \frac{\bar{x} - L}{s}$$

where L is a lower specification limit (specified time) and where Q_L is the quality index associated with L.

(4) Enter table B-5 with Q_L and n to find the estimated lot percent defective (p_L) associated with the lower specification limit.

(5) The decision rules are as follows:

(a) If $p_L \leq M$, accept the lot

(b) If $p_L > M$, reject the lot

d. To illustrate the selection of a life test sampling plan from MIL-STD-414, consider a lot of 60 components which has been submitted for acceptance relative to life characteristics. The failure times are known from past experience to be normally distributed. A life test sampling plan is to be selected from MIL-STD-414 using inspection level III, normal inspection, variability unknown - standard deviation method, single specification limit and form 2.

(1) Find the sampling plan, n and M, such that if the lot has a reliability of 0.975 for a 35 hour mission time, R(35) = 0.975, the probability of acceptance would be high.

(2) If the submitted lot has R(35) = 0.90 what is its probability of being accepted?

(3) Using the plan found in part (1), the failure times were as follows: 34, 38, 47, 51, 55, 59 and 62 hours (use only as many as required in the sampling plan). Determine whether the lot should be accepted.

e. The solution is as follows:

(1) An AQL of 2.5% defective for a lower specification limit L = 35 hours would correspond with R(35) = 0.975.

(a) Using a lot size of 60 and inspection level III, table A-2 yields code letter C.

(b) Using code letter C and AQL = 2.5%, table B-3 yields n = 4 and M = 10.92.

(2) Find the OC curve which is shown for code letter C and AQL = 2.5% on page 7. For a lot with $R(35) = 0.90$ (i.e., 10% failing before 35 hours) this OC curve shows a probability of acceptance of 62%.

(3) Since n = 4, the resulting failure times were 34, 38, 47, 51.

x	x^2
34	1156
38	1444
47	2209
51	2601
170	7410

(a) $$\bar{X} = \frac{\Sigma x}{n} = \frac{170}{4} = 42.5$$

(b) $$s = \sqrt{\frac{n\Sigma x^2 - (\Sigma x)^2}{n(n-1)}} = \sqrt{\frac{4(7410) - (170)^2}{4(3)}} = 7.85$$

(c) $$Q_L = \frac{\bar{X} - L}{s} = \frac{42.5 - 35}{7.85} = 0.96$$

(d) Using $Q_L = 0.96$ and $n = 4$, Table B-5 yields an estimated percent defective $p_L = 18.00$.

(e) Since $p_L = 18.00 > M = 10.92$, the lot is rejected, and conclude that the reliability of the lot is too low.

F-16. Nonparametric sampling plans.

a. MIL-STD-105D, Sampling Procedures and Tables for Inspection By Attributes, provides sampling plans relative to attributes

inspection. Attributes inspection is inspection whereby either the unit of product is classified simply as defective or nondefective, or the number of defects in the unit is counted, with respect to a given requirement or set of requirements. Thus, conformance can be expressed in terms of percent defective or defects per hundred units. For reliability testing purposes, we shall be concerned with percent defective.

b. These procedures apply directly to one shot items, i.e., those for which failures do not depend on time. However, these sampling plans may also be applied to life tests even if the distribution of failure times is unknown. In this case a defective item would be defined as an item which failed prior to a specified time t. Such a life test must be nonreplacement and must be terminated at t.

c. A sampling plan may be found from MIL-STD-105D if the lot size, inspection level, AQL, degree of inspection, and type of sampling are specified. The initial step is to find a code letter in table I using lot size and inspection level. If either normal or tightened inspection is specified, enter table X with the code letter and AQL to find the sampling plan. If reduced inspection is in effect, find the plan in table II-C for single sampling.

d. The single sampling plan consists of a sample size (n), and acceptance number (Ac) and rejection number (Re). The lot is to be accepted if the number of defectives found in the sample does not exceed the acceptance number. The lot is to be rejected if the number of sample defectives is equal to or greater than the rejection number.

e. For example, a lot of 125 components has been submitted for acceptance with respect to life characteristics. The distribution of failure times is unknown. A nonreplacement life test sampling plan is to be selected from MIL-STD-105D using inspection level III, normal inspection and single sampling.

(1) Find a sampling plan such that the probability of acceptance will be high if reliability for a 10 hour mission time is 0.96.

(2) If the submitted lot has R(10) = 0.92, what is its probability of its being accepted if this plan is used?

(3) A sample of the size found in part 1 was subjected to test and 2 items failed prior to the 10 hour termination time. Should the lot be accepted?

f. The solution follows:

(1) The life test is to be terminated at 10 hours. Then an AQL of 4% would correspond to R(10) = 0.96. The sampling plan is found as follows:

(a) For a lot size of 125 and inspection level III, table I yields code letter G.

(b) For code letter G, AQL = 4.0, normal inspection and single sampling, table X-G-2 indicates a sampling plan of n = 32, Ac = 3 and Re = 4.

(2) Find the OC curve for code letter G and AQL = 4.0 in chart G. For a lot with R(10) = 0.92 (i.e., 8% failing before 10 hours) this OC curve indicates a 74% probability of acceptance.

(3) Of the 32 sample items, 2 failed before test termination time of 10 hours. Since the number of failures did not exceed the acceptance number, the lot should be accepted because its reliability is high enough.

Section V. REGRESSION ANALYSIS

F-17. General. a. A major problem during the research and development of an equipment is the assurance of high functioning reliability of the final prototype model. Reliability analysis sometimes involves the determination of some reliability parameter for various known stress levels or design characteristics; for example, determination of the mean time to failure at different levels of stress, determination of the bursting pressure for different wall thicknesses of rocket motors, determination of vehicle fuel usage at different velocity levels, etc. Such determinations are sometimes useful for evaluating equipment design, identifying trouble areas and potential corrective activities, etc.

b. Regression analysis, applied to experimental data, may be used to provide information concerning the relationship between a

response (dependent) variable and various levels of an independent variable(s).

F-18. Simple Linear Regression.

a. The simplest form of regression involves determination of the expected value of the response or dependent variable from information concerning the independent variable value. Herein, we shall discuss simple linear regression where the expected response is a linear function of the independent variable. The regression model is

$$\mu_{y.x} = A + Bx$$

where

$\mu_{y.x}$ = expected response associated with x

A and B are parameters of the model

b. The parameter B indicates the amount of change in y which can be attributed to a unit change in x, i.e., the slope of the line. A is the value of $\mu_{y.x}$ evaluated at x = 0.

c. Since A and B are parameters, they may be estimated by analyzing n observations of paired values of (x_i, y_i) where i = 1, 2, ..., n. The regression equation for estimating the expected response is

$$\hat{y} = a + bx$$

where

$$a = \frac{\Sigma x^2 \Sigma y - \Sigma x \Sigma xy}{n \Sigma x^2 - (\Sigma x)^2} \quad \text{is an estimate of A}$$

$$b = \frac{n \Sigma xy - \Sigma x \Sigma y}{n \Sigma x^2 - (\Sigma x)^2} \quad \text{is an estimate of B}$$

and

$\hat{y}$ is an estimate of $\mu_{y.x}$

d. To illustrate the use of regression analysis techniques, suppose the data shown in figure F-25 represents observations of the bursting pressure and wall thickness of a sample of three rocket motors.

y, Bursting Pressure (dynes/cm^2)	x, Thickness (mm.)
10,000	1
23,000	2
30,000	3

Figure F-25
Observations of Rocket Motor Wall Thickness and Bursting Pressure

(1) Determine the sample regression line, plot a scattergram of the paired observations, and draw the line.

(2) Estimate the effect on bursting pressure of adding 1 mm. to the wall thickness by both a point estimate and a 90% confidence interval.

(3) A proposed motor must have an average bursting pressure of 25,000 dynes per/cm^2. What wall thickness should be used in the design?

(4) Using the wall thickness found in part c, estimate both the average bursting pressure and the bursting pressure of an individual motor by a 90% confidence interval.

e. The solution follows:

$n = 3$

y	x	x^2	xy	y^2
10000	1	1	10000	100000000
23000	2	4	46000	529000000
30000	3	9	90000	900000000
63000	6	14	146000	1529000000
Σy	Σx	Σx^2	Σxy	Σy^2

(1) $$a = \frac{14(63000) - 6(146000)}{3(14) - (6)^2} = \frac{6000}{6} = 1000$$

$$b = \frac{3(146000) - 6(63000)}{3(14) - (6)^2} = \frac{60000}{6} = 10000$$

Then, the sample regression line is

$$\hat{y} = 1000 + 10000x = 1000(1+10x)$$

Figure F-26 shows the scattergram of plotted points as well as the drawn sample regression line.

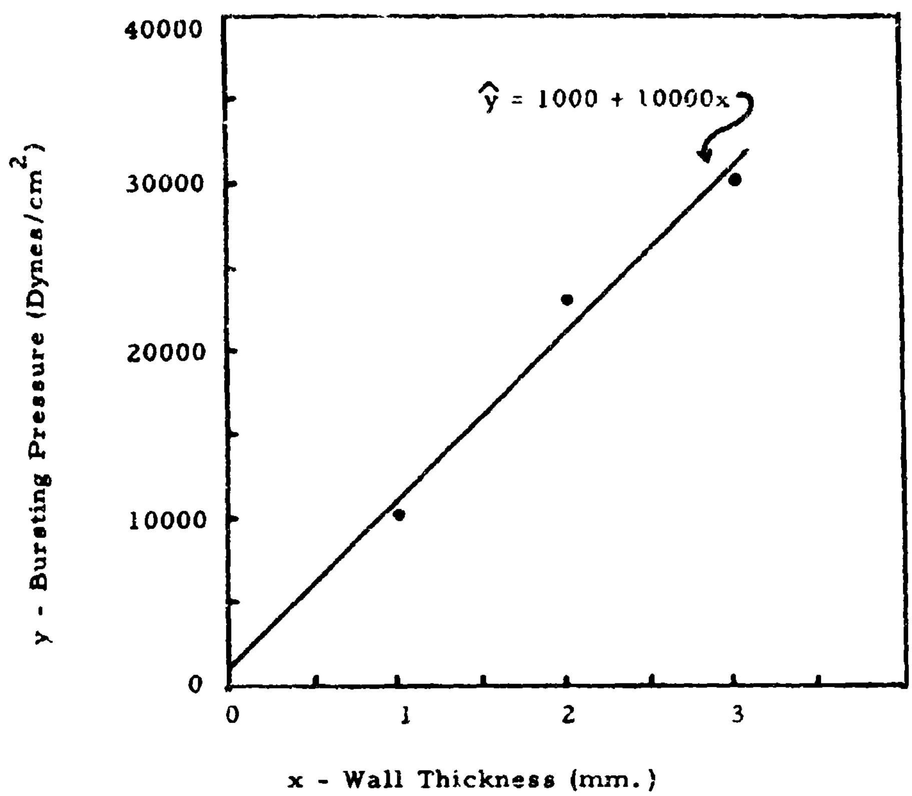


Figure F-26
Regression Scattergram

(2) The effect of wall thickness on bursting pressure is described by the parameter B which can be estimated from the sample value b = 10000 dynes/cm^2/mm..

A 100(1-α)% confidence interval for B is

$$b \pm t_{\alpha/2,n-2} \frac{s_{y \cdot x}}{\sqrt{\frac{n\Sigma x^2 - (\Sigma x)^2}{n}}}$$

where $t_{\alpha/2,n-2}$ may be evaluated from table H-4 appendix H.

$$s_{y \cdot x} = \sqrt{\frac{\Sigma y^2 - a\Sigma y - b\Sigma xy}{n-2}}$$

$$= \sqrt{\frac{1529000000 - (1000)(63000) - (10000)\ (146000)}{1}}$$

$$= \sqrt{6000000} = 2449$$

A 90% confidence interval for B is found as follows:

$$\alpha/2 = .05$$

$$t_{.05,1} = 6.314$$

$$10000 \pm 6.314 \frac{2449}{\sqrt{\frac{3(14) - (6)^2}{3}}}$$

$$10000 \pm 10935$$

$$-935 \leq B \leq 20935$$

Then there is 90% confidence that B falls within this interval.

(3) An estimate of the proper wall thickness to yield an average bursting pressure of 25000 dynes/cm^2 may be found by utilizing the sample regression equation to solve for x when y is replaced by 25000.

$$1000 + 10000x = 25000$$

$$x = 2.4 \text{ mm.}$$

(4) The average bursting pressure $\mu_{y.x}$ may be estimated with a 100 (1- α)% confidence interval as follows:

$$a + bx_k \pm t_{\alpha/2,n-2} \quad s_{y.x} \sqrt{\frac{1}{n} + \frac{n(x_k - \bar{x})^2}{n\Sigma x^2 - (\Sigma x)^2}}$$

where

$$\bar{x} = \frac{\Sigma x}{n} = \frac{6}{3} = 2$$

x_k is the particular value of x for which $\mu_{y.x}$ is to be estimated.

Then a 90% confidence interval of $\mu_{y.x}$ for $x_k = 2.4$

$$t_{.05,1} = 6.314$$

$$1000 + 10000(2.4) \pm (6.314)(2449) \sqrt{\frac{1}{3} + \frac{3(2.4-2)^2}{3(14) - (6)^2}}$$

$$25000 \pm 9896$$

$$15104 \leq \mu_{y.x} \leq 34896$$

We are 90% confident that the average bursting pressure will fall within this interval if the walls are constructed 2.4 mm. thick.

The bursting pressure of any individual rocket motor whose wall is x_k thick may also be estimated by a 100 $(1-\alpha)$ confidence interval.

$$a + bx_k \pm t_{\alpha/2,n-2} \quad s_{y.x} \sqrt{1 + \frac{1}{n} + \frac{n(x_k - \bar{x})^2}{n\Sigma x^2 - (\Sigma x)^2}}$$

Then if the process has been modified so that wall thickness will be 2.4 mm., a 90% confidence interval for the bursting pressure of the

next motor so produced is

$$1000 + 10000(2.4) \pm 6.314(2449)\sqrt{1 + \frac{1}{3} + \frac{3(2.4-2)^2}{3(14)-(6)^2}}$$

$$25000 \pm 18401$$

$$6599 \leq y \leq 43401$$

Thus, we are 90% sure that any particular motor produced with wall thickness of 2.4 mm. will have a bursting pressure within the above interval.

f. The sample size in the preceding example is unrealistically small, but is used to demonstrate the regression computation. The small sample size has contributed to the large intervals obtained.

APPENDIX G

RELIABILITY EVALUATION, FAILURE ANALYSIS AND CORRECTION -- THE FEEDBACK LOOP

(Not included in this issue.)

APPENDIX H

TABLES

This appendix consists of assorted basic tables used in reliability analysis. These tables are utilized and referenced in various statements and examples throughout the preceding appendices.

Table H-1a

Exponential Values: exp (-x)

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	1.00000	0.99005	0.98020	0.97045	0.96079	0.95123	0.94176	0.93239	0.92312	0.91393
0.1	0.90484	0.89583	0.88692	0.87810	0.86936	0.86071	0.85214	0.84366	0.83527	0.82696
0.2	0.81873	0.81058	0.80252	0.79453	0.78663	0.77880	0.77105	0.76338	0.75578	0.74826
0.3	0.74082	0.73345	0.72615	0.71892	0.71177	0.70469	0.69768	0.69073	0.68386	0.67706
0.4	0.67032	0.66365	0.65705	0.65051	0.64404	0.63763	0.63128	0.62500	0.61878	0.61263
0.5	0.60653	0.60050	0.59452	0.58860	0.58275	0.57695	0.57121	0.56553	0.55990	0.55433
0.6	0.54881	0.54335	0.53794	0.53259	0.52729	0.52205	0.51685	0.51171	0.50662	0.50158
0.7	0.49659	0.49164	0.48675	0.48191	0.47711	0.47237	0.46767	0.46301	0.45841	0.45384
0.8	0.44933	0.44486	0.44043	0.43605	0.43171	0.42741	0.42316	0.41895	0.41478	0.41066
0.9	0.40657	0.40252	0.39852	0.39455	0.39063	0.38674	0.38289	0.37908	0.37531	0.37158
1.0	0.36788	0.36422	0.36059	0.35701	0.35345	0.34994	0.34646	0.34301	0.33960	0.33622
1.1	0.33287	0.32956	0.32628	0.32303	0.31982	0.31664	0.31349	0.31037	0.30728	0.30422
1.2	0.30119	0.29820	0.29523	0.29229	0.28938	0.28650	0.28365	0.28083	0.27804	0.27527
1.3	0.27253	0.26982	0.26714	0.26448	0.26185	0.25924	0.25666	0.25411	0.25158	0.24908
1.4	0.24660	0.24414	0.24171	0.23931	0.23693	0.23457	0.23224	0.22993	0.22764	0.22537
1.5	0.22313	0.22091	0.21871	0.21654	0.21438	0.21225	0.21014	0.20805	0.20598	0.20393
1.6	0.20190	0.19989	0.19790	0.19593	0.19398	0.19205	0.19014	0.18825	0.18637	0.18452
1.7	0.18268	0.18087	0.17907	0.17728	0.17552	0.17377	0.17204	0.17033	0.16864	0.16696
1.8	0.16530	0.16365	0.16203	0.16041	0.15882	0.15724	0.15567	0.15412	0.15259	0.15107
1.9	0.14957	0.14808	0.14661	0.14515	0.14370	0.14227	0.14086	0.13946	0.13807	0.13670
2.0	0.13534	0.13399	0.13266	0.13134	0.13003	0.12873	0.12745	0.12619	0.12493	0.12369
2.1	0.12246	0.12124	0.12003	0.11884	0.11765	0.11648	0.11533	0.11418	0.11304	0.11192
2.2	0.11080	0.10970	0.10861	0.10753	0.10646	0.10540	0.10435	0.10331	0.10228	0.10127
2.3	0.10026	0.09926	0.09827	0.09730	0.09633	0.09537	0.09442	0.09348	0.09255	0.09163
2.4	0.09072	0.08982	0.08892	0.08804	0.08716	0.08629	0.08543	0.08458	0.08374	0.08291
2.5	0.08208	0.08127	0.08046	0.07966	0.07887	0.07808	0.07730	0.07654	0.07577	0.07502
2.6	0.07427	0.07353	0.07280	0.07208	0.07136	0.07065	0.06995	0.06925	0.06856	0.06788
2.7	0.06721	0.06654	0.06587	0.06522	0.06457	0.06393	0.06329	0.06266	0.06204	0.06142
2.8	0.06081	0.06020	0.05961	0.05901	0.05843	0.05784	0.05727	0.05670	0.05613	0.05558
2.9	0.05502	0.05448	0.05393	0.05340	0.05287	0.05234	0.05182	0.05130	0.05079	0.05029
3.0	0.04979	0.04929	0.04880	0.04832	0.04783	0.04736	0.04689	0.04642	0.04596	0.04550
3.1	0.04505	0.04460	0.04416	0.04372	0.04328	0.04285	0.04243	0.04200	0.04159	0.04117
3.2	0.04076	0.04036	0.03996	0.03956	0.03916	0.03877	0.03839	0.03801	0.03763	0.03725
3.3	0.03688	0.03652	0.03615	0.03579	0.03544	0.03508	0.03474	0.03439	0.03405	0.03371
3.4	0.03337	0.03304	0.03271	0.03239	0.03206	0.03175	0.03143	0.03112	0.03081	0.03050
3.5	0.03020	0.02990	0.02960	0.02930	0.02901	0.02872	0.02844	0.02816	0.02788	0.02760
3.6	0.02732	0.02705	0.02678	0.02652	0.02625	0.02599	0.02573	0.02548	0.02522	0.02497
3.7	0.02472	0.02448	0.02423	0.02399	0.02375	0.02352	0.02328	0.02305	0.02282	0.02260
3.8	0.02237	0.02215	0.02193	0.02171	0.02149	0.02128	0.02107	0.02086	0.02065	0.02045
3.9	0.02024	0.02004	0.01984	0.01964	0.01945	0.01925	0.01906	0.01887	0.01869	0.01850
4.0	0.01832	0.01813	0.01795	0.01777	0.01760	0.01742	0.01725	0.01708	0.01691	0.01674

Table H-1b

Exponential Values: exp (-x)

x	.00000	.00001	.00002	.00003	.00004	.00005	.00006	.00007	.00008	.00009
0.0000	1.00000	0.99999	0.99998	0.99997	0.99996	0.99995	0.99994	0.99993	0.99992	0.99991
0.0001	0.99990	0.99989	0.99988	0.99987	0.99986	0.99985	0.99984	0.99983	0.99982	0.99981
0.0002	0.99980	0.99979	0.99978	0.99977	0.99976	0.99975	0.99974	0.99973	0.99972	0.99971
0.0003	0.99970	0.99969	0.99968	0.99967	0.99966	0.99965	0.99964	0.99963	0.99962	0.99961
0.0004	0.99960	0.99959	0.99958	0.99957	0.99956	0.99955	0.99954	0.99953	0.99952	0.99951
0.0005	0.99950	0.99949	0.99948	0.99947	0.99946	0.99945	0.99944	0.99943	0.99942	0.99941
0.0006	0.99940	0.99939	0.99938	0.99937	0.99936	0.99935	0.99934	0.99933	0.99932	0.99931
0.0007	0.99930	0.99929	0.99928	0.99927	0.99926	0.99925	0.99924	0.99923	0.99922	0.99921
0.0008	0.99920	0.99919	0.99918	0.99917	0.99916	0.99915	0.99914	0.99913	0.99912	0.99911
0.0009	0.99910	0.99909	0.99908	0.99907	0.99906	0.99905	0.99904	0.99903	0.99902	0.99901
0.0010	0.99900	0.99899	0.99898	0.99897	0.99896	0.99895	0.99894	0.99893	0.99892	0.99891
0.0011	0.99890	0.99889	0.99888	0.99887	0.99886	0.99885	0.99884	0.99883	0.99882	0.99881
0.0012	0.99880	0.99879	0.99878	0.99877	0.99876	0.99875	0.99874	0.99873	0.99872	0.99871
0.0013	0.99870	0.99869	0.99868	0.99867	0.99866	0.99865	0.99864	0.99863	0.99862	0.99861
0.0014	0.99860	0.99859	0.99858	0.99857	0.99856	0.99855	0.99854	0.99853	0.99852	0.99851
0.0015	0.99850	0.99849	0.99848	0.99847	0.99846	0.99845	0.99844	0.99843	0.99842	0.99841
0.0016	0.99840	0.99839	0.99838	0.99837	0.99836	0.99835	0.99834	0.99833	0.99832	0.99831
0.0017	0.99830	0.99829	0.99828	0.99827	0.99826	0.99825	0.99824	0.99823	0.99822	0.99821
0.0018	0.99820	0.99819	0.99818	0.99817	0.99816	0.99815	0.99814	0.99813	0.99812	0.99811
0.0019	0.99810	0.99809	0.99808	0.99807	0.99806	0.99805	0.99804	0.99803	0.99802	0.99801
0.0020	0.99800	0.99799	0.99798	0.99797	0.99796	0.99795	0.99794	0.99793	0.99792	0.99791
0.0021	0.99790	0.99789	0.99788	0.99787	0.99786	0.99785	0.99784	0.99783	0.99782	0.99781
0.0022	0.99780	0.99779	0.99778	0.99777	0.99776	0.99775	0.99774	0.99773	0.99772	0.99771
0.0023	0.99770	0.99769	0.99768	0.99767	0.99766	0.99765	0.99764	0.99763	0.99762	0.99761
0.0024	0.99760	0.99759	0.99758	0.99757	0.99756	0.99755	0.99754	0.99753	0.99752	0.99751
0.0025	0.99750	0.99749	0.99748	0.99747	0.99746	0.99745	0.99744	0.99743	0.99742	0.99741
0.0026	0.99740	0.99739	0.99738	0.99737	0.99736	0.99735	0.99734	0.99733	0.99732	0.99731
0.0027	0.99730	0.99729	0.99728	0.99727	0.99726	0.99725	0.99724	0.99723	0.99722	0.99721
0.0028	0.99720	0.99719	0.99718	0.99717	0.99716	0.99715	0.99714	0.99713	0.99712	0.99711
0.0029	0.99710	0.99709	0.99708	0.99707	0.99706	0.99705	0.99704	0.99703	0.99702	0.99701
0.0030	0.99700	0.99699	0.99698	0.99697	0.99696	0.99695	0.99694	0.99693	0.99692	0.99691
0.0031	0.99690	0.99689	0.99688	0.99687	0.99686	0.99685	0.99684	0.99684	0.99683	0.99682
0.0032	0.99681	0.99680	0.99679	0.99678	0.99677	0.99676	0.99675	0.99674	0.99673	0.99672
0.0033	0.99671	0.99670	0.99669	0.99668	0.99667	0.99666	0.99665	0.99664	0.99663	0.99662
0.0034	0.99661	0.99660	0.99659	0.99658	0.99657	0.99656	0.99655	0.99654	0.99653	0.99652
0.0035	0.99651	0.99650	0.99649	0.99648	0.99647	0.99646	0.99645	0.99644	0.99643	0.99642
0.0036	0.99641	0.99640	0.99639	0.99638	0.99637	0.99636	0.99635	0.99634	0.99633	0.99632
0.0037	0.99631	0.99630	0.99629	0.99628	0.99627	0.99626	0.99625	0.99624	0.99623	0.99622
0.0038	0.99621	0.99620	0.99619	0.99618	0.99617	0.99616	0.99615	0.99614	0.99613	0.99612
0.0039	0.99611	0.99610	0.99609	0.99608	0.99607	0.99606	0.99605	0.99604	0.99603	0.99602
0.0040	0.99601	0.99600	0.99599	0.99598	0.99597	0.99596	0.99595	0.99594	0.99593	0.99592
0.0041	0.99591	0.99590	0.99589	0.99588	0.99587	0.99586	0.99585	0.99584	0.99583	0.99582
0.0042	0.99581	0.99580	0.99579	0.99578	0.99577	0.99576	0.99575	0.99574	0.99573	0.99572
0.0043	0.99571	0.99570	0.99569	0.99568	0.99567	0.99566	0.99565	0.99564	0.99563	0.99562
0.0044	0.99561	0.99560	0.99559	0.99558	0.99557	0.99556	0.99555	0.99554	0.99553	0.99552
0.0045	0.99551	0.99550	0.99549	0.99548	0.99547	0.99546	0.99545	0.99544	0.99543	0.99542
0.0046	0.99541	0.99540	0.99539	0.99538	0.99537	0.99536	0.99535	0.99534	0.99533	0.99532
0.0047	0.99531	0.99530	0.99529	0.99528	0.99527	0.99526	0.99525	0.99524	0.99523	0.99522
0.0048	0.99521	0.99520	0.99519	0.99518	0.99517	0.99516	0.99515	0.99514	0.99513	0.99512
0.0049	0.99511	0.99510	0.99509	0.99508	0.99507	0.99506	0.99505	0.99504	0.99503	0.99502

Table H-1c

Exponential Values: exp(-x)

x	.00000	.00001	.00002	.00003	.00004	.00005	.00006	.00007	.00008	.0000
0.0050	0.99501	0.99500	0.99499	0.99498	0.99497	0.99496	0.99495	0.99494	0.99493	0.99492
0.0051	0.99491	0.99490	0.99489	0.99488	0.99487	0.99486	0.99485	0.99484	0.99483	0.99482
0.0052	0.99481	0.99480	0.99479	0.99478	0.99477	0.99476	0.99475	0.99474	0.99473	0.99472
0.0053	0.99471	0.99470	0.99469	0.99468	0.99467	0.99466	0.99465	0.99464	0.99463	0.99462
0.0054	0.99461	0.99460	0.99459	0.99458	0.99457	0.99456	0.99455	0.99454	0.99453	0.99453
0.0055	0.99452	0.99451	0.99450	0.99449	0.99448	0.99447	0.99446	0.99445	0.99444	0.99443
0.0056	0.99442	0.99441	0.99440	0.99439	0.99438	0.99437	0.99436	0.99435	0.99434	0.99433
0.0057	0.99432	0.99431	0.99430	0.99429	0.99428	0.99427	0.99426	0.99425	0.99424	0.99423
0.0058	0.99422	0.99421	0.99420	0.99419	0.99418	0.99417	0.99416	0.99415	0.99414	0.99413
0.0059	0.99412	0.99411	0.99410	0.99409	0.99408	0.99407	0.99406	0.99405	0.99404	0.99403
0.0060	0.99402	0.99401	0.99400	0.99399	0.99398	0.99397	0.99396	0.99395	0.99394	0.99393
0.0061	0.99392	0.99391	0.99390	0.99389	0.99388	0.99387	0.99386	0.99385	0.99384	0.99383
0.0062	0.99382	0.99381	0.99380	0.99379	0.99378	0.99377	0.99376	0.99375	0.99374	0.99373
0.0063	0.99372	0.99371	0.99370	0.99369	0.99368	0.99367	0.99366	0.99365	0.99364	0.99363
0.0064	0.99362	0.99361	0.99360	0.99359	0.99358	0.99357	0.99356	0.99355	0.99354	0.99353
0.0065	0.99352	0.99351	0.99350	0.99349	0.99348	0.99347	0.99346	0.99345	0.99344	0.99343
0.0066	0.99342	0.99341	0.99340	0.99339	0.99338	0.99337	0.99336	0.99335	0.99334	0.99333
0.0067	0.99332	0.99331	0.99330	0.99329	0.99328	0.99327	0.99326	0.99325	0.99324	0.99323
0.0068	0.99322	0.99321	0.99320	0.99319	0.99318	0.99317	0.99316	0.99315	0.99314	0.99313
0.0069	0.99312	0.99311	0.99310	0.99309	0.99308	0.99307	0.99306	0.99305	0.99304	0.99303
0.0070	0.99302	0.99301	0.99300	0.99299	0.99298	0.99297	0.99296	0.99295	0.99295	0.99294
0.0071	0.99293	0.99292	0.99291	0.99290	0.99289	0.99288	0.99287	0.99286	0.99285	0.99284
0.0072	0.99283	0.99282	0.99281	0.99280	0.99279	0.99278	0.99277	0.99276	0.99275	0.99274
0.0073	0.99273	0.99272	0.99271	0.99270	0.99269	0.99268	0.99267	0.99266	0.99265	0.99264
0.0074	0.99263	0.99262	0.99261	0.99260	0.99259	0.99258	0.99257	0.99256	0.99255	0.99254
0.0075	0.99253	0.99252	0.99251	0.99250	0.99249	0.99248	0.99247	0.99246	0.99245	0.99244
0.0076	0.99243	0.99242	0.99241	0.99240	0.99239	0.99238	0.99237	0.99236	0.99235	0.99234
0.0077	0.99233	0.99232	0.99231	0.99230	0.99229	0.99228	0.99227	0.99226	0.99225	0.99224
0.0078	0.99223	0.99222	0.99221	0.99220	0.99219	0.99218	0.99217	0.99216	0.99215	0.99214
0.0079	0.99213	0.99212	0.99211	0.99210	0.99209	0.99208	0.99207	0.99206	0.99205	0.99204
0.0080	0.99203	0.99202	0.99201	0.99200	0.99199	0.99198	0.99197	0.99196	0.99195	0.99194
0.0081	0.99193	0.99192	0.99191	0.99190	0.99189	0.99188	0.99187	0.99186	0.99185	0.99184
0.0082	0.99183	0.99182	0.99181	0.99180	0.99179	0.99178	0.99177	0.99176	0.99175	0.99174
0.0083	0.99173	0.99172	0.99171	0.99170	0.99169	0.99168	0.99167	0.99166	0.99166	0.99165
0.0084	0.99164	0.99163	0.99162	0.99161	0.99160	0.99159	0.99158	0.99157	0.99156	0.99155
0.0085	0.99154	0.99153	0.99152	0.99151	0.99150	0.99149	0.99148	0.99147	0.99146	0.99145
0.0086	0.99144	0.99143	0.99142	0.99141	0.99140	0.99139	0.99138	0.99137	0.99136	0.99135
0.0087	0.99134	0.99133	0.99132	0.99131	0.99130	0.99129	0.99128	0.99127	0.99126	0.99125
0.0088	0.99124	0.99123	0.99122	0.99121	0.99120	0.99119	0.99118	0.99117	0.99116	0.99115
0.0089	0.99114	0.99113	0.99112	0.99111	0.99110	0.99109	0.99108	0.99107	0.99106	0.99105
0.0090	0.99104	0.99103	0.99102	0.99101	0.99100	0.99099	0.99098	0.99097	0.99096	0.99095
0.0091	0.99094	0.99093	0.99092	0.99091	0.99090	0.99089	0.99088	0.99087	0.99086	0.99085
0.0092	0.99084	0.99083	0.99082	0.99081	0.99080	0.99079	0.99078	0.99077	0.99076	0.99075
0.0093	0.99074	0.99073	0.99072	0.99071	0.99070	0.99069	0.99068	0.99067	0.99066	0.99065
0.0094	0.99064	0.99063	0.99062	0.99061	0.99060	0.99059	0.99058	0.99057	0.99056	0.99055
0.0095	0.99054	0.99054	0.99053	0.99052	0.99051	0.99050	0.99049	0.99048	0.99047	0.99046
0.0096	0.99045	0.99044	0.99043	0.99042	0.99041	0.99040	0.99039	0.99038	0.99037	0.99036
0.0097	0.99035	0.99034	0.99033	0.99032	0.99031	0.99030	0.99029	0.99028	0.99027	0.99026
0.0098	0.99025	0.99024	0.99023	0.99022	0.99021	0.99020	0.99019	0.99018	0.99017	0.99016
0.0099	0.99015	0.99014	0.99013	0.99012	0.99011	0.99010	0.99009	0.99008	0.99007	0.99006

Table H-1d
Exponential Values: exp(-x)

x	.00000	.00001	.00002	.00003	.00004	.00005	.00006	.00007	.00008	.00009
0.0100	0.99005	0.99004	0.99003	0.99002	0.99001	0.99000	0.98999	0.98998	0.98997	0.98996
0.0101	0.98995	0.98994	0.98993	0.98992	0.98991	0.98990	0.98989	0.98988	0.98987	0.98986
0.0102	0.98985	0.98984	0.98983	0.98982	0.98981	0.98980	0.98979	0.98978	0.98977	0.98976
0.0103	0.98975	0.98974	0.98973	0.98972	0.98971	0.98970	0.98969	0.98968	0.98967	0.98966
0.0104	0.98965	0.98964	0.98963	0.98962	0.98961	0.98960	0.98959	0.98958	0.98957	0.98956
0.0105	0.98955	0.98955	0.98954	0.98953	0.98952	0.98951	0.98950	0.98949	0.98948	0.98947
0.0106	0.98946	0.98945	0.98944	0.98943	0.98942	0.98941	0.98940	0.98939	0.98938	0.98937
0.0107	0.98936	0.98935	0.98934	0.98933	0.98932	0.98931	0.98930	0.98929	0.98928	0.98927
0.0108	0.98926	0.98925	0.98924	0.98923	0.98922	0.98921	0.98920	0.98919	0.98918	0.98917
0.0109	0.98916	0.98915	0.98914	0.98913	0.98912	0.98911	0.98910	0.98909	0.98908	0.98907
0.0110	0.98906	0.98905	0.98904	0.98903	0.98902	0.98901	0.98900	0.98899	0.98898	0.98897
0.0111	0.98896	0.98895	0.98894	0.98893	0.98892	0.98891	0.98890	0.98889	0.98888	0.98887
0.0112	0.98886	0.98885	0.98884	0.98883	0.98882	0.98881	0.98880	0.98879	0.98878	0.98877
0.0113	0.98876	0.98875	0.98874	0.98873	0.98872	0.98871	0.98870	0.98869	0.98868	0.98867
0.0114	0.98866	0.98865	0.98864	0.98864	0.98863	0.98862	0.98861	0.98860	0.98859	0.98858
0.0115	0.98857	0.98856	0.98855	0.98854	0.98853	0.98852	0.98851	0.98850	0.98849	0.98848
0.0116	0.98847	0.98846	0.98845	0.98844	0.98843	0.98842	0.98841	0.98840	0.98839	0.98838
0.0117	0.98837	0.98836	0.98835	0.98834	0.98833	0.98832	0.98831	0.98830	0.98829	0.98828
0.0118	0.98827	0.98826	0.98825	0.98824	0.98823	0.98822	0.98821	0.98820	0.98819	0.98818
0.0119	0.98817	0.98816	0.98815	0.98814	0.98813	0.98812	0.98811	0.98810	0.98809	0.98808
0.0120	0.98807	0.98806	0.98805	0.98804	0.98803	0.98802	0.98801	0.98800	0.98799	0.98798
0.0121	0.98797	0.98796	0.98795	0.98794	0.98793	0.98792	0.98791	0.98790	0.98789	0.98788
0.0122	0.98787	0.98786	0.98785	0.98784	0.98783	0.98782	0.98781	0.98780	0.98780	0.98779
0.0123	0.98778	0.98777	0.98776	0.98775	0.98774	0.98773	0.98772	0.98771	0.98770	0.98769
0.0124	0.98768	0.98767	0.98766	0.98765	0.98764	0.98763	0.98762	0.98761	0.98760	0.98759
0.0125	0.98758	0.98757	0.98756	0.98755	0.98754	0.98753	0.98752	0.98751	0.98750	0.98749
0.0126	0.98748	0.98747	0.98746	0.98745	0.98744	0.98743	0.98742	0.98741	0.98740	0.98739
0.0127	0.98738	0.98737	0.98736	0.98735	0.98734	0.98733	0.98732	0.98731	0.98730	0.98729
0.0128	0.98728	0.98727	0.98726	0.98725	0.98724	0.98723	0.98722	0.98721	0.98720	0.98719
0.0129	0.98718	0.98717	0.98716	0.98715	0.98714	0.98713	0.98712	0.98711	0.98710	0.98709
0.0130	0.98708	0.98707	0.98706	0.98705	0.98704	0.98703	0.98702	0.98702	0.98701	0.98700
0.0131	0.98699	0.98698	0.98697	0.98696	0.98695	0.98694	0.98693	0.98692	0.98691	0.98690
0.0132	0.98689	0.98688	0.98687	0.98686	0.98685	0.98684	0.98683	0.98682	0.98681	0.98680
0.0133	0.98679	0.98678	0.98677	0.98676	0.98675	0.98674	0.98673	0.98672	0.98671	0.98670
0.0134	0.98669	0.98668	0.98667	0.98666	0.98665	0.98664	0.98663	0.98662	0.98661	0.98660
0.0135	0.98659	0.98658	0.98657	0.98656	0.98655	0.98654	0.98653	0.98652	0.98651	0.98650
0.0136	0.98649	0.98648	0.98647	0.98646	0.98645	0.98644	0.98643	0.98642	0.98641	0.98640
0.0137	0.98639	0.98638	0.98637	0.98636	0.98635	0.98634	0.98633	0.98632	0.98631	0.98630
0.0138	0.98629	0.98628	0.98628	0.98627	0.98626	0.98625	0.98624	0.98623	0.98622	0.98621
0.0139	0.98620	0.98619	0.98618	0.98617	0.98616	0.98615	0.98614	0.98613	0.98612	0.98611
0.0140	0.98610	0.98609	0.98608	0.98607	0.98606	0.98605	0.98604	0.98603	0.98602	0.98601
0.0141	0.98600	0.98599	0.98598	0.98597	0.98596	0.98595	0.98594	0.98593	0.98592	0.98591
0.0142	0.98590	0.98589	0.98588	0.98587	0.98586	0.98585	0.98584	0.98583	0.98582	0.98581
0.0143	0.98580	0.98579	0.98578	0.98577	0.98576	0.98575	0.98574	0.98573	0.98572	0.98571
0.0144	0.98570	0.98569	0.98568	0.98567	0.98566	0.98565	0.98564	0.98563	0.98562	0.98561
0.0145	0.98560	0.98559	0.98558	0.98558	0.98557	0.98556	0.98555	0.98554	0.98553	0.98552
0.0146	0.98551	0.98550	0.98549	0.98548	0.98547	0.98546	0.98545	0.98544	0.98543	0.98542
0.0147	0.98541	0.98540	0.98539	0.98538	0.98537	0.98536	0.98535	0.98534	0.98533	0.98532
0.0148	0.98531	0.98530	0.98529	0.98528	0.98527	0.98526	0.98525	0.98524	0.98523	0.98522
0.0149	0.98521	0.98520	0.98519	0.98518	0.98517	0.98516	0.98515	0.98514	0.98513	0.98512

Table H-1
Exponential Values: exp(-x)

x	.00000	.00001	.00002	.00003	.00004	.00005	.00006	.00007	.00008	.00009
0.0150	0.98511	0.98510	0.98509	0.98508	0.98507	0.98506	0.98505	0.98504	0.98503	0.98502
0.0151	0.98501	0.98500	0.98499	0.98498	0.98497	0.98496	0.98495	0.98494	0.98493	0.98492
0.0152	0.98491	0.98491	0.98490	0.98489	0.98488	0.98487	0.98486	0.98485	0.98484	0.98483
0.0153	0.98482	0.98481	0.98480	0.98479	0.98478	0.98477	0.98476	0.98475	0.98474	0.98473
0.0154	0.98472	0.98471	0.98470	0.98469	0.98468	0.98467	0.98466	0.98465	0.98464	0.98463
0.0155	0.98462	0.98461	0.98460	0.98459	0.98458	0.98457	0.98456	0.98455	0.98454	0.98453
0.0156	0.98452	0.98451	0.98450	0.98449	0.98448	0.98447	0.98446	0.98445	0.98444	0.98443
0.0157	0.98442	0.98441	0.98440	0.98439	0.98438	0.98437	0.98436	0.98435	0.98434	0.98433
0.0158	0.98432	0.98431	0.98430	0.98429	0.98428	0.98427	0.98427	0.98426	0.98425	0.98424
0.0159	0.98423	0.98422	0.98421	0.98420	0.98419	0.98418	0.98417	0.98416	0.98415	0.98414
0.0160	0.98413	0.98412	0.98411	0.98410	0.98409	0.98408	0.98407	0.98406	0.98405	0.98404
0.0161	0.98403	0.98402	0.98401	0.98400	0.98399	0.98398	0.98397	0.98396	0.98395	0.98394
0.0162	0.98393	0.98392	0.98391	0.98390	0.98389	0.98388	0.98387	0.98386	0.98385	0.98384
0.0163	0.98383	0.98382	0.98381	0.98380	0.98379	0.98378	0.98377	0.98376	0.98375	0.98374
0.0164	0.98373	0.98372	0.98371	0.98370	0.98369	0.98368	0.98367	0.98366	0.98366	0.98365
0.0165	0.98364	0.98363	0.98362	0.98361	0.98360	0.98359	0.98358	0.98357	0.98356	0.98355
0.0166	0.98354	0.98353	0.98352	0.98351	0.98350	0.98349	0.98348	0.98347	0.98346	0.98345
0.0167	0.98344	0.98343	0.98342	0.98341	0.98340	0.98339	0.98338	0.98337	0.98336	0.98335
0.0168	0.98334	0.98333	0.98332	0.98331	0.98330	0.98329	0.98328	0.98327	0.98326	0.98325
0.0169	0.98324	0.98323	0.98322	0.98321	0.98320	0.98319	0.98318	0.98317	0.98316	0.98315
0.0170	0.98314	0.98313	0.98312	0.98311	0.98310	0.98309	0.98308	0.98307	0.98307	0.98306
0.0171	0.98305	0.98304	0.98303	0.98302	0.98301	0.98300	0.98299	0.98298	0.98297	0.98296
0.0172	0.98295	0.98294	0.98293	0.98292	0.98291	0.98290	0.98289	0.98288	0.98287	0.98286
0.0173	0.98285	0.98284	0.98283	0.98282	0.98281	0.98280	0.98279	0.98278	0.98277	0.98276
0.0174	0.98275	0.98274	0.98273	0.98272	0.98271	0.98270	0.98269	0.98268	0.98267	0.98266
0.0175	0.98265	0.98264	0.98263	0.98262	0.98261	0.98260	0.98259	0.98258	0.98257	0.98256
0.0176	0.98255	0.98254	0.98253	0.98252	0.98251	0.98250	0.98250	0.98249	0.98248	0.98247
0.0177	0.98246	0.98245	0.98244	0.98243	0.98242	0.98241	0.98240	0.98239	0.98238	0.98237
0.0178	0.98236	0.98235	0.98234	0.98233	0.98232	0.98231	0.98230	0.98229	0.98228	0.98227
0.0179	0.98226	0.98225	0.98224	0.98223	0.98222	0.98221	0.98220	0.98219	0.98218	0.98217
0.0180	0.98216	0.98215	0.98214	0.98213	0.98212	0.98211	0.98210	0.98209	0.98208	0.98207
0.0181	0.98206	0.98205	0.98204	0.98203	0.98202	0.98201	0.98200	0.98199	0.98198	0.98197
0.0182	0.98196	0.98195	0.98194	0.98194	0.98193	0.98192	0.98191	0.98190	0.98189	0.98188
0.0183	0.98187	0.98186	0.98185	0.98184	0.98183	0.98182	0.98181	0.98180	0.98179	0.98178
0.0184	0.98177	0.98176	0.98175	0.98174	0.98173	0.98172	0.98171	0.98170	0.98169	0.98168
0.0185	0.98167	0.98166	0.98165	0.98164	0.98163	0.98162	0.98161	0.98160	0.98159	0.98158
0.0186	0.98157	0.98156	0.98155	0.98154	0.98153	0.98152	0.98151	0.98150	0.98149	0.98148
0.0187	0.98147	0.98146	0.98145	0.98144	0.98143	0.98142	0.98141	0.98141	0.98140	0.98139
0.0188	0.98138	0.98137	0.98136	0.98135	0.98134	0.98133	0.98132	0.98131	0.98130	0.98129
0.0189	0.98128	0.98127	0.98126	0.98125	0.98124	0.98123	0.98122	0.98121	0.98120	0.98119
0.0190	0.98118	0.98117	0.98116	0.98115	0.98114	0.98113	0.98112	0.98111	0.98110	0.98109
0.0191	0.98108	0.98107	0.98106	0.98105	0.98104	0.98103	0.98102	0.98101	0.98100	0.98099
0.0192	0.98098	0.98097	0.98096	0.98095	0.98094	0.98093	0.98092	0.98091	0.98090	0.98089
0.0193	0.98089	0.98088	0.98087	0.98086	0.98085	0.98084	0.98083	0.98082	0.98081	0.98080
0.0194	0.98079	0.98078	0.98077	0.98076	0.98075	0.98074	0.98073	0.98072	0.98071	0.98070
0.0195	0.98069	0.98068	0.98067	0.98066	0.98065	0.98064	0.98063	0.98062	0.98061	0.98060
0.0196	0.98059	0.98058	0.98057	0.98056	0.98055	0.98054	0.98053	0.98052	0.98051	0.98050
0.0197	0.98049	0.98048	0.98047	0.98046	0.98045	0.98044	0.98043	0.98042	0.98041	0.98040
0.0198	0.98039	0.98038	0.98038	0.98037	0.98036	0.98035	0.98034	0.98033	0.98032	0.98031
0.0199	0.98030	0.98029	0.98028	0.98027	0.98026	0.98025	0.98024	0.98023	0.98022	0.98021

Table H-1f

Exponential Values: exp(-x)

x	.00000	.00001	.00002	.00003	.00004	.00005	.00006	.00007	.00008	.00009
0.0200	0.98020	0.98019	0.98018	0.98017	0.98016	0.98015	0.98014	0.98013	0.98012	0.98011
0.0201	0.98010	0.98009	0.98008	0.98007	0.98006	0.98005	0.98004	0.98003	0.98002	0.98001
0.0202	0.98000	0.97999	0.97998	0.97997	0.97996	0.97995	0.97994	0.97993	0.97992	0.97991
0.0203	0.97990	0.97989	0.97989	0.97988	0.97987	0.97986	0.97985	0.97984	0.97983	0.97982
0.0204	0.97981	0.97980	0.97979	0.97978	0.97977	0.97976	0.97975	0.97974	0.97973	0.97972
0.0205	0.97971	0.97970	0.97969	0.97968	0.97967	0.97966	0.97965	0.97964	0.97963	0.97962
0.0206	0.97961	0.97960	0.97959	0.97958	0.97957	0.97956	0.97955	0.97954	0.97953	0.97952
0.0207	0.97951	0.97950	0.97949	0.97948	0.97947	0.97946	0.97945	0.97944	0.97943	0.97942
0.0208	0.97941	0.97941	0.97940	0.97939	0.97938	0.97937	0.97936	0.97935	0.97934	0.97933
0.0209	0.97932	0.97931	0.97930	0.97929	0.97928	0.97927	0.97926	0.97925	0.97924	0.97923
0.0210	0.97922	0.97921	0.97920	0.97919	0.97918	0.97917	0.97916	0.97915	0.97914	0.97913
0.0211	0.97912	0.97911	0.97910	0.97909	0.97908	0.97907	0.97906	0.97905	0.97904	0.97903
0.0212	0.97902	0.97901	0.97900	0.97899	0.97898	0.97897	0.97896	0.97895	0.97894	0.97894
0.0213	0.97893	0.97892	0.97891	0.97890	0.97889	0.97888	0.97887	0.97886	0.97885	0.97884
0.0214	0.97883	0.97882	0.97881	0.97880	0.97879	0.97878	0.97877	0.97876	0.97875	0.97874
0.0215	0.97873	0.97872	0.97871	0.97870	0.97869	0.97868	0.97867	0.97866	0.97865	0.97864
0.0216	0.97863	0.97862	0.97861	0.97860	0.97859	0.97858	0.97857	0.97856	0.97855	0.97854
0.0217	0.97853	0.97852	0.97851	0.97850	0.97849	0.97848	0.97848	0.97847	0.97846	0.97845
0.0218	0.97844	0.97843	0.97842	0.97841	0.97840	0.97839	[illegible]	[illegible]	0.97836	0.97835
0.0219	0.97834	0.97833	0.97832	0.97831	0.97830	0.97829	[illegible]	[illegible]	0.97826	0.97825
0.0220	0.97824	0.97823	0.97822	0.97821	0.97820	0.97819	[illegible]	[illegible]	0.97816	0.97815
0.0221	0.97814	0.97813	0.97812	0.97811	0.97810	0.97809	[illegible]	0.97807	0.97806	0.97805
0.0222	0.97804	0.97803	0.97803	0.97802	0.97801	0.97800	0.97799	0.97798	0.97797	0.97796
0.0223	0.97795	0.97794	0.97793	0.97792	0.97791	0.97790	0.97789	0.97788	0.97787	0.97786
0.0224	0.97785	0.97784	0.97783	0.97782	0.97781	0.97780	0.97779	0.97778	0.97777	0.97776
0.0225	0.97775	0.97774	0.97773	0.97772	0.97771	0.97770	0.97769	0.97768	0.97767	0.97766
0.0226	0.97765	0.97764	0.97763	0.97762	0.97761	0.97760	0.97759	0.97759	0.97758	0.97757
0.0227	0.97756	0.97755	0.97754	0.97753	0.97752	0.97751	0.97750	0.97749	0.97748	0.97747
0.0228	0.97746	0.97745	0.97744	0.97743	0.97742	0.97741	0.97740	0.97739	0.97738	0.97737
0.0229	0.97736	0.97735	0.97734	0.97733	0.97732	0.97731	0.97730	0.97729	0.97728	0.97727
0.0230	0.97726	0.97725	0.97724	0.97723	0.97722	0.97721	0.97720	0.97719	0.97718	0.97717
0.0231	0.97716	0.97715	0.97715	0.97714	0.97713	0.97712	0.97711	0.97710	0.97709	0.97708
0.0232	0.97707	0.97706	0.97705	0.97704	0.97703	0.97702	0.97701	0.97700	0.97699	0.97698
0.0233	0.97697	0.97696	0.97695	0.97694	0.97693	0.97692	0.97691	0.97690	0.97689	0.97688
0.0234	0.97687	0.97686	0.97685	0.97684	0.97683	0.97682	0.97681	0.97680	0.97679	0.97678
0.0235	0.97677	0.97676	0.97675	0.97674	0.97673	0.97673	0.97672	0.97671	0.97670	0.97669
0.0236	0.97668	0.97667	0.97666	0.97665	0.97664	0.97663	0.97662	0.97661	0.97660	0.97659
0.0237	0.97658	0.97657	0.97656	0.97655	0.97654	0.97653	0.97652	0.97651	0.97650	0.97649
0.0238	0.97648	0.97647	0.97646	0.97645	0.97644	0.97643	0.97642	0.97641	0.97640	0.97639
0.0239	0.97638	0.97637	0.97636	0.97635	0.97634	0.97633	0.97632	0.97631	0.97631	0.97630
0.0240	0.97629	0.97628	0.97627	0.97626	0.97625	0.97624	0.97623	0.97622	0.97621	0.97620
0.0241	0.97619	0.97618	0.97617	0.97616	0.97615	0.97614	0.97613	0.97612	0.97611	0.97610
0.0242	0.97609	0.97608	0.97607	0.97606	0.97605	0.97604	0.97603	0.97602	0.97601	0.97600
0.0243	0.97599	0.97598	0.97597	0.97596	0.97595	0.97594	0.97593	0.97592	0.97591	0.97591
0.0244	0.97590	0.97589	0.97588	0.97587	0.97586	0.97585	0.97584	0.97583	0.97582	0.97581
0.0245	0.97580	0.97579	0.97578	0.97577	0.97576	0.97575	0.97574	0.97573	0.97572	0.97571
0.0246	0.97570	0.97569	0.97568	0.97567	0.97566	0.97565	0.97564	0.97563	0.97562	0.97561
0.0247	0.97560	0.97559	0.97558	0.97557	0.97556	0.97555	0.97554	0.97553	0.97552	0.97551
0.0248	0.97550	0.97550	0.97549	0.97548	0.97547	0.97546	0.97545	0.97544	0.97543	0.97542
0.0249	0.97541	0.97540	0.97539	0.97538	0.97537	0.97536	0.97535	0.97534	0.97533	0.97532

Table H-1g

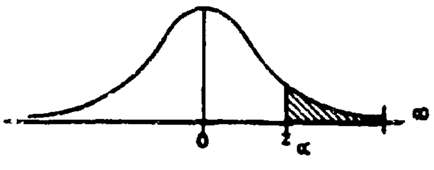
Exponential Values: exp(-x)

X	.00000	.00001	.00002	.00003	.00004	.00005	.00006	.00007	.00008	.00009
0.0250	0.97531	0.97530	0.97529	0.97528	0.97527	0.97526	0.97525	0.97524	0.97523	0.97522
0.0251	0.97521	0.97520	0.97519	0.97518	0.97517	0.97516	0.97515	0.97514	0.97513	0.97512
0.0252	0.97511	0.97511	0.97510	0.97509	0.97508	0.97507	0.97506	0.97505	0.97504	0.97503
0.0253	0.97502	0.97501	0.97500	0.97499	0.97498	0.97497	0.97496	0.97495	0.97494	0.97493
0.0254	0.97492	0.97491	0.97490	0.97489	0.97488	0.97487	0.97486	0.97485	0.97484	0.97483
0.0255	0.97482	0.97481	0.97480	0.97479	0.97478	0.97477	0.97476	0.97475	0.97474	0.97473
0.0256	0.97472	0.97472	0.97471	0.97470	0.97469	0.97468	0.97467	0.97466	0.97465	0.97464
0.0257	0.97463	0.97462	0.97461	0.97460	0.97459	0.97458	0.97457	0.97456	0.97455	0.97454
0.0258	0.97453	0.97452	0.97451	0.97450	0.97449	0.97448	0.97447	0.97446	0.97445	0.97444
0.0259	0.97443	0.97442	0.97441	0.97440	0.97439	0.97438	0.97437	0.97436	0.97435	0.97434
0.0260	0.97434	0.97433	0.97432	0.97431	0.97430	0.97429	0.97428	0.97427	0.97426	0.97425
0.0261	0.97424	0.97423	0.97422	0.97421	0.97420	0.97419	0.97418	0.97417	0.97416	0.97415
0.0262	0.97414	0.97413	0.97412	0.97411	0.97410	0.97409	0.97408	0.97407	0.97406	0.97405
0.0263	0.97404	0.97403	0.97402	0.97401	0.97400	0.97399	0.97398	0.97397	0.97396	0.97396
0.0264	0.97395	0.97394	0.97393	0.97392	0.97391	0.97390	0.97389	0.97388	0.97387	0.97386
0.0265	0.97385	0.97384	0.97383	0.97382	0.97381	0.97380	0.97379	0.97378	0.97377	0.97376
0.0266	0.97375	0.97374	0.97373	0.97372	0.97371	0.97370	0.97369	0.97368	0.97367	0.97366
0.0267	0.97365	0.97364	0.97363	0.97362	0.97361	0.97360	0.97359	0.97359	0.97358	0.97357
0.0268	0.97356	0.97355	0.97354	0.97353	0.97352	0.97351	0.97350	0.97349	0.97348	0.97347
0.0269	0.97346	0.97345	0.97344	0.97343	0.97342	0.97341	0.97340	0.97339	0.97338	0.97337
0.0270	0.97336	0.97335	0.97334	0.97333	0.97332	0.97331	0.97330	0.97329	0.97328	0.97327
0.0271	0.97326	0.97325	0.97324	0.97323	0.97322	0.97322	0.97321	0.97320	0.97319	0.97318
0.0272	0.97317	0.97316	0.97315	0.97314	0.97313	0.97312	0.97311	0.97310	0.97309	0.97308
0.0273	0.97307	0.97306	0.97305	0.97304	0.97303	0.97302	0.97301	0.97300	0.97299	0.97298
0.0274	0.97297	0.97296	0.97295	0.97294	0.97293	0.97292	0.97291	0.97290	0.97289	0.97288
0.0275	0.97287	0.97286	0.97286	0.97285	0.97284	0.97283	0.97282	0.97281	0.97280	0.97279
0.0276	0.97278	0.97277	0.97276	0.97275	0.97274	0.97273	0.97272	0.97271	0.97270	0.97269
0.0277	0.97268	0.97267	0.97266	0.97265	0.97264	0.97263	0.97262	0.97261	0.97260	0.97259
0.0278	0.97258	0.97257	0.97256	0.97255	0.97254	0.97253	0.97252	0.97251	0.97251	0.97250
0.0279	0.97249	0.97248	0.97247	0.97246	0.97245	0.97244	0.97243	0.97242	0.97241	0.97240
0.0280	0.97239	0.97238	0.97237	0.97236	0.97235	0.97234	0.97233	0.97232	0.97231	0.97230
0.0281	0.97229	0.97228	0.97227	0.97226	0.97225	0.97224	0.97223	0.97222	0.97221	0.97220
0.0282	0.97219	0.97218	0.97217	0.97216	0.97216	0.97215	0.97214	0.97213	0.97212	0.97211
0.0283	0.97210	0.97209	0.97208	0.97207	0.97206	0.97205	0.97204	0.97203	0.97202	0.97201
0.0284	0.97200	0.97199	0.97198	0.97197	0.97196	0.97195	0.97194	0.97193	0.97192	0.97191
0.0285	0.97190	0.97189	0.97188	0.97187	0.97186	0.97185	0.97184	0.97183	0.97182	0.97181
0.0286	0.97181	0.97180	0.97179	0.97178	0.97177	0.97176	0.97175	0.97174	0.97173	0.97172
0.0287	0.97171	0.97170	0.97169	0.97168	0.97167	0.97166	0.97165	0.97164	0.97163	0.97162
0.0288	0.97161	0.97160	0.97159	0.97158	0.97157	0.97156	0.97155	0.97154	0.97153	0.97152
0.0289	0.97151	0.97150	0.97149	0.97148	0.97147	0.97147	0.97146	0.97145	0.97144	0.97143
0.0290	0.97142	0.97141	0.97140	0.97139	0.97138	0.97137	0.97136	0.97135	0.97134	0.97133
0.0291	0.97132	0.97131	0.97130	0.97129	0.97128	0.97127	0.97126	0.97125	0.97124	0.97123
0.0292	0.97122	0.97121	0.97120	0.97119	0.97118	0.97117	0.97116	0.97115	0.97114	0.97113
0.0293	0.97113	0.97112	0.97111	0.97110	0.97109	0.97108	0.97107	0.97106	0.97105	0.97104
0.0294	0.97103	0.97102	0.97101	0.97100	0.97099	0.97098	0.97097	0.97096	0.97095	0.97094
0.0295	0.97093	0.97092	0.97091	0.97090	0.97089	0.97088	0.97087	0.97086	0.97085	0.97084
0.0296	0.97083	0.97082	0.97081	0.97080	0.97079	0.97079	0.97078	0.97077	0.97076	0.97075
0.0297	0.97074	0.97073	0.97072	0.97071	0.97070	0.97069	0.97068	0.97067	0.97066	0.97065
0.0298	0.97064	0.97063	0.97062	0.97061	0.97060	0.97059	0.97058	0.97057	0.97056	0.97055
0.0299	0.97054	0.97053	0.97052	0.97051	0.97050	0.97049	0.97048	0.97047	0.97046	0.97046

Table H-2

NORMAL DISTRIBUTION PROBABILITIES

$$\int_{z_\alpha}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz = \alpha$$



z_α	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43251	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23270	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15151	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

TABLE H-3a -- χ^2 DISTRIBUTION PROBABILITY VALUES

Values of χ^2_α

ν	$\chi^2_{.995}$	$\chi^2_{.99}$	$\chi^2_{.975}$	$\chi^2_{.95}$	$\chi^2_{.90}$	$\chi^2_{.80}$	$\chi^2_{.75}$	$\chi^2_{.70}$	ν
1	.0000393	.000157	.000982	.00393	.0158	.0642	.102	.148	1
2	.0100	.0201	.0506	.103	.211	.446	.575	.713	2
3	.0717	.115	.216	.352	.584	1.005	1.213	1.424	3
4	.207	.297	.484	.711	1.064	1.649	1.923	2.195	4
5	.412	.554	.831	1.145	1.610	2.343	2.675	3.000	5
6	.676	.872	1.237	1.635	2.204	3.070	3.455	3.828	6
7	.989	1.239	1.690	2.167	2.833	3.822	4.255	4.671	7
8	1.344	1.646	2.180	2.733	3.490	4.594	5.071	5.527	8
9	1.735	2.088	2.700	3.325	4.168	5.380	[illegible]	6.393	9
10	2.156	2.558	3.247	3.940	4.865	6.179	[illegible]	7.267	10
11	2.603	3.053	3.816	4.575	5.578	6.989	7.584	8.148	11
12	3.074	3.571	4.404	5.226	6.304	7.807	8.438	9.034	12
13	3.565	4.107	5.009	5.892	7.042	8.634	9.299	9.926	13
14	4.075	4.660	5.629	6.571	7.790	9.467	10.165	10.821	14
15	4.601	5.229	6.262	7.261	8.547	10.307	11.036	11.721	15
16	5.142	5.812	6.908	7.962	9.312	11.152	11.192	12.624	16
17	5.697	6.408	7.564	8.672	10.085	12.002	12.792	13.531	17
18	6.265	7.015	8.231	9.390	10.865	12.857	13.675	14.440	18
19	6.844	7.633	8.907	10.117	11.651	13.716	14.562	15.352	19
20	7.434	8.260	9.591	10.851	12.443	14.578	15.452	16.266	20
21	8.034	8.897	10.283	11.591	13.240	15.445	16.344	17.182	21
22	8.643	9.542	10.982	12.338	14.041	16.314	17.240	18.101	22
23	9.260	10.196	11.688	13.091	14.848	17.187	18.137	19.021	23
24	9.886	10.856	12.401	13.848	15.659	18.062	19.037	19.943	24
25	10.520	11.524	13.120	14.611	16.473	18.940	19.939	20.867	25
26	11.160	12.198	13.844	15.379	17.292	19.820	20.843	21.792	26
27	11.808	12.879	14.573	16.151	18.114	20.703	21.749	22.719	27
28	12.461	13.565	15.308	16.928	18.939	21.588	22.657	23.647	28
29	13.121	14.256	16.047	17.708	19.768	22.475	23.567	24.577	29
30	13.787	14.953	16.791	18.493	20.599	23.364	24.478	25.508	30
35	17.156	18.484	20.558	22.462	24.812	27.820	29.058	30.181	35
40	20.674	22.142	24.423	26.507	29.067	32.326	33.664	34.874	40
45	24.281	25.880	28.356	30.610	33.367	36.863	38.294	39.586	45
50	27.962	29.687	32.348	34.762	37.706	41.426	42.944	44.314	50
55	31.708	33.552	36.390	38.956	42.078	46.011	47.612	49.055	55
60	35.510	37.467	40.474	43.186	46.478	50.614	52.295	53.808	60
65	39.360	41.427	44.595	47.448	50.902	55.233	56.991	58.572	65
70	43.253	45.426	48.750	51.737	55.349	59.868	61.698	63.344	70
75	47.186	49.460	52.935	56.052	59.815	64.515	66.416	68.125	75
80	51.153	53.526	57.146	60.390	64.299	69.174	71.144	72.913	80
85	55.151	57.621	61.382	64.748	68.799	73.843	75.880	77.707	85
90	59.179	61.741	65.640	69.124	73.313	78.522	80.623	82.508	90
95	[illegible]	65.886	69.919	73.518	77.841	83.210	85.374	87.314	95
100	67.312	70.053	74.216	77.928	82.381	87.906	90.131	92.125	100
105	71.414	74.241	78.530	82.352	86.933	92.610	94.894	96.941	105
110	75.536	78.448	82.861	86.790	91.495	97.321	99.663	101.761	110
115	79.679	82.672	87.207	91.240	96.067	102.038	104.437	106.565	115
120	83.839	86.913	91.567	95.703	100.648	106.762	109.216	111.413	120

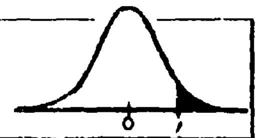
TABLE H-3b -- χ^2 DISTRIBUTION PROBABILITY VALUES
(continued)

Values of χ^2_α

ν	$\chi^2_{.50}$	$\chi^2_{.30}$	$\chi^2_{.25}$	$\chi^2_{.20}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	ν
1	.455	1.074	1.323	1.642	2.706	3.841	5.024	6.635	7.879	1
2	1.386	2.408	2.773	3.219	4.605	5.991	7.378	9.210	10.597	2
3	2.366	3.665	4.108	4.642	6.251	7.815	9.348	11.345	12.838	3
4	3.357	4.878	5.385	5.989	7.779	9.488	11.143	13.277	14.860	4
5	4.351	6.064	6.626	7.289	9.236	11.070	12.832	15.086	16.750	5
6	5.348	7.231	7.841	8.558	10.645	12.592	14.449	16.812	18.548	6
7	6.346	8.383	9.037	9.803	12.017	14.067	16.013	18.475	20.278	7
8	7.344	9.524	10.219	11.030	13.362	15.507	17.535	20.090	21.955	8
9	8.343	10.656	11.389	12.242	14.684	16.919	19.023	21.666	23.589	9
10	9.342	11.781	12.549	13.442	15.987	18.307	20.483	23.209	25.188	10
11	10.341	12.899	13.701	14.631	17.275	19.675	21.920	24.725	26.757	11
12	11.340	14.011	14.845	15.812	18.549	21.026	23.337	26.217	28.300	12
13	12.340	15.119	15.984	16.985	19.812	22.362	24.736	27.688	29.819	13
14	13.339	16.222	17.117	18.151	21.064	23.685	26.119	29.141	31.319	14
15	14.339	17.322	16.245	19.311	22.307	24.996	27.488	30.578	32.801	15
16	15.338	18.418	19.369	20.465	23.542	26.296	28.845	32.000	34.267	16
17	16.338	19.511	20.489	21.615	24.769	27.587	30.191	33.409	35.718	17
18	17.338	20.601	21.605	22.760	25.989	28.869	31.526	34.805	37.156	18
19	18.338	21.689	22.718	23.900	27.204	30.144	32.852	36.191	38.582	19
20	19.337	22.775	23.828	25.038	28.412	31.410	34.170	37.566	39.997	20
21	20.337	23.858	24.935	26.171	29.615	32.671	35.479	38.932	41.401	21
22	21.337	24.939	26.039	27.301	30.813	33.924	36.781	40.289	42.796	22
23	22.337	26.018	27.141	28.429	32.007	35.172	38.076	41.638	44.181	23
24	23.337	27.096	28.241	29.553	33.196	36.415	39.364	42.980	45.558	24
25	24.337	28.172	29.339	30.675	34.382	37.652	40.646	44.314	46.928	25
26	25.336	29.246	30.434	31.795	35.563	38.885	41.923	45.642	48.290	26
27	26.336	30.319	31.528	22.912	36.741	40.113	43.194	46.963	49.645	27
28	27.336	31.391	32.620	34.027	37.916	41.337	44.461	48.278	50.993	28
29	28.336	32.461	33.711	35.139	39.087	42.557	45.722	49.588	52.336	29
30	29.336	33.530	34.800	36.250	40.256	43.773	46.979	50.892	53.672	30
35	34.338	38.860	40.221	41.802	46.034	49.798	53.207	57.359	60.304	35
40	39.337	44.166	45.615	47.295	51.780	55.755	59.345	63.706	66.792	40
45	44.337	49.453	50.984	52.757	57.480	61.653	65.414	69.971	73.190	45
50	49.336	54.725	56.333	58.194	63.141	67.502	71.424	76.167	79.512	50
55	54.336	59.983	61.665	63.610	68.770	73.309	77.384	82.305	85.769	55
60	59.336	65.229	66.982	69.006	74.370	79.080	83.301	88.391	91.970	60
65	64.336	70.465	72.286	74.387	79.946	84.819	89.181	94.433	93.122	65
70	69.335	75.693	77.578	79.752	85.500	90.530	95.027	100.436	104.230	70
75	74.335	80.912	82.860	85.105	91.034	96.216	100.843	106.403	110.300	75
80	79.335	86.124	88.132	90.446	96.550	101.879	106.632	112.338	116.334	80
85	84.335	91.329	93.396	95.777	102.050	107.521	112.397	118.244	122.337	85
90	89.335	96.529	98.653	101.097	107.536	113.145	118.139	124.125	128.310	90
95	94.335	101.723	103.902	106.499	113.008	118.751	123.861	129.980	134.257	95
100	99.335	106.911	109.145	111.715	118.468	124.342	129.565	135.814	140.179	100
105	104.335	112.095	114.381	117.009	123.917	129.918	135.250	141.62[illegible]	146.078	105
110	109.335	117.275	119.612	112.299	129.355	135.480	140.920	147.421	151.956	110
115	114.335	122.451	124.838	127.581	134.752	141.030	146.574	153.197	157.814	115
120	119.335	127.623	130.059	132.858	140.201	146.568	152.215	158.956	163.654	120

Table H-4
t Distribution Probabilities

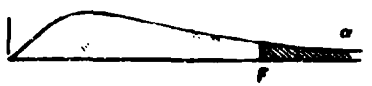
The first column lists the number of degrees of freedom (ν). The headings of the other columns give probabilities (P) for t to exceed numerically the entry value.



ν \ P	0.25	0.125	0.05	0.025	.00125	0.005	0.0025
1	1.00000	2.4142	6.3138	12.706	25.452	63.657	127.32
2	0.81650	1.6036	2.9200	4.3027	6.2053	9.9248	14.089
3	0.76489	1.4226	2.3534	3.1825	4.1765	5.8409	7.4533
4	0.74070	1.3444	2.1318	2.7764	3.4954	4.6041	5.5976
5	0.72669	1.3009	2.0150	2.5706	3.1634	4.0321	4.7733
6	0.71756	1.2733	1.9432	2.4469	2.9687	3.7074	4.3168
7	0.71114	1.2543	1.8946	2.3646	2.8412	3.4995	4.0293
8	0.70639	1.2403	1.8595	2.3060	2.7515	3.3554	3.8325
9	0.70272	1.2297	1.8331	2.2622	2.6850	3.2498	3.6897
10	0.69981	1.2213	1.8125	2.2281	2.6338	3.1693	3.5814
11	0.69745	1.2145	1.7959	2.2010	2.5931	3.1058	3.4966
12	0.69548	1.2089	1.7823	2.1788	2.5600	3.0545	3.4284
13	0.69384	1.2041	1.7709	2.1604	2.5326	3.0123	3.3725
14	0.69242	1.2001	1.7613	2.1448	2.5096	2.9768	3.3257
15	0.69120	1.1967	1.7530	2.1315	2.4899	2.9467	3.2860
16	0.69013	1.1937	1.7459	2.1199	2.4729	2.9208	3.2520
17	0.68919	1.1910	1.7396	2.1098	2.4581	2.8982	3.2225
18	0.68837	1.1887	1.7341	2.1009	2.4450	2.8784	3.1966
19	0.68763	1.1866	1.7291	2.0930	2.4334	2.8609	3.1737
20	0.68696	1.1848	1.7247	2.0860	2.4231	2.8453	3.1534
21	0.68635	1.1831	1.7207	2.0796	2.4138	2.8314	3.1352
22	0.68580	1.1816	1.7171	2.0739	2.4055	2.8188	3.1188
23	0.68531	1.1802	1.7139	2.0687	2.3979	2.8073	3.1040
24	0.68485	1.1789	1.7109	2.0639	2.3910	2.7969	3.0905
25	0.68443	1.1777	1.7081	2.0595	2.3846	2.7874	3.0782
26	0.68405	1.1766	1.7056	2.0555	2.3788	2.7787	3.0669
27	0.68370	1.1757	1.7033	2.0518	2.3734	2.7707	3.0565
28	0.68335	1.1748	1.7011	2.0484	2.3685	2.7633	3.0469
29	0.68304	1.1739	1.6991	2.0452	2.3638	2.7564	3.0380
30	0.68276	1.1731	1.6973	2.0423	2.3596	2.7500	3.0298
40	0.68066	1.1673	1.6839	2.0211	2.3289	2.7045	2.9712
60	0.67862	1.1616	1.6707	2.0003	2.2991	2.6603	2.9146
120	0.67656	1.1559	1.6577	1.9799	2.2699	2.6174	2.8599
∞	0.67449	1.1503	1.6449	1.9600	2.2414	2.5758	2.8070

Table H-5a

F DISTRIBUTION VALUES FOR $F_{.10,\nu_x,\nu_y}$



ν_x = degrees of freedom for numerator

ν_y = degrees of freedom for denominator

ν_y \ ν_x	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.05	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

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Table H-5b

F DISTRIBUTION VALUES FOR $F_{.05,\ \nu_x,\ \nu_y}$

ν_x = degrees of freedom for numerator

ν_y = degrees of freedom for denominator

$\nu_y \backslash \nu_x$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.50
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46	4.43	4.40	4.36
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77	3.74	3.70	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.30	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.90	2.86	2.83	2.79	2.75	2.71
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.40
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.30
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34	2.30	2.25	2.21
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	[illegible]	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99	1.95	1.90	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96	1.92	1.87	1.81
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.90	1.85	1.80	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.20	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22	2.18	2.10	2.03	1.94	1.90	1.85	1.81	1.75	1.70	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50	1.43	1.35	1.23
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1.00

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Table H-5c

F DISTRIBUTION VALUES FOR $F_{.025,\ \nu_x,\ \nu_y}$

ν_x = degrees of freedom for numerator

ν_y = degrees of freedom for denominator

ν_y \ ν_x	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00

Table H-5d

F DISTRIBUTION VALUES FOR $F_{.01,\ \nu_x,\ \nu_y}$

ν_x = degrees of freedom for numerator

ν_y = degrees of freedom for denominator

$\nu_y \backslash \nu_x$	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	4052	4999.5	5403	5625	5764	5859	5928	5982	6022	6056	6106	6157	6209	6235	6261	6287	6313	6339	6366
2	98.50	99.00	99.17	99.25	99.30	99.33	99.36	99.37	99.39	99.40	99.42	99.43	99.45	99.46	99.47	99.47	99.48	99.49	99.50
3	34.12	30.82	29.46	28.71	28.24	27.91	27.67	27.49	27.35	27.23	27.05	26.87	26.69	26.60	26.50	26.41	26.32	26.22	26.13
4	21.20	18.00	16.69	15.98	15.52	15.21	14.98	14.80	14.66	14.55	14.37	14.20	14.02	13.93	13.84	13.75	13.65	13.56	13.46
5	16.26	13.27	12.06	11.39	10.97	10.67	10.46	10.29	10.16	10.05	9.89	9.72	9.55	9.47	9.38	9.29	9.20	9.11	9.02
6	13.75	10.92	9.78	9.15	8.75	8.47	8.26	8.10	7.98	7.87	7.72	7.56	7.40	7.31	7.23	7.14	7.06	6.97	6.88
7	12.25	9.55	8.45	7.85	7.46	7.19	6.99	6.84	6.72	6.62	6.47	6.31	6.16	6.07	5.99	5.91	5.82	5.74	5.65
8	11.26	8.65	7.59	7.01	6.63	6.37	6.18	6.03	5.91	5.81	5.67	5.52	5.36	5.28	5.20	5.12	5.03	4.95	4.86
9	10.56	8.02	6.99	6.42	6.06	5.80	5.61	5.47	5.35	5.26	5.11	4.96	4.81	4.73	4.65	4.57	4.48	4.40	4.31
10	10.04	7.56	6.55	5.99	5.64	5.39	5.20	5.06	4.94	4.85	4.71	4.56	4.41	4.33	4.25	4.17	4.08	4.00	3.91
11	9.65	7.21	6.22	5.67	5.32	5.07	4.89	4.74	4.63	4.54	4.40	4.25	4.10	4.02	3.94	3.86	3.78	3.69	3.60
12	9.33	6.93	5.95	5.41	5.06	4.82	4.64	4.50	4.39	4.30	4.16	4.01	3.86	3.78	3.70	3.62	3.54	3.45	3.36
13	9.07	6.70	5.74	5.21	4.86	4.62	4.44	4.30	4.19	4.10	3.96	3.82	3.66	3.59	3.51	3.43	3.34	3.25	3.17
14	8.86	6.51	5.56	5.04	4.69	4.46	4.28	4.14	4.03	3.94	3.80	3.66	3.51	3.43	3.35	3.27	3.18	3.09	3.00
15	8.68	6.36	5.42	4.89	4.56	4.32	4.14	4.00	3.89	3.80	3.67	3.52	3.37	3.29	3.21	3.13	3.05	2.96	2.87
16	8.53	6.23	5.29	4.77	4.44	4.20	4.03	3.89	3.78	3.69	3.55	3.41	3.26	3.18	3.10	3.02	2.93	2.84	2.75
17	8.40	6.11	5.18	4.67	4.34	4.10	3.93	3.79	3.68	3.59	3.46	3.31	3.16	3.08	3.00	2.92	2.83	2.75	2.65
18	8.29	6.01	5.09	4.58	4.25	4.01	3.84	3.71	3.60	3.51	3.37	3.23	3.08	3.00	2.92	2.84	2.75	2.66	2.57
19	8.18	5.93	5.01	4.50	4.17	3.94	3.77	3.63	3.52	3.43	3.30	3.15	3.00	2.92	2.84	2.76	2.67	2.58	2.49
20	8.10	5.85	4.94	4.43	4.10	3.87	3.70	3.56	3.46	3.37	3.23	3.09	2.94	2.86	2.78	2.69	2.61	2.52	2.42
21	8.02	5.78	4.87	4.37	4.04	3.81	3.64	3.51	3.40	3.31	3.17	3.03	2.88	2.80	2.72	2.64	2.55	2.46	2.36
22	7.95	5.72	4.82	4.31	3.99	3.76	3.59	3.45	3.35	3.26	3.12	2.98	2.83	2.75	2.67	2.58	2.50	2.40	2.31
23	7.88	5.66	4.76	4.26	3.94	3.71	3.54	3.41	3.30	3.21	3.07	2.93	2.78	2.70	2.62	2.54	2.45	2.35	2.26
24	7.82	5.61	4.72	4.22	3.90	3.67	3.50	3.36	3.26	3.17	3.03	2.89	2.74	2.66	2.58	2.49	2.40	2.31	2.21
25	7.77	5.57	4.68	4.18	3.85	3.63	3.46	3.32	3.22	3.13	2.99	2.85	2.70	2.62	2.54	2.45	2.36	2.27	2.17
26	7.72	5.53	4.64	4.14	3.82	3.59	3.42	3.29	3.18	3.09	2.96	2.81	2.66	2.58	2.50	2.42	2.33	2.23	2.13
27	7.68	5.49	4.60	4.11	3.78	3.56	3.39	3.26	3.15	3.06	2.93	2.78	2.63	2.55	2.47	2.38	2.29	2.20	2.10
28	7.64	5.45	4.57	4.07	3.75	3.53	3.36	3.23	3.12	3.03	2.90	2.75	2.60	2.52	2.44	2.35	2.26	2.17	2.06
29	7.60	5.42	4.54	4.04	3.73	3.50	3.33	3.20	3.09	3.00	2.87	2.73	2.57	2.49	2.41	2.33	2.23	2.14	2.03
30	7.56	5.39	4.51	4.02	3.70	3.47	3.30	3.17	3.07	2.98	2.84	2.70	2.55	2.47	2.39	2.30	2.21	2.11	2.01
40	7.31	5.18	4.31	3.83	3.51	3.29	3.12	2.99	2.89	2.80	2.66	2.52	2.37	2.29	2.20	2.11	2.02	1.92	1.80
60	7.08	4.98	4.13	3.65	3.34	3.12	2.95	2.82	2.72	2.63	2.50	2.35	2.20	2.12	2.03	1.94	1.84	1.73	1.60
120	6.85	4.79	3.95	3.48	3.17	2.96	2.79	2.66	2.56	2.47	2.34	2.19	2.03	1.95	1.86	1.76	1.66	1.53	1.38
∞	6.63	4.61	3.78	3.32	3.02	2.80	2.64	2.51	2.41	2.32	2.18	2.04	1.88	1.79	1.70	1.59	1.47	1.32	1.00

Table H-6

Kolmogorov-Smirnov Critical Values (d_α)

Sample size (n)	Level of significance (α)				
	0.20	0.15	0.10	0.05	0.01
1	0.900	0.925	0.950	0.975	0.995
2	0.684	0.726	0.776	0.842	0.929
3	0.565	0.597	0.642	0.708	0.828
4	0.494	0.525	0.564	0.624	0.733
5	0.446	0.474	0.510	0.565	0.669
6	0.410	0.436	0.470	0.521	0.618
7	0.381	0.405	0.438	0.486	0.577
8	0.358	0.381	0.411	0.457	0.543
9	0.339	0.360	0.388	0.432	0.514
10	0.322	0.342	0.368	0.410	0.490
11	0.307	0.326	0.352	0.391	0.468
12	0.295	0.313	0.338	0.375	0.450
13	0.284	0.302	0.325	0.361	0.433
14	0.274	0.292	0.314	0.349	0.418
15	0.266	0.283	0.304	0.338	0.404
16	0.258	0.274	0.295	0.328	0.392
17	0.250	0.266	0.286	0.318	0.381
18	0.244	0.259	0.278	0.309	0.371
19	0.237	0.252	0.272	0.301	0.363
20	0.231	0.246	0.264	0.294	0.356
25	0.21	0.22	0.24	0.27	0.32
30	0.19	0.20	0.22	0.24	0.29
35	0.18	0.19	0.21	0.23	0.27
over 35	$\frac{1.07}{\sqrt{n}}$	$\frac{1.14}{\sqrt{n}}$	$\frac{1.22}{\sqrt{n}}$	$\frac{1.36}{\sqrt{n}}$	$\frac{1.63}{\sqrt{n}}$

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Table H-7a

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit.

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
n = 1				n = 2				n = 3			
0	.100	.050	.010	0	.316	.224	.100	0	.464	.368	.215+
1	0	0	0	1	.051	.025+	.005+	1	.196	.135+	.059
				2	0	0	0	2	.035	.017	.003
								3	0	0	0
n = 4				n = 5				n = 6			
0	.562	.473	.316	0	.631	.549	.398	0	.681	.607	.464
1	.320	.249	.141	1	.416	.343	.222	1	.490	.418	.294
2	.143	.098	.042	2	.247	.189	.106	2	.333	.271	.173
3	.026	.013	.003	3	.112	.076	.033	3	.201	.153	.085
4	0	0	0	4	.021	.010	.002	4	.093	.063	.027
				5	0	0	0	5	.017	.009	.002
								6	0	0	[illegible]
n = 7				n = 8							
0	.720	.652	.518	0	.750	.688	.562	0	.774	.717	.599
1	.547	.479	.357	1	.594	.529	.410	1	[illegible]	.571	.456
2	.404	.341	.256	2	.462	.400	.293	2	.510	.450	.344
3	.279	.225+	.142	3	.345	.289	.196	3	.401	.345	.250
4	.170	.129	.071	4	.240	.193	.121	4	.301	.251	.171
5	.079	.053	.023	5	.147	.111	.051	5	.210	.169	.105+
6	.015+	.007	.001	6	.069	.046	.020	6	.129	.098	.053
7	0	0	0	7	.013	.006	.001	7	.061	.041	.017
				8	0	0	0	8	.012	.006	.001
								9	0	0	0

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
	n = 10				n = 11				n = 12		
0	.794	.741	.631	0	.811	.762	.658	0	.825+	.779	.681
1	.663	.606	.496	1	.690	.636	.530	1	.713	.661	.560
2	.550	.493	.388	2	.585-	.530	.428	2	.614	.562	.463
3	.448	.393	.297	3	.489	.436	.340	3	.525-	.473	.378
4	.354	.304	.218	4	.401	.350	.262	4	.441	.391	.302
5	.267	.222	.150	5	.318	.271	.194	5	.362	.315+	.235-
6	.188	.150	.093	6	.241	.200	.134	6	.288	.245+	.175-
7	.116	.087	.048	7	.169	.135+	.650	7	.219	.181	.121
8	.055-	.037	.016	8	.105-	.079	.564	8	.154	.123	.076
9	.010	.005	.001	9	.049	.033	.014	9	.096	.072	.039
10	0	0	0	10	.010	.005	.001	10	.045+	.030	.013
				11	0	0	0	11	.009	.004	.001
								12	0	0	0
	n = 13				n = 14				n = 15		
0	.838	.794	.702	0	.848	.807	.720	0	.858	.819	.736
1	.732	.684	.587	1	.749	.703	.611	1	.764	.721	.632
2	.640	.590	.494	2	.663	.615	.522	2	.683	.637	.547
3	.556	.505+	.412	3	.583	.534	.443	3	.607	.560	.471
4	.477	.427	.339	4	.508	.460	.373	4	.536	.489	.403
5	.402	.355-	.273	5	.437	.390	.308	5	.468	.423	.340
6	.331	.287	.213	6	.369	.325+	.249	6	.404	.360	.282
7	.264	.224	.159	7	.305-	.264	.195-	7	.342	.300	.229
8	.201	.166	.111	8	.243	.206	.146	8	.282	.244	.179
9	.142	.113	.069	9	.185+	.153	.102	9	.226	.191	.135-
10	.088	.066	.036	10	.131	.104	.064	10	.172	.142	.094
11	.042	.028	.012	11	.081	.061	.033	11	.122	.097	.059
12	.008	.004	.001	12	.039	.026	.011	12	.076	.057	.031
13	0	0	0	13	.007	.004	.001	13	.036	.024	.010
				14	0	0	0	14	.007	.003	.001
								15	0	0	0

Table H-7c

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit.

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
n = 16				n = 17				n = 18			
0	.866	.829	.750	0	.873	.838	.763	0	.880	.847	.774
1	.778	.736	.651	1	.790	.750	.668	1	.801	.762	.684
2	.700	.656	.570	2	.716	.674	.590	2	.731	.690	.609
3	.629	.583	.497	3	.648	.604	.520	3	.666	.623	.542
4	.561	.516	.431	4	.584	.539	.457	4	.604	.561	.480
5	.496	.452	.370	5	.522	.478	.397	5	.545	.502	.423
6	.435-	.391	.313	6	.463	.420	.342	6	.488	.446	.369
7	.375+	.333	.261	7	.406	.364	.291	7	.433	.392	.319
8	.318	.279	.212	8	.350	.311	.242	8	.380	.341	.271
9	.263	.227	.166	9	.297	.260	.197	9	.329	.291	.226
10	.210	.178	.125+	10	.246	.212	.155+	10	.279	.244	.184
11	.161	.132	.088	11	.197	.166	.117	11	.231	.199	.145+
12	.114	.090	.055+	12	.151	.124	.082	12	.185+	.156	.110
13	.071	.053	.029	13	.107	.085	.052	13	.142	.116	.077
14	.034	.023	.010	14	.067	.050	.027	14	.101	.080	.049
15	.007	.003	.001	15	.032	.021	.009	15	.063	.047	.025+
16	0	0	0	16	.006	.003	.001	16	.030	.020	.008
				17	0	0	0	17	.006	.003	.001
								18	0	0	0

Table H-7d

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit.

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
n = 19				n = 20				n = 21			
0	.886	.854	.780	0	.891	.861	.793	0	.896	.867	.803
1	.810	.774	.698	1	.819	.784	.711	1	.827	.793	.723
2	.743	.704	.626	2	.755+	.717	.642	2	.766	.729	.656
3	.681	.641	.561	3	.696	.656	.579	3	.709	.671	.596
4	.622	.581	.502	4	.639	.599	.522	4	.655	.616	.540
5	.566	.524	.446	5	.585+	.544	.468	5	.603	.563	.488
6	.511	.470	.394	6	.533	.492	.417	6	.552	.513	.439
7	.459	.418	.345-	7	.482	.442	.369	7	.503	.464	.392
8	.408	.368	.298	8	.433	.394	.323	8	.456	.417	.347
9	.358	.320	.254	9	.385-	.347	.280	9	.410	.372	.305-
10	.310	.274	.212	10	.338	.302	.239	10	.364	.328	.264
11	.263	.230	.173	11	.293	.259	.200	11	.321	.286	.226
12	.218	.188	.137	12	.249	.217	.163	12	.278	.245-	.189
13	.175+	.147	.103	13	.207	.177	.129	13	.236	.206	.155-
14	.134	.110	.073	14	.166	.140	.098	14	.196	.168	.122
15	.095+	.075+	.046	15	.127	.104	.069	15	.158	.132	.092
16	.059	.044	.024	16	.090	.071	.044	16	.121	.099	.065+
17	.028	.019	.008	17	.056	.042	.023	17	.086	.068	.041
18	.006	.003	.001	18	.027	.018	.008	18	.054	.040	.022
19	0	0	0	19	.005+	.003	.001	19	.026	.017	.007
				20	0	0	0	20	.005+	.002	0
								21	0	0	0

Table H-7e

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit.

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
n = 22				n = 23				n = 24			
0	.901	.873	.811	0	.905-	.878	.819	0	.909	.883	.825+
1	.834	.802	.734	1	.841	.810	.744	1	.847	.817	.754
2	.776	.741	.670	2	.785-	.751	.682	2	.793	.760	.693
3	.721	.684	.611	3	.732	.696	.626	3	.742	.708	.639
4	.669	.631	.557	4	.682	.645+	.573	4	.694	.658	.588
5	.619	.580	.507	5	.634	.596	.524	5	.648	.611	.540
6	.570	.532	.459	6	.587	.549	.478	6	.602	.565+	.495+
7	.523	.485-	.413	7	.541	.504	.433	7	.558	.521	.452
8	.477	.439	.370	8	.497	.460	.391	8	.516	.479	.410
9	.432	.395+	.328	9	.454	.417	.350	9	.474	.437	.370
10	.389	.353	.288	10	.411	.375+	.311	10	.433	.397	.332
11	.346	.311	.250	11	.370	.335+	.273	11	.392	.358	.295+
12	.305-	.271	.214	12	.330	.296	.237	12	.353	.319	.260
13	.264	.233	.179	13	.290	.258	.203	13	.315	.282	.226
14	.225-	.196	.147	14	.252	.222	.171	14	.277	.246	.194
15	.187	.160	.116	15	.214	.186	.140	15	.241	.212	.163
16	.150	.126	.088	16	.178	.152	.111	16	.205-	.178	.133
17	.115-	.094	.062	17	.143	.120	.084	17	.170	.146	.106
18	.082	.065-	.039	18	.110	.090	.059	18	.137	.115-	.080
19	.051	.038	.021	19	.078	.062	.038	19	.105	.086	.057
20	.024	.016	.007	20	.049	.037	.020	20	.075	.059	.036
21	.005-	.002	0	21	.023	.016	.007	21	.047	.035-	.019
22	0	0	0	22	.005-	.002	0	22	.022	.015+	.006
				23	0	0	0	23	.004	.002	0
								24	0	0	0

Table H-7f

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit.

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
n = 25				n = 26				n = 27			
0	.912	.887	.832	0	.915+	.891	.838	0	.918	.895-	.843
1	.853	.824	.763	1	.858	.830	.771	1	.863	.836	.776
2	.801	.769	.704	2	.808	.777	.714	2	.815-	.765-	.723
3	.752	.718	.651	3	.761	.728	.663	3	.769	.737	.674
4	.705+	.670	.602	4	.716	.682	.615+	4	.725+	.692	.627
5	.660	.625-	.556	5	.672	.637	.570	5	.683	.649	.585
6	.617	.580	.512	6	.630	.595-	.527	6	.642	.608	.542
7	.574	.538	.469	7	.589	.553	.486	7	.603	.568	.502
8	.533	.496	.429	8	.549	.513	.446	8	.564	.529	.463
9	.492	.456	.390	9	.509	.474	.408	9	.525+	.491	.426
10	.452	.417	.352	10	.471	.436	372	10	.488	.453	.390
11	.413	.379	.316	11	.433	.398	.336	11	.451	.417	.355-
12	.375+	.341	.281	12	.396	.362	.302	12	.415+	.382	.321
13	.338	.305+	.248	13	.359	.327	.269	13	.380	.347	.289
14	.301	.270	.216	14	.324	.292	.237	14	.345-	.313	.257
15	.265+	.236	.185-	15	.289	.258	.206	15	.311	.280	.227
16	.230	.202	.155+	16	.254	.226	.177	16	.277	.248	.198
17	.196	.170	.127	17	.221	.194	.149	17	.244	.217	.169
18	.163	.139	.101	18	.188	.163	.122	18	.212	.186	.143
19	.131	.110	.077	19	.157	.134	.097	19	.181	.157	.117
20	.101	.082	.054	20	.126	.106	.073	20	.151	.129	.093
21	.072	.057	.034	21	.097	.079	.052	21	.121	.101	.070
22	.045-	.034	.018	22	.069	.054	.033	22	.093	.076	.050
23	.021	.014	.006	23	.043	.032	.017	23	.066	.052	.032
24	.004	.002	0	24	.021	.014	.006	24	.042	.031	.017
25	0	0	0	25	.004	.002	0	25	.020	.013	.006
				26	0	0	0	26	.004	.002	0
								27	0	0	0

Table H-7g

CONFIDENCE LIMITS FOR A PROPORTION (ONE-SIDED)

If the observed proportion is d/n, enter the table with n and γ for a lower one-sided limit.

d	90%	95%	99%	d	90%	95%	99%	d	90%	95%	99%
	n = 28				n = 29				n = 30		
0	.921	.899	.848	0	.924	.902	.853	0	.926	.905-	.858
1	.868	.841	.785+	1	.872	.847	.792	1	.876	.851	.798
2	.821	.792	.732	2	.827	.798	.740	2	.832	.805-	.748
3	.777	.746	.684	3	.784	.754	.693	3	.791	.761	.702
4	.735-	.702	.639	4	.743	.712	.650	4	.751	.720	.660
5	.694	.661	.596	5	.703	.671	.608	5	.713	.681	.619
6	.654	.620	.555+	6	.665+	.632	.568	6	.675+	.643	.580
7	.615+	.581	.516	7	.628	.594	.530	7	.639	.606	.543
8	.578	.543	.479	8	.591	.557	.493	8	.603	.570	.507
9	.541	.506	.442	9	.555-	.521	.458	9	.568	.535-	.473
10	.504	.470	.407	10	.519	.486	.423	10	.534	.501	.439
11	.468	.435-	.373	11	.485-	.451	.390	11	.500	.467	.406
12	.433	.400	.340	12	.450	.417	.357	12	.467	.434	.374
13	.399	.366	.308	13	.417	.384	.326	13	.434	.402	.343
14	.365-	.333	.277	14	.384	.352	.295+	14	.401	.370	.313
15	.331	.301	.247	15	.351	.320	.266	15	.370	.339	.284
16	.299	.269	.218	16	.319	.289	.237	16	.338	.308	.256
17	.267	.238	.190	17	.288	.259	.209	17	.308	.279	.228
18	.235+	.208	.163	18	.257	.229	.182	18	.277	.250	.201
19	.204	.179	.137	19	.226	.200	.157	19	.248	.221	.176
20	.174	.151	.112	20	.197	.172	.132	20	.218	.193	.151
21	.145-	.124	.089	21	.168	.145+	.108	21	.190	.166	.127
22	.117	.098	.068	22	.140	.119	.086	22	.162	.140	.104
23	.089	.073	.048	23	.112	.094	.065+	23	.135-	.115-	.083
24	.064	.050	.031	24	.086	.070	.046	24	.109	.091	.063
25	.040	.030	.016	25	.062	.049	.030	25	.083	.068	.045-
26	.019	.013	.005+	26	.039	.029	.015+	26	.059	.047	.028
27	.004	.002	0	27	.018	.012	.005+	27	.037	.028	.015
28	0	0	0	28	.004	.002	0	28	.018	.012	.005+
				29	0	0	0	29	.004	.002	0
								30	0	0	0

Table H-8a

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

The observed proportion in a random sample is d/n

d	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	d
n = 1													
0	[illegible]	.900	0	.925	0	.950	0	.975	0	.990	0	.995	0
1	[illegible]	1	[illegible]	1	.050	1	.025	1	[illegible]	1	.005	1	1
n = 2													
0	[illegible]	.684	[illegible]	[illegible]	0	[illegible]	[illegible]	.842	0	.900	0	.929	0
1	[illegible]	.949	[illegible]	.962	.025+	.975-	.013	.987	.005+	.995-	.003	.997	1
2	[illegible]	1	[illegible]	1	.224	1	.158	1	.100	1	.071	1	2
n = 3													
0	[illegible]	[illegible]	[illegible]	.576	0	.632	0	.708	0	.785-	0	.829	0
1	[illegible]	[illegible]	.026	[illegible]	.017	.865-	.008	.906	.003	.941	.002	.959	1
2	[illegible]	[illegible]	[illegible]	.974	.135+	.983	.094	.992	.059	.997	.041	.998	2
3	[illegible]	1	.422	1	.368	1	.292	1	.215+	1	.171	1	3
n = 4													
0	[illegible]	[illegible]	0	.477	0	.527	0	.602	0	.684	0	.734	0
1	[illegible]	[illegible]	[illegible]	.712	.013	.751	.006	.806	.003	.859	.001	[illegible]	1
2	[illegible]	[illegible]	.122	.878	.098	.902	.068	.932	.042	.958	.029	.971	2
3	[illegible]	[illegible]	[illegible]	[illegible]	.249	.987	.194	.994	.141	.997	.111	.999	3
4	[illegible]	1	.523	1	.473	1	[illegible]	1	.316	1	.266	1	4
n = 5													
0	[illegible]	[illegible]	0	.404	0	.451	0	.522	0	.602	0	.653	0
1	[illegible]	[illegible]	.015+	.516	.010	.657	.005+	.716	.002	.778	.001	[illegible]	1
2	[illegible]	[illegible]	[illegible]	.779	.076	.911	.053	.853	.033	.894	.023	.917	2
3	.247	[illegible]	.221	.904	.189	.924	.147	.947	.106	.967	.083	.977	3
4	[illegible]	[illegible]	[illegible]	.995-	.343	.990	.284	.995-	.222	.998	.165+	.999	4
5	[illegible]	1	.595	1	.549	1	.478	1	.398	1	.347	1	5
n = 6													
0	[illegible]	[illegible]	[illegible]	.351	0	.393	0	.459	0	.536	0	[illegible]	0
1	[illegible]	[illegible]	.013	.542	.009	.582	.004	.641	.002	.706	[illegible]	[illegible]	1
2	[illegible]	[illegible]	.079	.694	.063	.729	.043	.777	.027	.827	.019	[illegible]	2
3	[illegible]	[illegible]	.179	[illegible]	.153	.847	.118	[illegible]	.085-	.915+	[illegible]	[illegible]	3
4	[illegible]	[illegible]	.306	.921	.271	.937	.223	.957	.173	.973	.144	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	.418	.991	.359	.996	.294	.998	.254	[illegible]	5
6	[illegible]	1	[illegible]	1	.607	1	.541	1	[illegible]	1	[illegible]	1	6
n = 7													
0	[illegible]	[illegible]	0	.309	0	[illegible]	0	.410	0	[illegible]	0	[illegible]	0
1	[illegible]	[illegible]	.011	.432	.007	.521	.004	.579	.001	.643	.001	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	.053	[illegible]	[illegible]	.710	.023	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	.745+	.129	.775-	[illegible]	.816	.071	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	.849	.225+	[illegible]	[illegible]	.901	.142	[illegible]	[illegible]	.945+	4
5	[illegible]	[illegible]	[illegible]	.933	[illegible]	[illegible]	[illegible]	.963	.236	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	.479	.993	.421	.996	.357	[illegible]	[illegible]	.999	6
7	[illegible]	1	[illegible]	1	[illegible]	1	.590	1	[illegible]	1	[illegible]	1	7
n = 8													
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.312	0	.369	[illegible]	[illegible]	0	[illegible]	0
1	.013	[illegible]	[illegible]	[illegible]	[illegible]	.471	.003	.527	[illegible]	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.600	[illegible]	.651	[illegible]	[illegible]	[illegible]	.742	2
3	.147	[illegible]	[illegible]	[illegible]	[illegible]	.721	[illegible]	.755+	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	.193	[illegible]	.157	.843	.121	[illegible]	.100	[illegible]	4
5	.345-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.939	[illegible]	.953	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	.594	[illegible]	[illegible]	[illegible]	.529	[illegible]	.473	.997	[illegible]	[illegible]	[illegible]	.999	7
8	[illegible]	1	[illegible]	1	[illegible]	1	.631	1	.562	1	.516	1	8
n = 9													
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.445-	0
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.555-	1
2	.061	[illegible]	[illegible]	.516	[illegible]	[illegible]	[illegible]	[illegible]	.017	[illegible]	.012	.693	2
3	.129	[illegible]	[illegible]	[illegible]	[illegible]	.655+	.075-	[illegible]	[illegible]	[illegible]	[illegible]	.781	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.137	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.219	[illegible]	6
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.972	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	9

Table H-8b

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

d	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	d
n = 10													
0	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	.369	0	.411	0
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.445+	.001	.504	.001	.544	1
2	.055-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.025+	.556	.010	[illegible]	.011	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	.067	[illegible]	[illegible]	.652	.044	.733	.037	.735+	3
4	[illegible]	.646	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.093	[illegible]	.077	.809	4
5	.257	[illegible]	.247	.753	[illegible]	[illegible]	[illegible]	[illegible]	.150	[illegible]	.129	.872	5
6	.354	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.213	[illegible]	.191	.923	6
7	[illegible]	[illegible]	[illegible]	[illegible]	.393	.913	.348	.933	.297	[illegible]	.265-	.963	7
8	[illegible]	[illegible]	.525+	[illegible]	.493	.953	.444	.975-	[illegible]	.984	[illegible]	.989	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.945-	.555-	.997	.495	.999	[illegible]	.999	9
10	[illegible]	1	.772	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	10
n = 11													
0	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	.352	0
1	[illegible]	[illegible]	[illegible]	.333	[illegible]	.354	[illegible]	[illegible]	.001	[illegible]	0	.509	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.014	.572	.010	[illegible]	2
3	.105-	.511	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.610	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	.154	[illegible]	.135+	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.114	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	10
11	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	11
n = 12													
0	0	.175+	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	.030	[illegible]	[illegible]	[illegible]	[illegible]	.537	[illegible]	.573	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.527	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.791	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	11
12	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	12
n = 13													
0	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	.335-	0
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.541	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.139	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	[illegible]	[illegible]	.311	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	.799	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	12
13	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	13
n = 14													
0	0	.15	0	.169	0	[illegible]	0	[illegible]	0	[illegible]	0	.315+	0
1	[illegible]	.251	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1
2	[illegible]	.337	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	.171	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	.243	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.127	[illegible]	6
7	.305-	.595+	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	.437	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	13
14	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	14

Table H-8c

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

d	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	d
						n = 15							
0	0	.142	0	.159	0	.181	0	.218	0	.264	0	.299	0
1	.007	.206	.005+	.254	.003	.279	.002	.319	.001	.368	0	.402	1
2	.036	.317	[illegible]	.307	.024	.363	.017	.405-	.010	.453	.007	.496	2
3	.075	.393	[illegible]	.413	[illegible]	.440	.043	.481	.031	.529	.024	.561	3
4	.122	.462	.113	.474	.097	.511	.078	.551	.059	.597	.049	.627	4
5	[illegible]	.538	[illegible]	.537	[illegible]	.577	.115	.615	.094	.660	[illegible]	[illegible]	5
6	[illegible]	[illegible]	.217	[illegible]	.191	.640	.163	.677	.135-	.713	.117	.744	6
7	.257	.559	.265+	[illegible]	.244	[illegible]	.213	.734	.179	.771	.159	.795-	7
8	.340	.71	.323	.735-	.300	.756	.266	.787	.229	.821	.205+	.841	8
9	.424	.772	.386	.789	.360	.809	.323	.837	.282	.865+	[illegible]	[illegible]	9
10	.509	[illegible]	[illegible]	[illegible]	.423	[illegible]	.384	[illegible]	.340	.906	.312	.920	10
11	.538	.878	.515	.896	.489	.903	.449	.922	.403	.941	.373	.951	11
12	.597	.926	.577	.933	.560	.943	.519	.957	.471	.969	.439	.975	12
13	.653	[illegible]	.663	.977	.637	.976	.595+	.983	.547	.990	.514	.993	13
14	.764	.973	.745	.995+	.721	.997	.681	.998	.632	.999	.599	1	14
15	.858	1	.841	1	.819	1	[illegible]	1	.736	1	.702	1	15
						n = 16							
0	0	[illegible]	0	.149	0	.171	0	.206	0	.250	0	.282	0
1	.007	[illegible]	[illegible]	[illegible]	.003	.264	.002	.362	.001	.349	0	.381	1
2	.034	[illegible]	[illegible]	[illegible]	.023	.344	.015	.383	.010	.430	.007	[illegible]	2
3	.071	.371	.063	.391	.053	.417	.040	.456	.029	.503	[illegible]	.534	3
4	.114	.439	.103	[illegible]	.090	.484	.073	.524	.055+	.569	.045+	.599	4
5	[illegible]	[illegible]	[illegible]	.523	.132	.546	.110	.587	.088	.630	.075-	[illegible]	5
6	[illegible]	.565+	[illegible]	.575-	.179	.609	.152	.646	.125+	.687	.109	.713	6
7	.263	.625-	[illegible]	.643	.227	.667	.199	.701	.156	.739	.147	.764	7
8	[illegible]	.682	[illegible]	[illegible]	.279	.721	.247	.753	.212	.788	.190	.810	8
9	.375+	.737	.357	.763	.333	.773	.299	.802	.251	.834	.236	.853	9
10	.435-	[illegible]	[illegible]	[illegible]	.391	.822	.354	.848	.313	.875-	[illegible]	.891	10
11	[illegible]	[illegible]	[illegible]	[illegible]	.452	.868	.413	.890	.370	.912	.342	.925+	11
12	.551	[illegible]	.541	[illegible]	.515	.910	.475	.927	.431	.945+	.401	.955-	12
13	[illegible]	[illegible]	[illegible]	[illegible]	.553	.947	.544	.960	.497	.971	[illegible]	[illegible]	13
14	[illegible]	[illegible]	[illegible]	.971	[illegible]	.977	.617	.984	.570	.990	.537	.993	14
15	[illegible]	[illegible]	[illegible]	[illegible]	.736	.997	.695	.995	.651	.999	.619	1	15
16	[illegible]	1	[illegible]	1	.829	1	.794	1	.750	1	.718	1	16
						n = 17							
0	0	[illegible]	0	.141	0	.152	0	.195+	0	.237	0	[illegible]	0
1	[illegible]	[illegible]	[illegible]	[illegible]	.003	.250	.001	.287	.001	.332	0	[illegible]	1
2	[illegible]	.284	[illegible]	[illegible]	.021	.326	.013-	.364	.009	.410	[illegible]	[illegible]	2
3	[illegible]	[illegible]	.059	[illegible]	[illegible]	.395	.038	.434	.027	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.461	.069	.499	.052	.543	.043	.573	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.522	.103	.560	.082	.603	[illegible]	[illegible]	5
6	[illegible]	.637	[illegible]	[illegible]	[illegible]	[illegible]	.142	.617	.117	.655	.101	[illegible]	6
7	[illegible]	[illegible]	[illegible]	.713	[illegible]	[illegible]	.184	.671	.155+	.709	.137	.734	7
8	.297	[illegible]	[illegible]	[illegible]	[illegible]	.679	.230	.722	.197	.758	.175	.791	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.740	.279	.770	.242	.803	[illegible]	[illegible]	9
10	[illegible]	.754	[illegible]	.769	[illegible]	.785	.329	.815	.291	.845-	[illegible]	[illegible]	10
11	.453	[illegible]	[illegible]	[illegible]	.420	.834	.383	.858	.342	.883	.315+	[illegible]	11
12	[illegible]	[illegible]	[illegible]	.891	[illegible]	.876	.440	.897	.397	.915	[illegible]	[illegible]	12
13	.554	[illegible]	[illegible]	.923	.539	.915+	.501	.932	.457	.945	.427	.957	13
14	[illegible]	[illegible]	[illegible]	.942	[illegible]	.950	.566	.952	.520	.973	[illegible]	.979	14
15	.715	[illegible]	[illegible]	.973	.674	.979	.636	.985+	.590	.991	[illegible]	.994	15
16	.792	.946	[illegible]	.965	[illegible]	.997	.713	.999	.666	.999	.637	1	16
17	.873	1	[illegible]	1	.848	1	[illegible]	1	.763	1	.734	1	17
						n = 18							
0	0	[illegible]	0	.134	0	.157	0	.185+	0	.226	0	[illegible]	0
1	[illegible]	.149	[illegible]	.216	.003	.235	.001	.273	.001	[illegible]	0	[illegible]	1
2	[illegible]	[illegible]	[illegible]	.287	.020	[illegible]	.014	.347	.009	.392	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	.353	[illegible]	.377	.036	.414	.025+	.459	.020	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.433	.064	[illegible]	.049	.520	[illegible]	.549	4
5	.142	[illegible]	[illegible]	.476	.116	.493	.097	.535-	.077	.577	.065+	[illegible]	5
6	[illegible]	[illegible]	[illegible]	.530	[illegible]	[illegible]	.133	.590	[illegible]	.632	[illegible]	[illegible]	6
7	[illegible]	.557	[illegible]	.575-	[illegible]	[illegible]	.173	.643	.145+	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.659	[illegible]	.692	[illegible]	.729	[illegible]	[illegible]	8
9	[illegible]	[illegible]	.312	.665	.291	[illegible]	.260	.740	[illegible]	.774	[illegible]	.795+	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.309	.785-	.271	.815	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	.773	[illegible]	[illegible]	[illegible]	.827	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.302	[illegible]	12
13	[illegible]	.757	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.451	[illegible]	14
15	[illegible]	.737	[illegible]	.842	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.975-	.517	[illegible]	15
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.975	[illegible]	.992	[illegible]	[illegible]	16
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.727	.999	[illegible]	[illegible]	[illegible]	1	17
18	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	18

Table H-8d

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

d	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	d
							n = 19						
0	0	.114	0	.127	0	.144	0	.175	0	.220	0	.241	0
1	.005	.190	.004	.205+	.003	[illegible]	.001	.260	.001	.302	0	.331	1
2	.028	.257	.024	.273	.019	.296	.013	.331	.008	.374	.005	.404	2
3	.059	.319	.053	.336	.044	.359	.034	.396	.024	.439	.019	.463	3
4	.095+	.375	.086	.396	.075+	.419	.061	.456	.046	.498	.038	.527	4
5	.134	.434	.123	.452	.110	.476	.091	.512	.073	.554	.062	.582	5
6	.175+	.489	.163	.507	.147	.530	.126	.566	.103	.606	.090	.613	6
7	.218	.541	.205-	.559	.188	.582	.163	.616	.137	.655+	.121	.671	7
8	.263	.592	.248	.610	.230	.632	.203	.665+	.173	.702	.155-	.726	8
9	.310	.642	.294	.659	.274	.680	.244	.711	.212	.746	.192	[illegible]	9
10	.358	.690	.341	.706	.320	.726	.289	.756	.254	.788	.232	.808	10
11	.405	.737	.390	.752	.368	.770	.335-	.797	.298	.827	.274	.845+	11
12	.459	.782	.441	.795+	.418	.812	.384	.837	.345-	.853	.319	.879	12
13	.511	.825-	.493	.837	.470	.853	.434	.874	.394	.897	.367	.910	13
14	.566	.866	.548	.877	.524	.890	.488	.909	.446	.927	.419	.938	14
15	[illegible]	.905-	.604	.914	.581	.925-	.544	.939	.502	.954	.473	.962	15
16	.681	.941	.665	.947	.641	.956	.604	.966	.561	.976	.532	.981	16
17	.743	.972	.727	.976	.704	.981	.669	.987	.626	.992	.595	.994	17
18	.810	.994	.795+	.996	.774	.997	.740	.999	.698	.999	.669	1	18
19	[illegible]	1	.873	1	.854	1	.824	1	.780	1	.757	1	19
							n = 20						
0	0	.109	0	.121	0	.139	0	.168	0	.207	0	.233	0
1	.005+	.181	.004	.190	.003	.215	.001	.249	.001	.289	0	.317	1
2	.027	.245-	.023	.261	.018	.283	.012	.317	.008	.359	.005+	.387	2
3	.056	.304	.050	.321	.042	.344	.032	.379	.023	.421	.018	.449	3
4	.090	.361	.082	.378	.071	.401	.057	.437	.044	.478	.036	.507	4
5	.127	.415-	.117	.433	.104	.456	.087	.491	.069	.532	.052	.560	5
6	.166	.467	.154	.485-	.140	.508	.119	.543	.095	.583	.095-	.610	6
7	.207	.519	.194	.535+	.177	.558	.154	.592	.129	.631	.114	.657	7
8	.249	.567	.235-	.584	.217	.606	.191	.639	.163	.677	.146	.701	8
9	.293	.615+	.278	.632	.259	.653	.231	.685-	.200	.720	.181	.743	9
10	.339	.662	.322	.678	.302	.698	.272	.728	.239	.761	.219	.782	10
11	.385-	.707	.369	.722	.347	.741	.315+	.769	.280	.800	.257	.819	11
12	.433	.751	.415	.765+	.394	.783	.361	.809	.323	.837	.299	.854	12
13	.482	.793	.465-	.806	.442	.823	.408	.846	.369	.871	.343	.886	13
14	.533	.834	.515+	.846	.492	.860	.457	.881	.417	.902	.390	.915+	14
15	.585+	.873	.567	.883	.544	.896	.509	.913	.469	.931	.440	.942	15
16	.639	.910	.622	.918	.599	.929	.553	.943	.522	.956	.493	.964	16
17	.695	.944	.679	.950	.656	.955	.621	.968	.579	.977	.551	.982	17
18	.755+	.973	.739	.977	.717	.982	.683	.988	.642	.992	.613	.995-	18
19	.819	.995-	.804	.996	.784	.997	.751	.999	.711	.999	.683	1	19
20	.891	1	.879	1	.861	1	.832	1	.793	1	.767	1	20
							n = 21						
0	0	[illegible]	0	.115	0	.133	0	.161	0	.197	0	.223	0
1	.005+	.173	.004	.187	.002	.207	.001	.238	0	.277	0	.304	1
2	.025	.234	.022	.250	.017	.271	.012	.304	.007	.344	.005+	.372	2
3	.054	.292	.047	.307	.040	.329	.030	.363	.022	.404	.017	.432	3
4	[illegible]	.345+	.075	.362	.063	.384	.054	.419	.041	.460	.034	.488	4
5	.121	.397	.111	.415-	.099	.437	.052	.472	.065+	.512	.055+	.539	5
6	.158	.446	.146	.465-	.132	.487	.113	.522	.092	.561	[illegible]	.588	6
7	.194	.492	.184	.514	.167	.536	.146	.570	.122	[illegible]	.109	.634	7
8	[illegible]	.544	.223	.561	.206	.583	.181	.615	.155-	.653	.139	.677	8
9	.273	.590	.263	.607	.245-	[illegible]	.218	.660	.189	.695+	.171	.715	9
10	.311	[illegible]	.305+	.652	.286	.672	.257	.702	.226	.736	.205+	.754	10
11	.354	.677	[illegible]	.695-	.325	.714	.298	.743	.264	.774	[illegible]	.795-	11
12	[illegible]	[illegible]	.393	.737	.372	.755+	.340	.782	.305-	.811	.282	.829	12
13	[illegible]	.764	.439	.777	.417	.794	.384	.819	.347	.845+	.323	.862	13
14	[illegible]	[illegible]	[illegible]	.816	.464	.832	.430	.854	.392	.878	.366	.892	14
15	[illegible]	[illegible]	.531+	.854	.513	.868	.478	.887	.439	.908	.412	.920	15
16	.603	.879	[illegible]	[illegible]	.563	.901	.526	.918	[illegible]	.935-	[illegible]	.945-	16
17	.655-	.914	[illegible]	[illegible]	[illegible]	.932	[illegible]	.946	.540	[illegible]	[illegible]	[illegible]	17
18	.709	[illegible]	[illegible]	[illegible]	.671	.960	[illegible]	.970	[illegible]	.978	[illegible]	.983	18
19	[illegible]	.974	.750	[illegible]	[illegible]	.983	[illegible]	.988	.656	.993	[illegible]	.995+	19
20	.827	.995-	.813	.996	.793	.998	.762	.999	.723	1	[illegible]	1	20
21	[illegible]	1	.884	1	.897	1	.839	1	.803	1	.777	1	21

Table H-8e

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

d	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	d
n = 22													
0	0	[illegible]	0	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.001	[illegible]	0	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	.021	[illegible]	[illegible]	[illegible]	.011	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	.051	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.684	[illegible]	.719	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	.353	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.407	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.451	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.651	.971	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	1	[illegible]
22	[illegible]	1	[illegible]	1	[illegible]	1	.846	1	[illegible]	1	[illegible]	1	22
n = 23													
0	0	[illegible]	[illegible]	[illegible]	0	.122	0	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.219	0	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.011	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.350	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	1	[illegible]
23	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]
n = 24													
0	0	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	[illegible]
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
12	.353	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]	1	[illegible]	1	[illegible]	[illegible]	[illegible]	1	[illegible]	1	[illegible]

Table H-8c

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

J	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	J
n = 25													
0	0	[illegible]	0	.090	0	[illegible]	0	[illegible]	[illegible]	[illegible]	0	[illegible]	0
1	.004	[illegible]	.003	.159	[illegible]	[illegible]	.001	[illegible]	0	[illegible]	0	[illegible]	1
2	.021	[illegible]	.018	.213	.012	[illegible]	.013	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
3	.046-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	.295+	.065-	.313	[illegible]	[illegible]	[illegible]	.361	.036	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	.110	[illegible]	[illegible]	.451	[illegible]	[illegible]	[illegible]	.518	6
7	.163	.475	.163	[illegible]	.137	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	.195	[illegible]	.195-	.483	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.571	[illegible]	[illegible]	8
9	.230	.508	[illegible]	.524	.202	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	.265+	[illegible]	[illegible]	.553	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.671	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	.323	.660	[illegible]	.659	[illegible]	.697	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	.360	.677	.341	.695-	.313	[illegible]	.251	[illegible]	[illegible]	.722	13
14	.413	[illegible]	.393	.713	.379	.730	.349	.756	[illegible]	[illegible]	[illegible]	[illegible]	14
15	[illegible]	.735-	.437	.749	.417	.756	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
16	[illegible]	[illegible]	.470	.752	.456	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
17	[illegible]	[illegible]	.512	.815+	.496	[illegible]	[illegible]	.851	[illegible]	[illegible]	[illegible]	[illegible]	17
18	.576	[illegible]	[illegible]	.847	[illegible]	.861	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18
19	[illegible]	[illegible]	[illegible]	[illegible]	.590	.890	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	19
20	[illegible]	[illegible]	[illegible]	[illegible]	.625-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20
21	.705+	[illegible]	[illegible]	[illegible]	.670	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21
22	[illegible]	[illegible]	[illegible]	[illegible]	.715	[illegible]	[illegible]	.975-	[illegible]	[illegible]	[illegible]	[illegible]	22
23	.871	[illegible]	[illegible]	[illegible]	.769	.976	.760	.990	.704	[illegible]	[illegible]	[illegible]	23
24	[illegible]	[illegible]	[illegible]	.997	.924	[illegible]	[illegible]	.999	.753	1	[illegible]	1	24
25	[illegible]	1	[illegible]	1	.937	1	.863	1	[illegible]	1	[illegible]	1	25
n = 26													
0	0	[illegible]	0	[illegible]	0	.109	0	.132	[illegible]	[illegible]	[illegible]	[illegible]	0
1	[illegible]	[illegible]	[illegible]	[illegible]	.002	[illegible]	[illegible]	[illegible]	0	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	[illegible]	.205+	.014	[illegible]	.019	.251	[illegible]	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.024	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	.054	[illegible]	.044	.349	[illegible]	.375-	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.363	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	.117	[illegible]	.106	.405+	[illegible]	[illegible]	[illegible]	.473	[illegible]	[illegible]	6
7	[illegible]	[illegible]	.146	.427	.134	.447	.115	.475	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	.551	.177	[illegible]	.163	.487	.163	.513	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	[illegible]	.209	[illegible]	.196	[illegible]	[illegible]	.557	[illegible]	.592	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	.545-	.226	.564	.202	[illegible]	.177	.629	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	.259	[illegible]	.234	.631	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	.619	.292	[illegible]	.265	.666	.237	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	.345-	.655+	[illegible]	[illegible]	[illegible]	[illegible]	.269	[illegible]	[illegible]	[illegible]	13
14	[illegible]	[illegible]	[illegible]	.690	.362	[illegible]	[illegible]	.736	.302	[illegible]	[illegible]	[illegible]	14
15	[illegible]	.711	[illegible]	[illegible]	[illegible]	[illegible]	.367	[illegible]	.335	[illegible]	[illegible]	[illegible]	15
16	.471	[illegible]	.455+	[illegible]	[illegible]	.774	.405	.798	.372	[illegible]	[illegible]	[illegible]	16
17	[illegible]	.779	[illegible]	[illegible]	.474	[illegible]	[illegible]	.828	[illegible]	[illegible]	[illegible]	[illegible]	17
18	[illegible]	[illegible]	.533	[illegible]	.513	[illegible]	[illegible]	.857	[illegible]	[illegible]	[illegible]	[illegible]	18
19	[illegible]	[illegible]	.573	[illegible]	.553	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.915-	19
20	[illegible]	.574	[illegible]	[illegible]	.595-	[illegible]	[illegible]	.910	[illegible]	[illegible]	[illegible]	[illegible]	20
21	[illegible]	[illegible]	[illegible]	[illegible]	.637	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.946	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.553	[illegible]	[illegible]	[illegible]	23
24	[illegible]	.979	[illegible]	[illegible]	.777	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.771	1	[illegible]	1	25
26	.915-	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	.816	1	26
n = 27													
0	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	0	[illegible]	[illegible]	.178	0
1	[illegible]	.137	.003	[illegible]	.002	[illegible]	.001	[illegible]	[illegible]	[illegible]	0	[illegible]	1
2	[illegible]	[illegible]	.017	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	.317	[illegible]	.331	[illegible]	.351	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	.121	[illegible]	.112	.372	.101	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	.151	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	.157	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.551	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	.527	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10

Table H-8g

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

d	[illegible] L	U	[illegible] L	U	90% L	U	95% L	U	98% L	U	99% L	U	d
n = 27													
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.731	13
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	19
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.517	[illegible]	21
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	23
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	25
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	[illegible]	1	26
27	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	27
n = 28													
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	19
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	21
22	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22
23	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	23
24	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	24
25	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	25
26	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	26
27	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1	27
28	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	[illegible]	1	28
n = 29													
0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	0
1	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1
2	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2
3	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3
4	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	4
5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5
6	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	6
7	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	7
8	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	8
9	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	9
10	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	10
11	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	11
12	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	12
13	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13
14	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	14
15	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	15
16	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	16
17	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	17
18	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	18
19	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	19
20	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	20

Table H-8h

CONFIDENCE LIMITS FOR A PROPORTION (TWO-SIDED)

d	80% L	80% U	85% L	85% U	90% L	90% U	95% L	95% U	98% L	98% U	99% L	99% U	d
n = 29													
21	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	.483	[illegible]	[illegible]	[illegible]	21
22	[illegible]	[illegible]	.613	[illegible]	.594	[illegible]	.555-	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	22
23	.665+	[illegible]	.652	[illegible]	.632	[illegible]	[illegible]	[illegible]	[illegible]	.935-	.545-	[illegible]	23
24	.701	.916	[illegible]	.921	.671	.930	.642	[illegible]	.605	.964	[illegible]	[illegible]	24
25	.743	.939	.730	.944	.712	.952	[illegible]	[illegible]	.650	[illegible]	[illegible]	[illegible]	25
26	[illegible]	.961	.771	[illegible]	.754	.971	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	26
27	[illegible]	[illegible]	.815-	.984	.798	.988	.772	.992	.740	[illegible]	[illegible]	[illegible]	27
28	[illegible]	[illegible]	.861	.997	.847	.998	.822	[illegible]	[illegible]	1	[illegible]	1	28
29	[illegible]	1	.915-	1	[illegible]	1	.881	1	[illegible]	1	[illegible]	1	29
n = 30													
0	[illegible]	.074	0	.083	0	.095+	0	.116	0	[illegible]	[illegible]	[illegible]	0
1	[illegible]	.124	.003	[illegible]	.002	.149	.001	.172	0	[illegible]	[illegible]	[illegible]	1
2	[illegible]	.158	.015+	.160	[illegible]	.195+	.008	.221	.005+	.252	[illegible]	[illegible]	2
3	.037	.209	.033	[illegible]	.029	.239	.021	.265+	.015-	[illegible]	[illegible]	[illegible]	3
4	.059	.249	.054	.262	.047	.280	.038	.307	.028	.340	[illegible]	[illegible]	4
5	.083	.287	.076	.301	.068	.319	.056	.347	.045-	.381	[illegible]	[illegible]	5
6	.109	[illegible]	.101	.339	.091	.357	.077	.386	.063	.420	[illegible]	[illegible]	6
7	.135-	.361	.126	.375+	.115-	.394	.099	.423	.083	.457	[illegible]	[illegible]	7
8	[illegible]	.397	.152	.411	.140	.430	[illegible]	.459	.104	.493	[illegible]	[illegible]	8
9	.190	.432	.179	.446	.166	.465+	.147	.494	.127	.527	[illegible]	[illegible]	9
10	[illegible]	.465	.207	.481	.193	.499	.173	.528	.151	.561	[illegible]	[illegible]	10
11	.248	.500	.236	.514	.221	.533	.199	.561	.175	.594	[illegible]	[illegible]	11
12	[illegible]	.533	.265+	.548	.250	.566	.227	.594	.201	.626	[illegible]	[illegible]	12
13	[illegible]	.566	.295-	.580	.279	.598	.255-	.626	.228	.657	[illegible]	[illegible]	13
14	[illegible]	.599	.325+	.612	.308	.630	.283	.657	.256	[illegible]	[illegible]	[illegible]	14
15	[illegible]	.630	.356	.644	.339	.661	.313	[illegible]	.284	.716	[illegible]	[illegible]	15
16	.401	.662	[illegible]	.675-	.370	.692	.343	.717	.313	[illegible]	[illegible]	[illegible]	16
17	.434	.692	.420	.705+	.402	.721	.374	.745+	.343	.772	[illegible]	[illegible]	17
18	.467	.723	.452	.735-	.434	.750	.406	.773	.374	.799	[illegible]	[illegible]	18
19	.500	.752	.486	.764	.467	.779	.439	.801	[illegible]	.824	[illegible]	[illegible]	19
20	[illegible]	[illegible]	.519	.793	.501	.807	.472	.827	.439	[illegible]	.417	[illegible]	20
21	[illegible]	.810	.554	.821	.535-	.834	.506	.853	.473	.873	[illegible]	[illegible]	21
22	[illegible]	.838	.589	.848	.570	.860	.541	[illegible]	.507	[illegible]	[illegible]	[illegible]	22
23	.639	.865+	.625-	.874	.606	.885+	.577	.901	.543	.917	[illegible]	[illegible]	23
24	.675+	.891	[illegible]	.897	.643	.909	.614	[illegible]	.580	.937	[illegible]	[illegible]	24
25	.713	.917	.699	.924	.681	.932	.653	.944	.619	.955+	[illegible]	[illegible]	25
26	[illegible]	.941	.738	.945	.720	.953	.693	.962	.660	[illegible]	.637	[illegible]	26
27	.791	.963	[illegible]	.967	.751	.972	.735-	.979	.702	.985+	[illegible]	[illegible]	27
28	[illegible]	[illegible]	.820	.985-	.805-	.988	.779	.992	.749	.995-	[illegible]	[illegible]	28
29	.876	[illegible]	.865	.997	.851	.998	.828	.999	.793	1	[illegible]	1	29
30	.925	1	.917	1	.905-	1	.884	1	.856	1	[illegible]	1	30

Prepared by: J. C. Pierce, Captain, USAF, Chanute Technical Training Center, Chanute AFB, Ill.

METHOD OF COMPUTATION

For a $1-\alpha$ confidence interval (two-sided) the lower limit (L) is the solution (for $\underline{p}$) of

$$\sum_{i=d}^{n} \binom{n}{i} (\underline{p})^i (1-\underline{p})^{n-i} = \alpha/2$$

The upper limit (U) is the solution (for $\bar{p}$) of

$$\sum_{i=0}^{d} \binom{n}{i} (\bar{p})^i (1-\bar{p})^{n-i} = \alpha/2$$

Values were computed to ±0, ±1 in the fifth place and then rounded to three places.

Values for $\underline{p}$ at d=0 and $\bar{p}$ at d=n were set at 0 and 1 respectively.

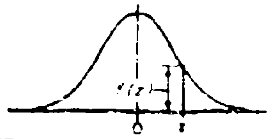


Table H-9
Ordinates f(z) of the
Standard Normal Curve at z

z	0	1	2	3	4	5	6	7	8	9
0.0	.3989	.3989	.3989	.3988	.3986	.3984	.3982	.3980	.3977	.3973
0.1	.3970	.3965	.3961	.3956	.3951	.3945	.3939	.3932	.3925	.3918
0.2	.3910	.3902	.3894	.3885	.3876	.3867	.3857	.3847	.3836	.3825
0.3	.3814	.3802	.3790	.3778	.3765	.3752	.3739	.3725	.3712	.3697
0.4	.3683	.3668	.3653	.3637	.3621	.3605	.3589	.3572	.3555	.3538
0.5	.3521	.3503	.3485	.3467	.3448	.3429	.3410	.3391	.3372	.3352
0.6	.3332	.3312	.3292	.3271	.3251	.3230	.3209	.3187	.3166	.3144
0.7	.3123	.3101	.3079	.3056	.3034	.3011	.2989	.2966	.2943	.2920
0.8	.2897	.2874	.2850	.2827	.2803	.2780	.2756	.2732	.2709	.2685
0.9	.2661	.2637	.2613	.2589	.2565	.2541	.2516	.2492	.2468	.2444
1.0	.2420	.2396	.2371	.2347	.2323	.2299	.2275	.2251	.2227	.2203
1.1	.2179	.2155	.2131	.2107	.2083	.2059	.2036	.2012	.1989	.1965
1.2	.1942	.1919	.1895	.1872	.1849	.1826	.1804	.1781	.1758	.1736
1.3	.1714	.1691	.1669	.1647	.1626	.1604	.1582	.1561	.1539	.1518
1.4	.1497	.1476	.1456	.1435	.1415	.1394	.1374	.1354	.1334	.1315
1.5	.1295	.1276	.1257	.1238	.1219	.1200	.1182	.1163	.1145	.1127
1.6	.1109	.1092	.1074	.1057	.1040	.1023	.1006	.0989	.0973	.0957
1.7	.0940	.0925	.0909	.0893	.0878	.0863	.0848	.0833	.0818	.0804
1.8	.0790	.0775	.0761	.0748	.0734	.0721	.0707	.0694	.0681	.0669
1.9	.0656	.0644	.0632	.0620	.0608	.0596	.0584	.0573	.0562	.0551
2.0	.0540	.0529	.0519	.0508	.0498	.0488	.0478	.0468	.0459	.0449
2.1	.0440	.0431	.0422	.0413	.0404	.0396	.0387	.0379	.0371	.0363
2.2	.0355	.0347	.0339	.0332	.0325	.0317	.0310	.0303	.0297	.0290
2.3	.0283	.0277	.0270	.0264	.0258	.0252	.0246	.0241	.0235	.0229
2.4	.0224	.0219	.0213	.0208	.0203	.0198	.0194	.0189	.0184	.0180
2.5	.0175	.0171	.0167	.0163	.0158	.0154	.0151	.0147	.0143	.0139
2.6	.0136	.0132	.0129	.0126	.0122	.0119	.0116	.0113	.0110	.0107
2.7	.0104	.0101	.0099	.0096	.0093	.0091	.0088	.0086	.0084	.0081
2.8	.0079	.0077	.0075	.0073	.0071	.0069	.0067	.0065	.0063	.0061
2.9	.0060	.0058	.0056	.0055	.0053	.0051	.0050	.0048	.0047	.0046
3.0	.0044	.0043	.0042	.0040	.0039	.0038	.0037	.0036	.0035	.0034
3.1	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026	.0025	.0025
3.2	.0024	.0023	.0022	.0022	.0021	.0020	.0020	.0019	.0018	.0018
3.3	.0017	.0017	.0016	.0016	.0015	.0015	.0014	.0014	.0013	.0013
3.4	.0012	.0012	.0012	.0011	.0011	.0010	.0010	.0010	.0009	.0009
3.5	.0009	.0008	.0008	.0008	.0008	.0007	.0007	.0007	.0007	.0006
3.6	.0006	.0006	.0006	.0005	.0005	.0005	.0005	.0005	.0005	.0004
3.7	.0004	.0004	.0004	.0004	.0004	.0004	.0003	.0003	.0003	.0003
3.8	.0003	.0003	.0003	.0003	.0003	.0002	.0002	.0002	.0002	.0002
3.9	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0001	.0001

Table H-10
GAMMA FUNCTION VALUES

n	Γ(n)	n	Γ(n)	n	Γ(n)	n	Γ(n)
1.00	1.000 00	1.25	.906 40	1.50	.886 23	1.75	.919 06
1.01	.994 33	1.26	.904 40	1.51	.886 59	1.76	.921 37
1.02	.988 84	1.27	.902 50	1.52	.887 04	1.77	.923 76
1.03	.983 55	1.28	.900 72	1.53	.887 57	1.78	.926 23
1.04	.978 44	1.29	.899 04	1.54	.888 18	1.79	.928 77
1.05	.973 50	1.30	.897 47	1.55	.888 87	1.80	.931 38
1.06	.968 74	1.31	.896 00	1.56	.889 64	1.81	.934 08
1.07	.964 15	1.32	.894 64	1.57	.890 49	1.82	.936 85
1.08	.959 73	1.33	.893 38	1.58	.891 42	1.83	.939 69
1.09	.955 46	1.34	.892 22	1.59	.892 43	1.84	.942 61
1.10	.951 35	1.35	.891 15	1.60	.893 52	1.85	.945 61
1.11	.947 40	1.36	.890 18	1.61	.894 68	1.86	.948 69
1.12	.943 59	1.37	.889 31	1.62	.895 92	1.87	.951 84
1.13	.939 93	1.38	.888 54	1.63	.897 24	1.88	.955 07
1.14	.936 42	1.39	.887 85	1.64	.898 64	1.89	.958 38
1.15	.933 04	1.40	.887 26	1.65	.900 12	1.90	.961 77
1.16	.929 80	1.41	.886 76	1.66	.901 67	1.91	.965 23
1.17	.926 70	1.42	.886 36	1.67	.903 30	1.92	.968 77
1.18	.923 73	1.43	.886 04	1.68	.905 00	1.93	.972 40
1.19	.920 89	1.44	.885 81	1.69	.906 78	1.94	.976 10
1.20	.918 17	1.45	.885 66	1.70	.908 64	1.95	.979 88
1.21	.915 58	1.46	.885 60	1.71	.910 57	1.96	.983 74
1.22	.913 11	1.47	.885 63	1.72	.912 58	1.97	.987 68
1.23	.910 75	1.48	.885 75	1.73	.914 67	1.98	.991 71
1.24	.908 52	1.49	.885 95	1.74	.916 83	1.99	.995 81
						2.00	1.000 00

Table H-11a

NATURAL LOGARITHMS OF NUMBERS—0.00 to 5.99

(Base e = 2.718 . . .)

Take tabular value −10 (0.0 to 0.9)

N	0	1	2	3	4	5	6	7	8	9
0.0		5.395	6.088	6.493	6.781	7.004	7.187	7.341	7.474	7.592
0.1	7.697	7.793	7.880	7.960	8.034	8.103	8.167	8.228	8.285	8.339
0.2	8.391	8.439	8.486	8.530	8.573	8.614	8.653	8.691	8.727	8.762
0.3	8.796	8.829	8.861	8.891	8.921	8.950	8.978	9.006	9.032	9.058
0.4	9.084	9.108	9.132	9.156	9.179	9.201	9.223	9.245	9.266	9.287
0.5	9.307	9.327	9.346	9.365	9.384	9.402	9.420	9.438	9.455	9.472
0.6	9.489	9.506	9.522	9.538	9.554	9.569	9.584	9.600	9.614	9.629
0.7	9.643	9.658	9.671	9.685	9.699	9.712	9.726	9.739	9.752	9.764
0.8	9.777	9.789	9.802	9.814	9.826	9.837	9.849	9.861	9.872	9.883
0.9	9.895	9.906	9.917	9.927	9.938	9.949	9.959	9.970	9.980	9.990
1.0	0.0 0000	0995	1980	2956	3922	4879	5827	6766	7696	8618
1.1	9531	*0436	*1333	*2222	*3103	*3976	*4842	*5700	*6551	*7395
1.2	0.1 8232	9062	9885	*0701	*1511	*2314	*3111	*3902	*4686	*5464
1.3	0.2 6236	7003	7763	8518	9267	*0010	*0748	*1481	*2208	*2930
1.4	0.3 3647	4359	5066	5767	6464	7156	7844	8526	9204	9878
1.5	0.4 0547	1211	1871	2527	3178	3825	4469	5108	5742	6373
1.6	7000	7623	8243	8858	9470	*0078	*0682	*1282	*1879	*2473
1.7	0.5 3063	3649	4232	4812	5389	5962	6531	7098	7661	8222
1.8	8779	9333	9884	*0432	*0977	*1519	*2058	*2594	*3127	*3658
1.9	0.6 4185	4710	5233	5752	6269	6783	7294	7803	8310	8813
2.0	9315	9813	*0310	*0804	*1295	*1784	*2271	*2755	*3237	*3716
2.1	0.7 4194	4669	5142	5612	6081	6547	7011	7473	7932	8390
2.2	8846	9299	9751	*0200	*0648	*1093	*1536	*1978	*2418	*2855
2.3	0.8 3291	3725	4157	4587	5015	5442	5866	6289	6710	7129
2.4	7547	7963	8377	8789	9200	9609	*0016	*0422	*0826	*1228
2.5	0.9 1629	2028	2426	2822	3216	3609	4001	4391	4779	5166
2.6	5551	5935	6317	6698	7078	7456	7833	8208	8582	8954
2.7	9325	9695	*0063	*0430	*0796	*1160	*1523	*1885	*2245	*2604
2.8	1.0 2962	3318	3674	4028	4380	4732	5082	5431	5779	6126
2.9	6471	6815	7158	7500	7841	8181	8519	8856	9192	9527
3.0	9861	*0194	*0526	*0856	*1186	*1514	*1841	*2168	*2493	*2817
3.1	1.1 3140	3462	3783	4103	4422	4740	5057	5373	5688	6002
3.2	6315	6627	6938	7248	7557	7865	8173	8479	8784	9089
3.3	9392	9695	9996	*0297	*0597	*0896	*1194	*1491	*1788	*2083
3.4	1.2 2378	2671	2964	3255	3547	3837	4127	4415	4703	4990
3.5	5276	5562	5846	6130	6413	6695	6976	7257	7536	7815
3.6	8093	8371	8647	8923	9198	9473	9746	*0019	*0291	*0563
3.7	1.3 0833	1103	1372	1641	1909	2176	2442	2708	2972	3237
3.8	3500	3763	4025	4286	4547	4807	5067	5325	5584	5841
3.9	6098	6354	6609	6864	7118	7372	7624	7877	8128	8379
4.0	8629	8879	9128	9377	9624	9872	*0118	*0364	*0610	*0854
4.1	1.4 1099	1342	1585	1828	2070	2311	2552	2792	3031	3270
4.2	3508	3746	3984	4220	4456	4692	4927	5161	5395	5629
4.3	5862	6094	6326	6557	6787	7018	7247	7476	7705	7933
4.4	8160	8387	8614	8840	9065	9290	9515	9739	9962	*0185
4.5	1.5 0408	0630	0851	1072	1293	1513	1732	1951	2170	2388
4.6	2606	2823	3039	3256	3471	3687	3902	4116	4330	4543
4.7	4756	4969	5181	5393	5604	5814	6025	6235	6444	6653
4.8	6862	7070	7277	7485	7691	7898	8104	8309	8515	8719
4.9	8924	9127	9331	9534	9737	9939	*0141	*0342	*0543	*0744
5.0	1.6 0944	1144	1343	1542	1741	1939	2137	2334	2531	2728
5.1	2924	3120	3315	3511	3705	3900	4094	4287	4481	4673
5.2	4866	5058	5250	5441	5632	5823	6013	6203	6393	6582
5.3	6771	6959	7147	7335	7523	7710	7896	8083	8269	8455
5.4	8640	8825	9010	9194	9378	9562	9745	9928	*0111	*0293
5.5	1.7 0475	0656	0838	1019	1199	1380	1560	1740	1919	2098
5.6	2277	2455	2633	2811	2988	3165	3342	3519	3695	3871
5.7	4047	4222	4397	4572	4746	4920	5094	5267	5440	5613
5.8	5786	5958	6130	6302	6473	6644	6815	6985	7156	7326
5.9	7495	7665	7834	8002	8171	8339	8507	8675	8842	9009
N	0	1	2	3	4	5	6	7	8	9

$\log_e 0.10 = 7.69741\ 49070 - 10$

Table H-11b

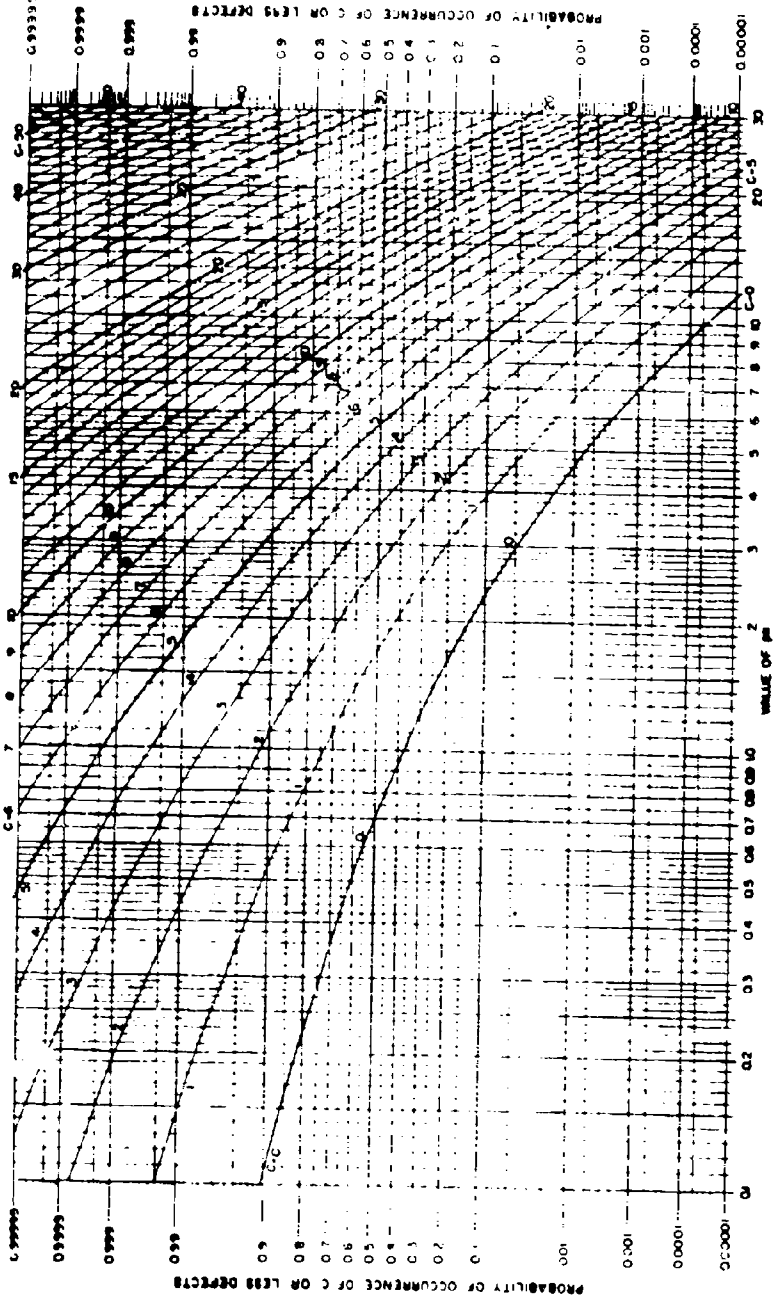
NATURAL LOGARIGHMS OF NUMBERS - 6.00 to 10.09

N	0	1	2	3	4	5	6	7	8	9
6.0	1.7 9176	9342	9509	9675	9840	*0006	*0171	*0335	*0500	*0665
6.1	1.8 0829	0993	1156	1319	1482	1645	1808	1970	2132	2294
6.2	2455	2616	2777	2938	3098	3258	3418	3578	3737	3896
6.3	4055	4214	4372	4530	4688	4845	5003	5160	5317	5473
6.4	5630	5786	5942	6097	6253	6408	6563	6718	6872	7026
6.5	7180	7334	7487	7641	7794	7947	8099	8251	8403	8555
6.6	8707	8858	9010	9160	9311	9462	9612	9762	9912	*0061
6.7	1.9 0211	0360	0509	0658	0806	0954	1102	1250	1398	1545
6.8	1692	1839	1986	2132	2279	2425	2571	2716	2862	3007
6.9	3152	3297	3442	3586	3730	3874	4018	4162	4305	4448
7.0	4591	4734	4876	5019	5161	5303	5445	5586	5727	5869
7.1	6009	6150	6291	6431	6571	6711	6851	6991	7130	7269
7.2	7408	7547	7685	7824	7962	8100	8238	8376	8513	8650
7.3	8787	8924	9061	9198	9334	9470	9606	9742	9877	*0013
7.4	2.0 0148	0283	0418	0553	0687	0821	0956	1089	1223	1357
7.5	1490	1624	1757	1890	2022	2155	2287	2419	2551	2683
7.6	2815	2946	3078	3209	3340	3471	3601	3732	3862	3992
7.7	4122	4252	4381	4511	4640	4769	4898	5027	5156	5284
7.8	5412	5540	5668	5796	5924	6051	6179	6306	6433	6560
7.9	6686	6813	6939	7065	7191	7317	7443	7568	7694	7819
8.0	7944	8069	8194	8318	8443	8567	8691	8815	8939	9063
8.1	9186	9310	9433	9556	9679	9802	9924	*0047	*0169	*0291
8.2	2.1 0413	0535	0657	0779	0900	1021	1142	1263	1384	1505
8.3	1626	1746	1866	1986	2106	2226	2346	2465	2585	2704
8.4	2823	2942	3061	3180	3298	3417	3535	3653	3771	3889
8.5	4007	4124	4242	4359	4476	4593	4710	4827	4943	5060
8.6	5176	5292	5409	5524	5640	5756	5871	5987	6102	6217
8.7	6332	6447	6562	6677	6791	6905	7020	7134	7248	7361
8.8	7475	7589	7702	7816	7929	8042	8155	8267	8380	8493
8.9	8605	8717	8830	8942	9054	9165	9277	9389	9500	9611
9.0	9722	9834	9944	*0055	*0166	*0276	*0387	*0497	*0607	*0717
9.1	2.2 0827	0937	1047	1157	1266	1375	1485	1594	1703	1812
9.2	1920	2029	2138	2246	2354	2462	2570	2678	2786	2894
9.3	3001	3109	3216	3324	3431	3538	3645	3751	3858	3965
9.4	4071	4177	4284	4390	4496	4601	4707	4813	4918	5024
9.5	5129	5234	5339	5444	5549	5654	5759	5863	5968	6072
9.6	6176	6280	6384	6488	6592	6696	6799	6903	7006	7109
9.7	7213	7316	7419	7521	7624	7727	7829	7932	8034	8136
9.8	8238	8340	8442	8544	8646	8747	8849	8950	9051	9152
9.9	9253	9354	9455	9556	9657	9757	9858	9958	*0058	*0158
10.0	2.3 0259	0358	0458	0558	0658	0757	0857	0956	1055	1154
N	0	1	2	3	4	5	6	7	8	9

Table H-11c

NATURAL LOGARIGHMS OF NUMBERS - 10 to 99

N	0	1	2	3	4	5	6	7	8	9
1	2.30259	39790	48491	56495	63906	70805	77259	83321	89037	94444
2	99573	*04452	*09104	*13549	*17805	*21888	*25810	*29584	*33220	*36730
3	3.40120	43399	46574	49651	52636	55535	58352	61092	63759	66356
4	68888	71357	73767	76120	78419	80666	82864	85015	87120	89182
5	91202	93183	95124	97029	98898	*00733	*02535	*04305	*06044	*07754
6	4.09434	11087	12713	14313	15888	17439	18965	20469	21951	23411
7	24850	26268	27667	29046	30407	31749	33073	34381	35671	36945
8	38203	39445	40672	41884	43082	44265	45435	45591	47734	48864
9	49981	51086	52179	53260	54329	55388	56435	57471	58497	59512



Cumulative probability curves of the Poisson exponential distribution (a modification of a chart given by Miss F. Thorndike in *The Bell System Technical Journal*, October 1926). These curves may be used for determining the probability of occurrence of *c* or less defects in a sample of *n* pieces selected from an infinite universe in which the fraction defective is *p*. They may also be used as an approximation under certain conditions [Equations (2.1b) and (2.4a)] for determining the probability of occurrence of *c* or less *defectives* for a given *p* and *n*. Further, they serve as a generalized set of OC curves for single sampling plans, when the Poisson distribution is applicable.

Figure H-12

Cumulative Probability Curves for the Poisson Distribution

APPENDIX I

DEFINITION OF ACRONYMS AND SYMBOLS

I-1. General. The more frequently used symbols and acronyms as contained in this handbook are defined below. Some are standard definitions and others may be uniquely used herein. In either case, the definitions apply to usage in this handbook.

I-2. Acronyms.

Ac	Acceptance number
AFDP	Air Force Development Plan
AFSCM	Air Force Systems Command Manual
AFSC-TR	Air Force Systems Command-Technical Report
AGREE	Advisory Group on Reliability for Electronic Equipment
AIAA	American Institute of Aeronautics and Astronautics
AIchE	American Institute of Chemical Engineers
AMC	Army Materiel Command
AMCP	Army Materiel Command Pamphlet
AMCR	Army Materiel Command Regulation
APE	Advance Production Engineering
AQL	Acceptable Quality Level

AR	Army Regulation
ASME	American Society of Mechanical Engineers
ASP	Army Strategic Plan
ASQC	American Society for Quality Control
ASTM	American Society for Testing Materials
BASE	Basic Army Strategic Estimate
CDC	Combat Developments Command
CDP	Contract Definition Phase
CONUS	Continental United States
CPFF	Cost-Plus-Fixed-Fee
CPM	Critical Path Method
CRD	Chief of Research and Development
CTP	Coordinated Test Plan
DA	Department of the Army
DCR	Design Characteristics Review
DoD	Department of Defense
ECP	Engineering Change Proposal
ECR	Engineering Concept Review
EIA	Electronics Industries Association
ET	Engineering Test
ET/ST	Engineering Test/Service Test
FACS	Failure Analysis and Control System
FARADA	Failure Rate Data

FMEA	Failure Mode and Effects Analysis
FMSAEG	Fleet Missile Systems Analysis and Evaluation Group
GFE	Government Furnished Equipment
ICAO	International Civil Aviation Organization
ICT	Inspection Comparison Test
IDEP	Inter-service Data Exchange Program
IEEE	Institute of Electrical and Electronic Engineers, Inc.
IFB	Invitation for Bid
IPR	In-Process Review
IPT	Initial Production Test
MDT	Mean Down Time
MIL-SPEC	Military Specification
MIL-STD	Military Standard
MOS	Military Occupational Specialty
MRBF	Mean Rounds Between Failure
MTBF	Mean Time Between Failures
MTF	Mean Time to Failure
MWO	Modification Work Order
OASD	Office of the Assistant Secretary of Defense
OC	Operating Characteristic
OSD	Office of the Secretary of Defense

PCP	Program Change Proposal
PERT	Program Evaluation and Review Technique
PM^2P	Project Manager's Master Plan
PSI	Pounds per Square Inch
PSR	Prototype System Review
QMA	Qualitative Materiel Approach
QMDO	Qualitative Materiel Development Objective
QMR	Qualitative Materiel Requirements
RADC	Rome Air Development Center
R&D	Research and Development
RDTE	Research, Development, Test and Evaluation
Re	Rejection number
RFP	Request for Proposal
SAE	Society of Automotive Engineers
SDR	Small Development Requirements
SNT	Society for Nondestructive Testing
SPR	Special Review
ST	Service Test
STR	Service Test Review
TAERS	The Army Equipment Record System
TCR	Technical Characteristic Review
TDP	Technical Development Plan

TOE	Table of Organization and Equipment
USAMC	United States Army Materiel Command
USATECOM	United States Army Test and Evaluation Command
WSEIAC	Weapon System Effectiveness Industry Advisory Committee

I-3. Symbols.

α	Shape parameter for the gamma distribution
α	Level of significance
a	Estimated intercept of the linear regression line
A	Intercept of the linear regression line
$\overline{A}$	Complement of the set A
β	Shape parameter of the Weibull distribution
$\hat{\beta}$	Estimate of the parameter β
b	Estimated slope of the linear regression line
B	Slope of the linear regression line
χ^2_ν	Chi squared random variable with ν degrees of freedom
$\chi^2_{\alpha,\nu}$	Chi squared value such that $P\left(\chi^2_\nu > \chi^2_{\alpha,\nu}\right) = \alpha$
d	Number of items surviving a time terminated test
d	Maximum deviation between hypothesized and sample distribution functions as used for the Kolmogorov-Smirnov test.

d_α	Critical value for the Kolmogorov-Smirnov test
d_k	Decision alternative at stage k (Dynamic Programming)
D_k	Set of all possible decision alternatives at stage k for the dynamic programming formulation
η	Scale parameter for the Weibull Distribution
$\hat{\eta}$	Estimate of the parameter η
e	Base of natural logarithms (2.71828...)
ϵ	Element of
exp(-x)	e^{-x}
E_i	Expected number of observations falling in the i^{th} interval
E(X)	Expected value of the random variable X
$E_\theta(r)$	Expected number of failures required to reach a decision for a sequential test
$E_\theta(t)$	Expected time duration of a sequential test
f(s)	Probability density function for a stress random variable
f(S)	Probability density function for a strength random variable
f(x)	Probability density function for the random variable X
f(z)	Probability density function for the standard normal random variable
F(x)	Probability distribution function for the random variable X
$F(x_i)$	Probability distribution function evaluated at x_i
$\hat{F}(x_i)$	Sample estimate of $F(x_i)$

F_{ν_x, ν_y}	F random variable with ν_x and ν_y degrees of freedom
F_{α, ν_x, ν_y}	F value such that $P\left(F_{\nu_x, \nu_y} > F_{\alpha, \nu_x, \nu_y}\right) = \alpha$
$\Gamma(b)$	Gamma function evaluated at b
γ	Location parameter for the Weibull Distribution
G	$\int_s^{\infty} f(S)dS$
$G(x_i, y_i)$	Amount of effort required to increase the reliability level of subsystem i from x_i to y_i
h_o	Intercept of the acceptance line for a sequential test
h_1	Intercept of the rejection line for a sequential test
H	$\int_{-\infty}^{s} f(s)ds$
H_o	Null hypothesis
H_1	Alternative hypothesis
h(x)	Hazard function or instantaneous failure rate
∞	Infinite
$\int$	Integration operator
k	Number of failures as counted throughout a sequential life test
k	Number of intervals in a frequency table
k	A typical stage number as used in the dynamic programming formulation
K	Discrete random variable (e.g., number of failure)

λ	Scale parameter of the gamma distribution
λ	Constant failure rate
λ^*	Required system failure rate
λ_i^*	Failure rate allocated to subsystem i
L	Lower specification limit (MIL-STD-414)
L_i	Lower limit of the i^{th} interval
ln(x)	Natural logarithm of x
μ	Population mean
$\mu_{y.x}$	Expected response associated with x as included in a linear regression model
M	Allowable percent defective (MIL-STD-414)
ν	Number of degrees of freedom
N	Total number of modules in the system
n	Sample size
n_i	Number of modules in the i^{th} subsystem
n!	$n(n-1)(n-2) \ldots (1) = \Gamma(n+1)$
O_i	Number of sample observations falling in the i^{th} interval
ϕ	Empty set
Π	Multiplication operator

$\prod_{i=1}^{n} x_i$	$x_1 x_2 x_3 \cdots x_n$
p	Probability of failure on a single trial (a parameter of the binomial distribution)
p	Probability that a single item will successfully fulfill a given mission
$\hat{p}$	Estimate of the parameter p
p_i	Probability that the i^{th} element succeeds
p_L	Estimated lot percent defective with respect to the lower specification limit (MIL-STD-414)
p_w	Probability that the switch will operate when it should operate
p'_w	Probability that the switch will not operate prematurely
P_a	Probability of acceptance
$P(A)$	Probability that the event A will occur
$P(B/A)$	Probability that the event B will occur if it is known that event A will occur
q_i	Probability that the i^{th} element will fail
q_w	Probability that the switch will fail to operate when it should operate
q'_w	Probability that the switch will operate prematurely
Q	System unreliability
Q_i	Unreliability of the i^{th} subsystem

Q_L	Quality index pertaining to the lower specification limit (MIL-STD-414)
r	Number of failures occurring during a life test
r_o	Rejection number
R	Reliability
R^*	System reliability requirement
R_i^*	The reliability requirement apportioned to subsystem i
R_i	Reliability level of subsystem i
$\overline{R}_i$	Unreliability of subsystem i
$R(x)$	Reliability function for the random variable X
$\hat{R}(x)$	Estimated reliability function
$R_s(x)$	System reliability function
R_s	System reliability
$R(T)$	Reliability for a mission of T units duration
$\hat{R}(T)$	Estimate of R(T)
$R_k(s_k, d_k)$	Return function pertaining to stage k in the dynamic programming formulation
ρ/ρ_o	Unit of air density
$\sum$	Summation operator

$\sum_{i=1}^{n} x_i$	$x_1 + x_2 + x_3 + \ldots + x_n$
σ^2	Variance
σ	Standard deviation
s	Stress random variable
s	Slope of the sequential test decision lines
s	Estimate of the parameter σ
s^2	Estimate of the parameter σ^2
s_k	State at stage k (Dynamic Programming)
$s_{y.x}$	Standard error of estimate for the regression model
S	Strength random variable
S_k	Set of all possible states at stage k in the dynamic programming formulation
θ	Mean time between failures
$\hat{\theta}$	Estimate of the parameter θ
θ_i	Minimum acceptable mean life of the i^{th} subsystem
θ_o	Acceptable mean life
θ_1	Unacceptable mean life
t	System mission time
t	Length of time a sequential life test has been operative
t_i	Mission time of the i^{th} subsystem

t_ν	t random variable with ν degrees of freedom
$t_{\alpha,\nu}$	A value of t such that $P\left(t_\nu > t_{\alpha,\nu}\right) = \alpha$
T	Test termination time
$T_k\left(s_k, d_k\right)$	Transformation at stage k for the dynamic programming formulation
U_i	Upper limit of the $i^{\underline{th}}$ interval
V(t)	Cumulative item-hours of operation at time t for the sequential test
V(X)	Variance of the random variable X
w	Number of parameters estimated when using the chi squared goodness-of-fit test
w_i	weighting factor for subsystem i
w_i	Importance factor for subsystem i
x_i	Present reliability level of subsystem i
x_t	Total number of operative item-hours accumulated during a life test
$\bar{x}$	Sample mean
X	Random variable such as time to failure
y_i	Reliability level apportioned to subsystem i
y_i^*	Optimal reliability goal apportioned to subsystem i
$\bar{y}$	System reliability requirement or goal
$\hat{y}$	Estimate of the regression parameter $\mu_{y.x}$

Z	Standard normal random variable
z_α	z value such that $P(Z > z_\alpha) = \alpha$
$100(1-\alpha)\%$	Confidence level expressed as a percentage
$\lim_{h \to 0}$	Limit as h approaches zero
$\binom{n}{k}$	Combination of n things taken k at a time $\left(\frac{n!}{k!(n-k)!}\right)$
$<$	Less than
$\leq$	Less than or equal to
$>$	Greater than
$\geq$	Greater than or equal to
$\subset$	Subset
$\cup$	Union
$\cap$	Intersection

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